LINEAR PROGRAMMING AND ITS APPLICATION TO THE LOCATIONAL PLANNING OF WOOD-PROCESSING INDUSTRIES

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ABSTRACT

Linear programming techniques can be used by industry to optimise the allocation of resources to processing units and to rationalise the distribution of production from these processing units to markets. Since transportation costs are a significant variable cost factor in wood-processing industries, linear programming methods are a valuable tool for assisting in the planning of the location and functional integration of individual plants. Although there is little published evidence of these procedures being adopted in New Zealand, it is apparent from this study that such objective approaches could be usefully applied to many of the location problems of major forest owners and wood-processors.

INTRODUCTION

Transportation costs are a significant variable cost factor in wood-processing industries, often constituting over twenty per cent of total costs (Abel, 1971). Functionally integrated wood-processors are located either in a single complex adjacent to a vast forest resource, or separated, by a scattered resource base, into a number of individual plants that contribute to centrally located processing units. For the large wood-processing enterprise the choice of location for the central processing units, and the most economic allocation of resources from each contributing plant, presents itself as a fundamental problem. Similarly, the most economic distribution of production from each of the processing units to the final consumer poses a further complication.

In this paper such a problem of locational planning is taken from the viewpoint of a large wood-processing firm that requires, in the first instance, an objective economic assessment of the merits for a relocation of a single processing unit. A device particularly suitable for solving this type of location problem is the simplex transportation method of linear programming. The concepts of linear programming and their broad applications to forestry and forest industry locational planning are reviewed first, as an introduction to the case study.

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LINEAR PROGRAMMING AND LOCATIONAL PLANNING IN FORESTRY AND FOREST INDUSTRIES

In the past decade a number of studies have used linear programming techniques to optimise commodity movements between the establishments of wood-processing enterprises, and on a much larger scale, the inter-regional movements of forest products (Barciok, Köhler and Rauschenbach, 1965; Barr, 1970; Corcoran, 1962; Glotov, 1967; Holley, 1968, 1970; Holland and Judge, 1962; Klocek, 1966; Muszynski, 1966). Apart from the work of Glotov (1967), each of these studies has assumed transportation costs as the principal determinant of product transfer and the location of wood-processing plants. Under this assumption the simplex transportation method of linear programming can be used to minimise transportation costs between specified origins and destinations. Following the work of Holland and Judge (1962) and Rimmer (1968), the problem can be stated algebraically as finding a set of x (i, j)'s (flows) such that:

$$\Sigma_i \Sigma_j \; x(i, j) \; c(i, j) = \text{Minimum} \quad (1)$$

subject to the constraints:

$$\Sigma_j \; x(i,j) = a(i); \; (i) = 1, 2, \ldots n \quad (2)$$

$$\Sigma_i \; x(i,j) = b(j); \; j = 1, 2, \ldots m \quad (3)$$

$$\Sigma_i \; a(i) = \Sigma_j \; b(j) \quad (4)$$

$$\text{and } x(i,j) \geqq 0 \text{ for all } (i) \text{ and } (j) \quad (5)$$

where x (i, j) the flow of timber from (i)th surplus region to (j)th deficit region;

a (i) the amount of timber available for export from the (i)th surplus region;

b (j) the amount of timber demanded by the (j)th deficit region;

c (i, j) the unit transportation costs for shipping timber between surplus region (i) and deficit region (j).

By use of the duality theorem of linear programming, a set of regional price differentials can be derived which correspond to the optimum set of flows. The problem may then be posed as one of maximising gross returns:

$$\Sigma_j \; b(j) \; v(j) - \Sigma_i \; a(i) \; u(i) = S = \text{Maximum} \quad (6)$$

subject to the constraints:

$$v(j) - u(i) \leqq c(i,j) \quad (7)$$

$$u(i), \; v(j) \geqq 0 \quad (8)$$

where u (i) the value of timber at the supply origin (i);

v (j) the value of timber delivered at the destination (j).

