

# STAND DYNAMICS AND DENSITY IN RADIATA PINE PLANTATIONS

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## ABSTRACT

Data were obtained from unthinned plots of radiata pine which had reached maximum density, as indicated by substantial and continuing mortality. A simple model of stand dynamics was estimated from these data. One of the equations in this model enables an estimate to be made of the maximum basal area a stand of given site, age and initial stocking could carry, thus providing a datum for a relative measure of stand density. Reineke's stand density index and modifications of it were shown to involve a source of bias which could lead to biased estimates of density.

## INTRODUCTION

Stand density is generally believed to have a powerful influence on stand growth and yield. However its use as an explanatory variable in thinning research, growth models, and in the field, has been the subject of considerable debate because of the lack of a satisfactory measure of stand density (Smith, 1962; Bickford *et al.* 1957).

One measure, used by Cosco (1971) in a simulation model for radiata pine (*Pinus radiata* D. Don) in New South Wales, is based on the ratio of standing basal area to the maximum basal area per unit area which a stand of given site, age and initial stocking could support. The maximum basal area per unit area is clearly the critical component of Cosco's measure. However it was not developed from a thorough examination of stand dynamics under conditions of maximum density.

The aim of this paper was to examine and estimate maximum basal area per unit area for radiata pine under conditions of maximum density. In doing so, it was also possible to throw some new light on another measure of stand density, Reineke's stand density index.

Curtin (1968), in defining density as the average intensity of competition between the individual trees in the stand, pointed out that substantial natural mortality provided a clear indication that a stand had reached maximum density. Admittedly, climatic fluctuations from year to year introduce a stochastic element into this condition of maximum density since the resources available to support the stand change in accord

with the climate. Moreover the occurrence of mortality may not be a sufficient condition to identify stands which have reached maximum density. Mortality can arise from causes other than intense competition. These other causes, such as insect and pathogen attack, are often superimposed on, or interact with, climatic fluctuations. Thus mortality does not provide a completely unambiguous criterion of maximum density. Nevertheless the occurrence of *substantial and continuing mortality* provides a reasonably consistent and objective means of identifying stands which have reached maximum density.

#### DATA

Some 280 permanent plots have been established by the Woods and Forests Department in the radiata pine plantations located in the south-east of South Australia. These plots form the basis of an extensive system of yield regulation covering an area of 68 000 ha (Lewis, 1954, 1957; Keeves, 1970). Some of these plots have been maintained as unthinned controls, providing an excellent opportunity to select stands which have reached maximum density.

The plots included in Fig. 1 were selected to illustrate the problems which arise in applying the criterion of substantial and continuing mortality. Plot 369 presented no problem because of the sharp break in the trends in mortality before and after the fourth measurement. However other plots, of which SP513 and EP24C were typical examples, did not exhibit a sharp break and a more detailed definition of substantial

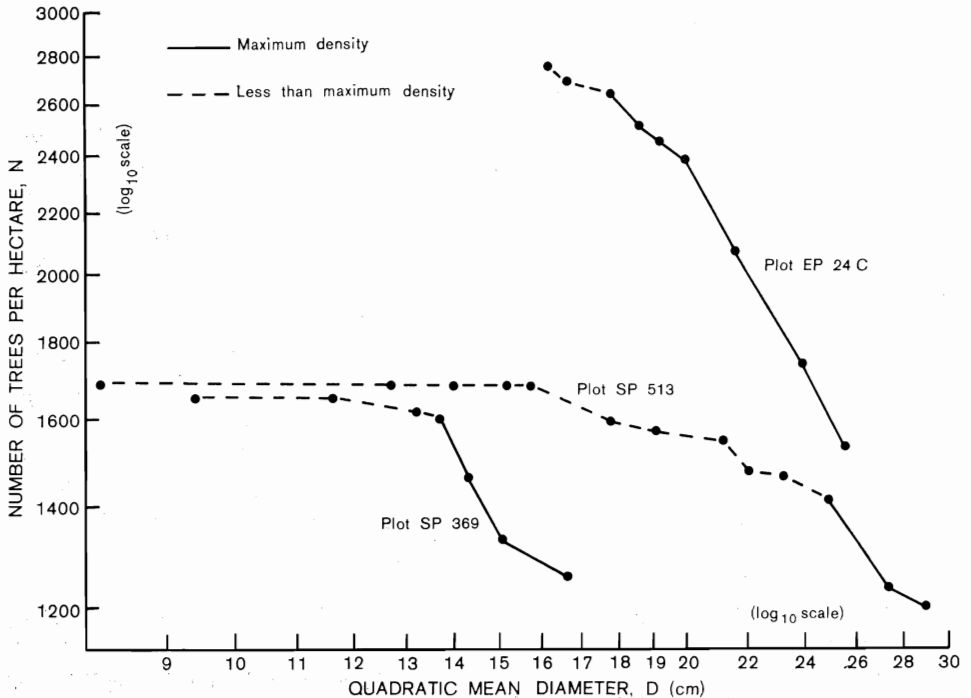


FIG. 1—Selection of measurements at maximum density.