Equation (7) therefore can be restated as:

$$v(j) \leqq u(i) + c(i,j). \quad (9)$$

Corcoran (1962, p. 46), when studying the optimum allocation of wood from a number of forests, extended the basic framework to incorporate a number of other variables. He stated the problem as:

$$\text{Maximise } Z = \Sigma_j \; p(j) \; x(j) \quad (10)$$

subject to the constraints:

$$\Sigma_j \; a(i,j) \; x(j) \leqq b(i); \; (i) = 1, 2, \ldots m \quad (11)$$

$$x(j) \geqq 0 \text{ for all } (j) \quad (12)$$

where m — the number of restricted resources;
j — the product to market activity;
i — the restricted resource type;
x (j) — the number of 100 cubic feet units of output to be sold in (j)th product to market activity;
b (i) — the amount of (i)th input available per period;
p (j) — the contribution to profit by one unit of output in the (j)th product to market activity;
a (i, j) — the amount of (i)th input to produce one unit of output in (j)th product to market activity;
Z — the profit function to be maximised.

In contrast, Barciok *et al.* (1965) attempted to minimise the costs of sawn timber delivered to the consumer. They assumed as the optimum criterion a minimum transport product (tonnes per kilometre). The statement of Barciok *et al.* (pp. 858-62), although displayed in table form, was based on the following equations:

$$\min_k \left[c(i,k) + c(k,j)\right]^{\text{"}} = c(i,l) + c(l,j) = c(i,l,j) \qquad (13)$$

the minimum is reached when k = 1.

where A (i) — the forest district $[(i) = 1, 2, \ldots n]$;
B (j) — the timber consumer $[(j) = 1, 2, \ldots m]$;
C (k) — the sawmill $[(k) = 1, 2, \ldots m]$;
c (i, k) — the transport distance from district (i) to sawmill (k) (km); the actual distance is multiplied by 1.3 or 1.4 because normally one-third of each logging route is on forest tracks, whereas sawn timber is carried exclusively on sealed or metal roads;
c (k, j) — the transport distance from sawmill (k) to consumer (j) (km); in this case the actual distance is divided by 2.27 to account for the loss of log-weight during conversion to sawn timber (as the problem can only be solved if the mass to be transported is equal on either side, the mass of sawn timber is multiplied by 2.27 and the corresponding distance to be transported is divided by 2.27);
c (i, l, j) — the minimum distance from district (i) to consumer (j) via sawmill (l) (km).

After the determination of c (i, l, j) the optimum routes for transporting the products can be derived. Following an alternative method, the optimum amounts to be transported can be obtained:

$$m(i,k) = \Sigma_i\ m(i,k,j) \qquad m(k,j) = \Sigma_j\ m(i,k,j) \qquad (14)$$

where m (i, k) — the amount of logs to be delivered from district (i) to sawmill (k);
m (k, j) — the amount of sawn timber to be delivered from sawmill (k) to consumer (j);
m (i, k, j) — the total amount of forest products to be transported from district (i) to consumer (j) via sawmill (k) (t), $m(i,k,j) \geqq 0$.

From this, the optimum productive capacities of the individual sawmills can also be derived:

$$c(k) = \Sigma_i \; m(i,k) = \Sigma_j \; m(k,j) \qquad (15)$$

where $c(k)$ the optimum production capacity of sawmill (k) (t).

These equations enable the sites and productive capacities of sawmills to be determined, as well as the movements of logs and sawn timber that are required to minimise transport outlay. To achieve the optimum solution the following function should tend to a minimum:

$$\Sigma_i \; \Sigma_k \; c(i,k) \, . \, m(i,k) + \Sigma_k \; \Sigma_j \; c(k,j) \, . \, m(k,j) \rightarrow \text{Minimum!} \qquad (16)$$

It is interesting to observe that the results of Barciok *et al.* (1965, pp. 862-3) showed clearly that the transport outlay rose with increasing concentration of sawmill production, deterioration of the traffic situation, and increasing distance of the sawmills from the mass-centre point of the forest product transport system.

THE CASE STUDY*

In 1970 the direct sales of sawn timber by the Company totalled some 194,000 m^3 (solid volume equivalent). During the same year nearly 100,000 m^3 (solid volume equivalent) of sawn timber were transferred between company operations. Table 1 illustrates that the company sawmills despatched over 57,000 m^3 to the processing unit (planing, joinery and timber preservation), and a further 8,000 m^3 to distributors situated in three urban centres. The sawmill located at Site G supplied more than half the raw material requirements of the processing unit. The processing unit transferred over 33,000 m^3 of finished timber to seven locationally separate markets.

TABLE 1—Intra-company movement of sawn timber (processed and unprocessed) in cubic metres (solid volume equivalent) for 1970

Destination / Origin	PA	MH	MI	MJ	MK	MG	MM	MN	MO	Total
SA (I)	12,305									12,305
SB (I)	840		19							859
SC (I)	906		1,975						224	3,105
SD (I)	4,314		210						389	4,913
SE (I)	42						4,069			4,111
SF (E)	8,088		1,421							9,509
SG (E)	30,550		73							30,623
PA		17,117	7,181	2,150	529	828	4,432	1,090		33,327
Total	57,045	17,117	10,879	2,150	529	828	8,501	1,090	613	98,752

Notes: Sawmills designated (I) predominantly operate with indigenous species, while those marked (E) predominantly operate with exotic species.

* As data for the case study are derived from an existing company the locations of plants and markets are designated alphabetically to ensure confidentiality. The wood-processing functions represented at the alphabetically-designated locations are sawmills (S), processors (P) and markets (M). Thus, SA refers to a sawmill sited at location A, whereas PA refers to a processing unit sited at location A. Throughout the article the case study is termed as the Company.

A LOCATION-DECISION PROBLEM

The problem is to evaluate the transportation costs and benefits of a relocation at site G of the processing unit presently located at site A. The objective of the analysis is to minimise total transfer costs of intra-company movements of processed and unprocessed sawn timber. All other costs, such as those for labour, management and social obligations, are held to be constant.

METHOD

The costs of a restructured allocation system, to allow for the relocation at site G of the processing unit, can be estimated, in part, by use of the simplex transportation method of linear programming. The problem can be stated algebraically in terms of equations (1), (2), (3), (4) and (5). The present allocation of unprocessed sawn timber is displayed in Fig. 1. The unit costs of transportation for all possible movements of unprocessed sawn timber from each origin to each destination are shown in Table 2.

TABLE 2—Estimated transport costs in dollars per cubic metre (solid volume equivalent) of sawn timber for the Company

Destination / Origin	PA	PG	MI	MO	MM
	$	$	$	$	$
SA	0	2.65	7.70	10.45	12.68
SB	4.02	1.62	10.59	7.77	10.63
SC	2.26	4.31	8.93	11.58	13.59
SD	3.46	4.10	11.83	14.44	16.45
SE	12.04	14.34	12.08	5.79	1.59
SF	2.01	2.54	8.58	—	—
SG	2.65	0	10.10	—	—

From this information a matrix of minimum transportation costs can be established to calculate an initial feasible solution for a restructured transportation system. In this solution, indigenous timber demand and supply is separated from exotic timber demand and supply. Differences in timber species and qualities are not taken into account.

Following the method outlined by Rimmer (1968), the transportation costs displayed in Table 2 are ranked from smallest to largest as shown in Tables 3 and 4.

TABLE 3—Ranked transportation costs for indigenous sawn timber (with demand and supply)

Destination / Origin	PG	MI	MO	MM	Supply (in cubic metres)
SA	3	7	10	16	12,305
SB	2	11	8	12	859
SC	5	9	13	17	3,105
SD	4	14	19	20	4,913
SE	18	15	6	1	4,111
Demand (in cubic metres)	18,407	2,204	613	4,069	25,293

TABLE 4—Ranked transportation costs for exotic sawn timber (with demand and supply)

Destination / Origin	PG	MI	Supply (in cubic metres)
SF	2	3	9,509
SG	1	4	30,623
Demand (in cubic metres)	38,638	1,494	40,132

Included in these matrices are statements of the quantity of resource at each origin and the size of demand at each destination. An initial feasible solution can be calculated from the matrices of rankings by proceeding as follows:

1. Beginning with the lowest ranked cell in Table 3, at the intersection of MM and SE, the demand for sawn timber at market M (4,069 m^3) is supplied fully by the sawmill at E, with a minor surplus of resource;
2. The second lowest ranked cell is then taken and the supply at SB is allocated completely to the demand at PG;
3. By following this method all sawn timber is allocated to all demand (Tables 5 and 6).