and continuing mortality had to be framed to enable the classification to be made objectively.

Two criteria were employed to define the onset of substantial and continuing mortality. The first measurement period following the accepted point of onset had to have mortality at least 0.5% p.a. greater than that in the previous period. This ensured the selection of a relatively pronounced break in the trend of mortality. Mortality during subsequent periods had to average at least 1% p.a. more than that before the onset. This ensured that the initial break in trend was both sustained and substantial. Relative to the average mortality of 0.5% p.a. over the entire data set prior to the onset of maximum density, and about 2.2% p.a. after it, these changes were quite large. This definition provided a clear and objective identification of the onset of maximum density although probably a conservative one for plots such as Plot SP513 in Fig. 1.

Some 168 measurements at maximum density were available from the 34 plots which had been left unthinned, but these data had to be further culled to yield the final set of 58 observations. The first measurement at the onset of maximum density in each plot could not be used because it lacked a value for stocking at maximum density from the previous measurement; a variable required for some of the models to be estimated.

For the 24 plots with four or more measurements, only the second and last measurements were included in the final set of observations to reduce the statistical problems arising from serial correlation between successive measurements. Serial correlation in basal area, for example, was significant based on a test using the Durbin-Watson statistic at the 95% probability level. This would violate the assumption of independence between the error terms which underlies regression analysis using ordinary least squares. On its own, this problem would result in under-estimation of the variances attached to the regression coefficients, although the estimated coefficients would still be unbiased. In combination with multicollinearity and/or the use of lagged variables, such as stocking at the previous measurement, it can give rise to biased or even inconsistent estimates of the regression coefficients (Goldberger, 1964). Use of the second and last measurements for each plot ensured that the data spanned as wide a range of age as possible, while minimizing serial correlation.

One of the remaining plots only had two measurements available at maximum density and thus only the second was included in the final data set. For the nine plots which had three measurements available, the second or third was chosen at random for inclusion in the final set of observations.

Most variables included in the data conformed with conventional definitions and usage for this type of study. However the use of stocking at age 10 years as a variable warrants some explanation. Initial stocking at the time these plots were established varied widely, reflecting differences in initial spacing and/or subsequent survival. These differences seemed likely to have an influence on maximum basal area, but no accurate data were available for stocking at the time of planting or for the early years thereafter.

An age of 10 years was chosen as the datum for a measure of "initial" stocking. This is close to the standard age (9½ years) at which the first inventory is normally carried out in South Australia and is therefore appropriate for subsequent application

of the functions from this study. Moreover some 24 plots had been measured at age 10 years, or within 2 years of it, so that its estimation presented no problems for the majority of the plots.

The remaining 10 plots had first been measured at age 22, 25 or 27 years and occurred on poor sites. At the time of establishment of these plots it was (and still is) standard practice to record the number of dead trees. Six of the plots had no dead trees so the stocking at age 10 years may safely be assumed to be that at the time of the first measurement. Three plots had the equivalent of about 25 dead trees per hectare, while the remaining plot had the equivalent of 12 dead trees per hectare. Previous experience and examination of data from four permanent plots not included in the test data suggested that most if not all of these deaths occurred after age 10 years. These other plots were also located on poor sites and had been established at or near age 10 years and remeasured at various times up to age 25 to 28 years. None had any dead trees at the time of establishment but subsequent deaths were of similar order to the plots in question. Thus it was assumed that all deaths on the plots in question occurred after age 10 years and the stocking at age 10 years was estimated accordingly.

The main explanatory variables used in this study are summarized in Table 1, in terms of their means, estimated standard deviations and maximum and minimum values. Evidently the data were not uniformly distributed across the ranges of all the variables concerned. Nevertheless the distribution was such as to provide an excellent basis for analysis, especially in relation to site, age and initial stocking.