TABLE 5—Initial feasible solution of indigenous sawn timber allocation in cubic metres (solid volume equivalent)

Destination / Origin	PG	MI	MO	MM	Total
SA	12,305				12,305
SB	859				859
SC	330	2,204	571		3,105
SD	4,913				4,913
SE			42	4,069	4,111
Total	18,407	2,204	613	4,069	25,293

TABLE 6—Initial feasible solution of exotic sawn timber allocation in cubic metres (solid volume equivalent)

Destination / Origin	PG	MI	Total
SF	8,015	1,494	9,509
SG	30,623		30,623
Total	38,638	1,494	40,132

The next step is to test whether or not the initial feasible solution is the least-cost system. This can be accomplished by use of the transportation method of linear programming outlined by Ferguson and Sargent (1968, pp. 30-38). By this method each unallocated or empty cell in the matrix is measured, in turn, as a possible means of a more economic allocation.

A minor improvement indicated by the first test of the initial system, constructed for a processing unit located at site G, was that it would be more economic for the sawmill located at B rather than the sawmill sited at C to supply the market at location 0. After adjustment of the initial solution a second test of the table indicated that no further improvements were possible and that a least-cost system had been attained as shown in Table 7.

TABLE 7—Re-calculated least-cost system of indigenous sawn timber allocation in cubic metres (solid volume equivalent)

Destination / Origin	PG	MI	MO	MM	Total
SA	12,305				12,305
SB	283		571		859
SC	901	2,204			3,105
SD	4,913				4,913
SE			42	4,069	4,111
Total	18,407	2,204	613	4,069	25,293

RESULTS

The existing intra-company movements of unprocessed sawn timber are illustrated in Fig. 1. The total transportation cost of this unprocessed sawn timber allocation system is estimated at $166,003. A new system, developed around a relocated plant at site G (Fig. 2) is estimated to cost $121,109, a reduction of approximately $45,000 on existing costs. The optimised system developed for site G reduces the number of shipments used by some 27% and eliminates cross-haulage of consignments. The principal economy of the system is that a shipment of 30,550 m^3 of unprocessed sawn timber a year, from the sawmill at G to the processing unit at A, is no longer required.

Processing site A, however, has a considerable locational advantage over site G in the cost of processed timber distribution. In Table 8 the estimated transportation costs of distribution from these two sites demonstrates the locational advantage for site A over site G in supplying important processed timber outlets at locations H, I and J. The advantage is a result of both the on-rail site of the plant at A and a closer locational proximity to the market at H. The total estimated cost of processed timber distribution from site A is $175,148, in comparison with a processed timber distribution cost of $226,272 from site G. The greater costs of processed timber distribution from site G (+$51,000) outweigh the monetary savings to be gained from relocation at G of the processing at present situated in A.

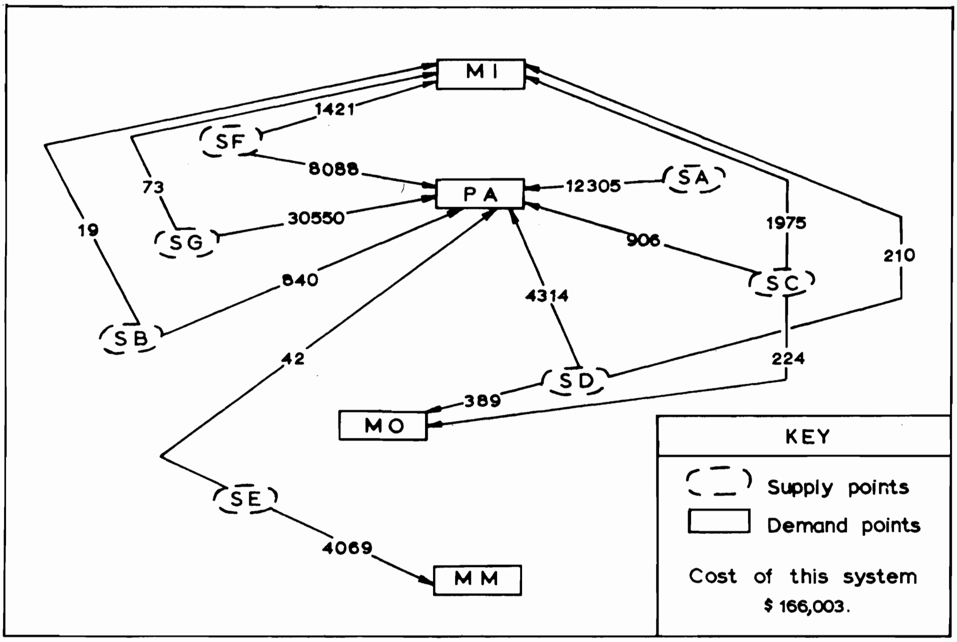

FIG. 1—1970 intra-company movements of unprocessed sawn timber in cubic metres (solid volume equivalent).

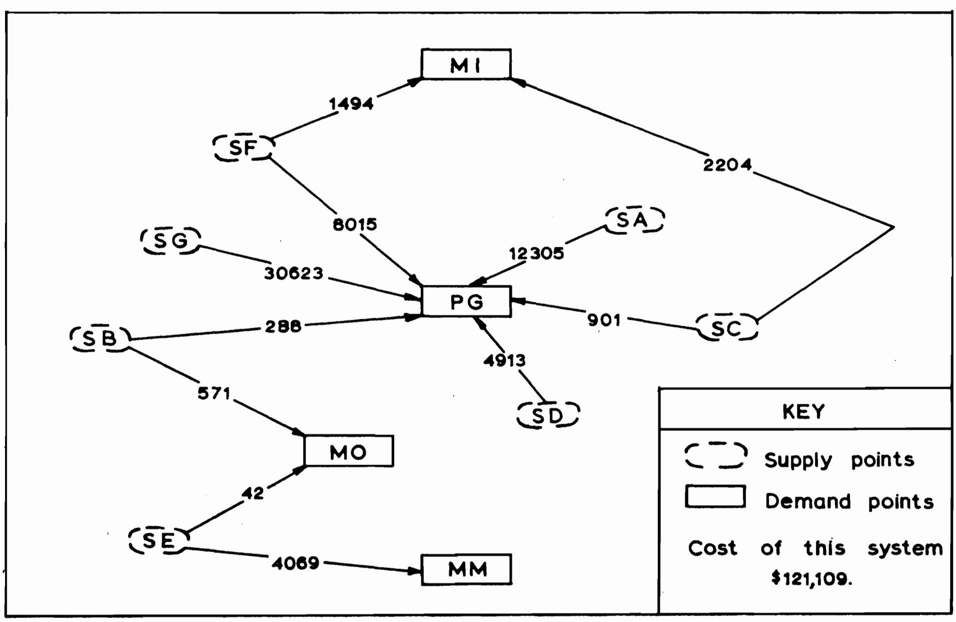

FIG. 2—Optimised intra-company movements of unprocessed sawn timber in cubic metres (solid volume equivalent) assuming processing plant located at G.

TABLE 8—Estimated transport costs in dollars per cubic metre (solid volume equivalent) of sawn timber from processing plants located at A and G to specified markets for the Company.

Origin / Destination	PA	PG
	$	$
MH	2.05	4.52
MI	7.70	10.03
MJ	5.61	7.91
MK	1.73	4.13
MG	2.65	0
MM	12.68	10.49
MN	12.29	10.27
MA	0	2.65

It is apparent from this examination that, in terms of total transportation cost minimisation, the present location of the processing unit at A is preferable to a location at G. Furthermore, when the costs of the existing allocation system of unprocessed sawn timber (Fig. 1) are compared with an optimised version of the existing system (Fig. 3), the small reduction of costs of only $5,000 indicated by the optimised system may, in reality, be unobtainable as this analysis does not take into consideration variation in demand for different species and qualities of timber.