TABLE 1—Summary statistics for sample data

Variable	Mean	Standard Deviation	Maximum	Minimum
Diameter (cm)	25.8	4.5	34.1	14.3
Basal area (m <sup>2</sup> /ha)	69.8	15.9	100.9	23.6
Stocking (stems/ha)				
Current	1336	269	2505	936
† years previously	1445	271	2642	1008
At age 10 years	1686	282	2815	1208
Site index (m)	31.9	5.6	38.4	17.4
Age (years)	34.4	9.1	54	16
Years since last measurement	3.52	1.44	6	2
Survival ratio	.977	.011	.994	.949

### GROWTH

Basal area is the integral of net increment in basal area over time. In this study it was simpler to estimate the function using basal area as the dependent variable, rather than net increment, because of the nature of the data. Nevertheless growth is the causal process underlying the function estimated in this section, a point to be taken up again in a later section.

The general form of the model used to estimate basal area at maximum density follows that used in a number of other studies (e.g. Sullivan and Clutter, 1972), being

a linear model using the logarithm of basal area as the dependent variable and the reciprocal of age as an independent variable. Non-linear models were not fitted because satisfactory significance tests were not available for them, and testing was a critical aspect of the present study. Moreover non-linear models seemed unlikely to offer much advantage over this linear model, given the range of ages involved.

A number of variants of the basic model were tested; such as a quadratic versus a logarithmic form for site index and a linear versus a logarithmic form for stocking at age 10 years. The following function had the lowest residual mean squared error and F tests showed it was significantly better than each of the other variants at the 95% probability level:—

$$\log B = -.217 - 4.669/A + .057S - .00062S^2 + .315 \log E \quad (1)$$

(R<sup>2</sup> = .91)      (.540)      (.009)      (.00015)      (.076)

where B denotes basal area (m<sup>2</sup>/ha), and logarithm is to base 10,

A denotes age (years),

S denotes site index (m), defined as the mean height of the tallest 75 trees/ha at age 30 years,

E denotes stocking at age 10 years (stems/ha).

The multiple coefficient of determination (R<sup>2</sup>) is shown immediately below the dependent variable, and the standard errors are shown in brackets below their respective coefficients. At the 95% probability level, 't' tests showed that each of the estimated coefficients was significantly different from zero. The high level of R<sup>2</sup> also suggested that this equation provided a good fit to the data. The value of the Durbin-Watson statistic (1.776) for this equation was above the upper limit of the critical region at the 95% probability level, indicating that the steps taken to avoid the problem of serial correlation seemed to have been successful. The scatter plot of residuals gave no indication of heterogeneity.

The quadratic form of site index in equation (1) implies that basal area reaches a maximum at a site index of about 46 m and then declines. This is well outside the range of the sample data (Table 1). Nevertheless it would obviously be foolish to extrapolate this function beyond a site index of about 40 m or, indeed, outside the range of the other variables in Table 1.

The implications of this function for measurement of stand density will be taken up in a later section, following consideration of further components in the system of stand dynamics. The first component involves further examination of basal area.

Basal area per hectare is, by definition, a function of the number of stems per acre and quadratic mean diameter, an identity expressed as follows:

$$\log B \equiv \log N + 2.0 \log D + \log (\pi/40,000) \quad (2)$$

where N denotes the number of stems per hectare,

D denotes the quadratic mean diameter (cm), defined as

the diameter corresponding to the value of mean basal area per tree.

Substituting equation (2) in equation (1), the following relationship between the number of stems per hectare, quadratic mean diameter and other variables was obtained for stands of maximum density:—

$$\log N = 3.888 - 2.0 \log D - 4.669/A + .057S - .00062 S^2 + .315 \log E \quad (3)$$

Equation (3) estimates the locus of all combinations of N and D for any given

age, site index, and stocking at age 10, within the range of the sample data. However it does not define the dynamic path which a particular stand would take in terms of the values of N and D at each point in time.

### MORTALITY

In order to examine the path of N and D for a particular stand at each point in time, it is necessary to examine mortality, the other dynamic process involved. To be consistent with the manner of measurement and the nature of the process, the mortality variable was defined as the geometric mean annual survival ratio, P, given by:

$$P = [N/N(t)]^{1/t} \quad (4)$$

where t denotes the number of years between the observation used in the final data set and the preceding measurement of that plot,

N denotes the number of living stems per hectare of the observation in the final data set,

N(t) denotes the number of living stems per hectare at the time of the preceding measurement.