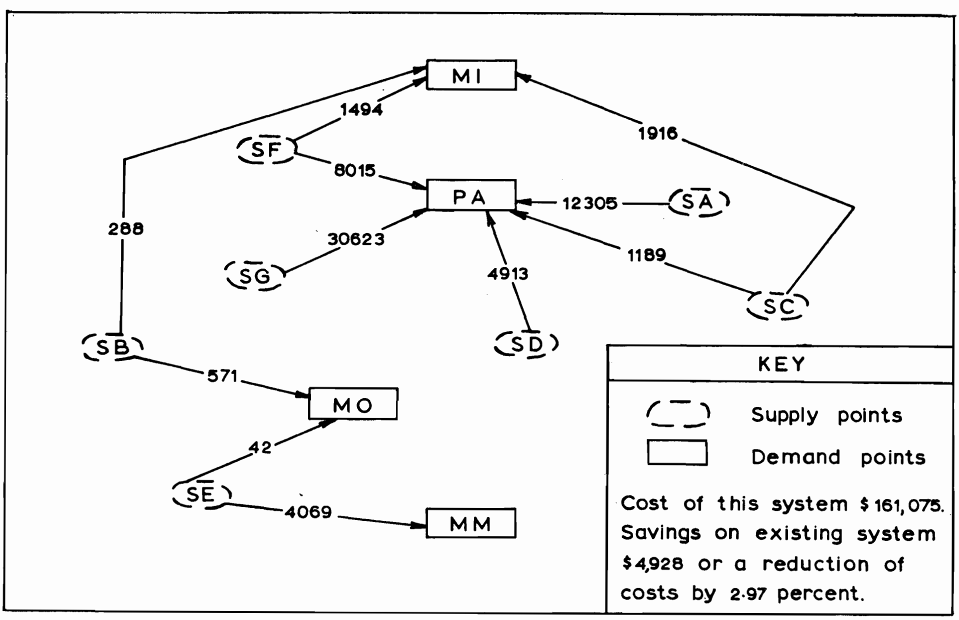

FIG. 3—Optimised intra-company movements of unprocessed sawn timber in cubic metres (solid volume equivalent) assuming present location of plants.

The optimised system, as in the case of the system developed for a processing unit at site G (Fig. 2), reduces the number of shipments considerably and eliminates cross-haulage of consignments. Given the present situation of sawmills and intermediate demand, and a choice of location for the processing unit at either site A or site G, it is evident that, in terms of minimising total transfer costs, the present intra-company movements of processed and unprocessed sawn timber are quite adequate, and that the processing unit should remain at site A.

SUMMARY AND CONCLUSIONS

The case study used in this paper demonstrates the application of an objective method to a problem of locational planning. An assessment of the transportation costs and benefits incurred by a relocation of an existing processing unit constituted the location-decision problem. All other cost factors were held to be constant. While it is obvious that these other cost factors, including social costs, are of importance to the location-decision, it is also clear that the initial suggestion to relocate the processing unit rested upon a wish to spatially juxtapose plants linked by functional interdependence. The analysis demonstrated that substantial economies could be gained in unprocessed sawn timber transportation costs by a relocation at site G, but, that distribution cost savings on processed timber from the existing plant at site A (in comparison to G) outweighed the advantages of such a relocation. The results indicated, therefore, that the existing processing unit should remain at its present location in A. It is interesting to note that in this case study the optimum solution, under the constraints of the problem, was to separate by some 90 km two plants that were functionally interdependent. If, however, the examination had revealed that substantial transportation economies were possible by relocation of the processing unit, then it would have been necessary to consider in detail other cost factors, including social and environmental costs. The basic relocation problem could have been further refined by examining alternative possible future conditions of timber supply and demand.

The practical value of linear programming in assisting the locational planner and in formulating schemes of spatial integration is evident from the case study. It is apparent that such objective methods could be usefully applied to many of the location problems of major forest owners and wood-processors in New Zealand.

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