This definition is relevant because survival is a multiplicative process: a 90% survival in the first year followed by a 70% survival in the second year gives an overall survival after two years of 63%. Successive measurements on any one plot were made at irregular intervals varying from 2 to 6 years so that explicit recognition of the time between measurements was essential.

The mortality variable was transformed to obtain a form consistent with that used in the other equations. Thus the difference between the logarithms of current stocking and stocking at the previous measurement became the dependent variable and time since the previous measurement an independent variable. A number of models using different combinations of independent variables were tested; the most complex involving the interactions between time since the previous measurement and the inverse of age and time and site, as well as the time variable itself. The only model in which all estimated coefficients were significantly different from zero, based on 't' tests at the 95% probability level, was the simple form in equation (5):—

$$\log N - \log N(t) = -.00975t \quad (5)$$

(R<sup>2</sup> = .81)      (.0006)

The R<sup>2</sup> value was calculated relative to the variation around the origin, since this equation was constrained to pass through the origin. The form of the dependent variable also implies that an exact constraint was imposed on the coefficient for stocking at the previous measurement. Both these constraints were consistent with the definition of the mortality variable.

Equation (5) was somewhat surprising because annual survival seems to be unrelated to age, site index or stocking at age 10 years; the estimated mean survival ratio being .978. Meyer's (1938) studies suggested that survival in ponderosa pine (*Pinus ponderosa* Dougl.) increased with increasing age and, to a lesser degree, with increasing site index. However these results were based on graphical analyses of data covering a much wider range of ages, sites and climatic conditions than the present study.

The second term in equation (5) was transposed to provide the second component of a model of stand dynamics:—

$$\log N = \log N(t) - .00975t \tag{6}$$

Given the stocking at some time previous under conditions of maximum density, equation (6) enables the current stocking to be predicted.

### A SIMPLE MODEL OF STAND DYNAMICS

Equations (3) and (6) represent the simultaneous processes of growth and mortality whose interaction traces out the dynamic path of the relationship between the number of stems per hectare and quadratic mean diameter over time under conditions of maximum density. This is illustrated for a given site index and initial stocking in Fig. 2.

The sloping lines are the loci traced out by equation (3) at different points in time while the horizontal lines are the loci traced out by equation (6) for those same points in time. The intersection points show the dynamic path. Comparison of the

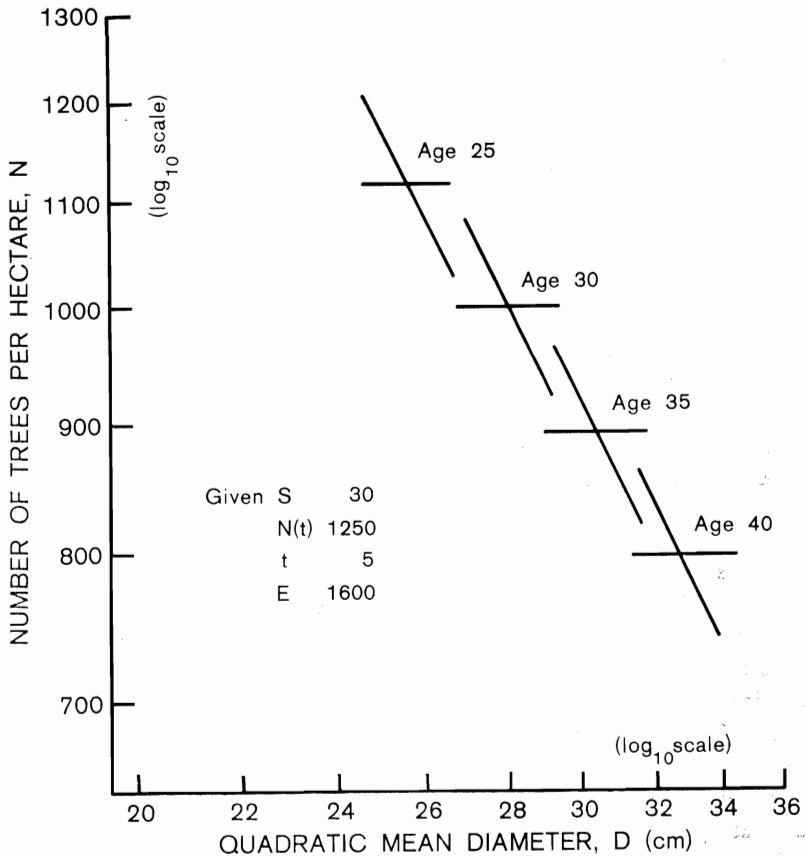


FIG. 2—Loci traced by growth and mortality processes.

trends in Fig. 2 with the observations classed as representing maximum density in Fig. 1 shows that the estimated trends conform with the general behaviour of the data.

Independent data were not available to enable an objective test of predictions from the model to be carried out. However the evidence available is encouraging. The  $R^2$  values were high. There was no evidence of serial correlation in the values of the Durbin-Watson statistic, nor of heterogeneity of variance in the scatters of the residual values.

Mortality also has a direct effect on basal area, since basal area is the integral of *net* increment in basal area over time. Thus the relationships in this model can be completed by deriving the equation which relates the rate of change in basal area to the rates of change in diameter and in stocking. Taking the derivative of equation (2) with respect to age, the following expression was obtained:—

$$\frac{d \log B}{dA} = \frac{2.d \log D}{dA} + \frac{d \log N}{dA} \quad (7)$$

The first term can be eliminated by taking the derivative of equation (1) with respect to age and substituting the result for it. The other two terms can be approximated by the periodic changes in the logarithms (to base 10) of diameter and stocking. The latter can be estimated using equation (6). Substituting accordingly and transposing terms, the following equation was obtained:—

$$\frac{\log D - \log D(t)}{t} = \frac{2.334}{A^2} - \frac{0.5 \log N - \log N(t)}{t} \quad (8a)$$

$$= \frac{2.334}{A^2} - .00488 \quad (8b)$$

where  $D(t)$  and  $N(t)$  denote the quadratic mean diameter and stocking  $t$  years prior to the current age.

The first term on the right hand side of equation (8) approaches zero as age approaches infinity. Thus the periodic rate of increase in diameter approaches 1.12% per annum as age approaches infinity. However the contribution of the first term remains significant over the entire range of ages of practical interest, certainly up to 60 years, and it should not be ignored.

### STAND DENSITY

The values of the maximum basal area per hectare estimated from equation (1) for various combinations of stocking at age 10 years, site and age are shown in Table 2.

For a given stocking at age 10 years, the maximum basal area a particular site can support changes markedly with age. Similar results were obtained by Gingrich (1967) using a different approach. However the results in Table 2 provide another illustration of why standing basal area in thinned stands is not a very satisfactory measure of density. A thinned stand with a standing basal area of 25 m<sup>2</sup>/ha at age 20 is much closer to the upper limit it can carry than it would be given the same standing basal area at age 30.

A relative measure of density could be expressed in terms of the difference between the standing basal area in a thinned stand and the maximum basal area, the ratio of



TABLE 2—Maximum basal area (m<sup>2</sup>/ha) for given site, age and initial stocking

Site Index (m)	Age (yrs)				
	20	25	30	35	40
Initial stocking = 1500 (stems/ha)					
25	38.7	43.1	46.3	48.7	50.6
30	50.4	56.1	60.2	63.4	65.9
35	61.0	68.0	73.0	76.8	79.8
Initial stocking = 1750					
25	40.6	45.2	48.6	51.1	53.1
30	52.9	58.9	63.2	66.6	69.2
35	64.1	71.3	76.6	80.7	83.8
Initial stocking = 2000					
25	42.4	47.2	50.7	53.3	55.4
30	55.1	61.4	65.9	69.4	72.1
35	66.8	74.4	79.9	84.1	87.4
Initial stocking = 2250					
25	44.0	48.9	52.6	55.3	57.5
30	57.2	63.7	68.4	72.0	74.9
35	69.3	77.2	82.9	87.3	90.7
Initial stocking = 2500					
25	45.4	50.6	54.4	57.2	59.4
30	59.1	65.9	70.7	74.5	77.4
35	71.7	79.8	85.7	90.3	93.8

these parameters, or some transform such as the logarithm of the ratio of these two parameters. Such measures are widely used in the United States and Europe.

The best form of this proposed measure of density can only be determined by further research into the relationship between volume (or basal area) increment, alternative forms of this measure, and other relevant variables. But this type of measure appears suitable for both field use and research work because it involves parameters which are easily determined and which would normally be available.

#### *Reineke's Stand Density Index*

Equation (3) bears a strong resemblance to the "reference curve" for even-aged stands which had reached "full density" (Reineke, 1933):—

$$\log N = \log a + b \log D \quad (9)$$

The variables N and D were defined as current stocking and "average diameter (by basal area)" i.e. quadratic mean diameter, while a and b were constants estimated from sample data. Reineke found that the slope coefficient seemed to be constant (-1.605) for 12 of the 14 species examined. Although Reineke concluded that this reference curve was not affected by age or site, the inclusion of these or other variables on the right-hand side of equation (9) would not affect his basic approach in developing a reference curve for use in a stand density index. Curtis (1971) has also shown that the "tree-area" approach of Chisman and Schumacher (1940) is closely related to Reineke's approach.

Our investigations actually started from the estimation of Reineke's reference curve. A number of variants of the model in equation (9), including various combinations of the independent variables in equation (1), were estimated using the same sample data described earlier. The following equation had the lowest residual mean squared error and each of the estimated coefficients was significantly different from zero, based on 't' tests at the 95% probability level:—

$$\log N = 1.471 - 1.077 \log D + .040S - .00052S^2 + .512 \log E \quad (10)$$

$$(R^2 = .87) \quad (.078) \quad (.007) \quad (.00012) \quad (.065)$$

The variant of this equation which included the inverse of age was not significantly different. The most striking point, however, is that the estimated coefficient attached to  $\log D$  in this equation is significantly different from  $-2.0$ , based on a 't' test at the 95% probability level. This was also true of *all* the other variants of this basic model. There was no evidence of serial correlation or marked collinearity between  $\log D$  and other independent variables which could have contributed to this result.

The system of simultaneous equations formulated earlier in this paper provided an explanation for this anomaly. Only equations (3) and (6) need be considered since they provide a complete description of the dynamic path of stocking and diameter. Clearly, stocking and diameter are jointly dependent variables since any change in stocking will be accompanied by a change in diameter; growth and mortality being simultaneous processes.  $\log N$  and  $\log D$  are called endogenous variables because they are determined by processes within the system being estimated. The remaining variables ( $A^{-1}$ ,  $S$ ,  $S^2$ ,  $\log E$ ,  $\log N(t)$ ,  $t$ ) are called exogenous variables because their values are determined outside the system being estimated or (in the case of  $\log N(t)$ ) by the past history of the system, not by the current processes.

There are two endogenous variables in equation (10) and two exogenous variables ( $\log N(t)$  and  $t$ ) are excluded from it. Thus according to the order condition of identifiability (Goldberger, 1964), this equation is over-identified. It would still be over-identified if further exogenous variables were dropped from equation (10), even to the point of Reineke's reference curve (equation 9). However the omission of these further variables introduces a further source of bias into the equation due to the specification error (Goldberger, 1964).

The use of ordinary least squares to estimate an over-identified equation results in an estimate of the coefficient of the endogenous variable on the right-hand side of the equation\* (i.e.  $\log D$ ) which is biased and inconsistent (Goldberger, 1964). This bias arises because the dependent variable and the error term in an over-identified equation are correlated, violating one of the basic assumptions underlying ordinary least squares.

It follows that equation (10), or variants of it such as Reineke's reference curve (equation 9), will yield biased forecasts of stocking for a given diameter, if estimated by ordinary least squares.

The effect of over-identification can be illustrated visually by reference to Fig. 2. Suppose equations (3) and (6) were exact but unknown descriptions of the processes of growth and mortality for a hypothetical plot with the characteristics shown in Fig. 2.

\* The identification problem would also have arisen had equation (8a) been estimated using ordinary least squares instead of deriving it from the other equations.

If data for N and D were collected from this plot at ages 25, 30, 35 and 40 years, they would plot at the four points of intersection in Fig. 2. These four points would be the data used by an analyst to estimate equation (10), which is equivalent to Reineke's equation (9) for the given site and stocking at age 10 years. Suppose this equation was estimated using ordinary least squares. The estimated coefficient for log D in this equation can be gauged visually by the slope of the line of best fit to the four points of intersection in Fig. 2. Its absolute value will obviously be less than 2.0, which is the slope of the inclined lines.

Reineke's reference curve, or modifications of it, is therefore an artifact of the data which approximates the path followed by this particular stand over time, but which would yield biased forecasts for other stands. This can be illustrated by considering another hypothetical plot from the same system as that in Fig. 2, differing only in the value of diameter at age 25 years. In order to obey the mensurational definition of basal area, this second plot must lie somewhere on the inclined line for age 25 years. Thus its true stocking would differ from that obtained by estimation from Reineke's reference curve.

The anomaly between equation (3) and the Reineke-type equations (9) or (10) arises because Reineke and other workers following him did not take the identity relating basal area, diameter and stocking (equation 2) explicitly into account. The use of ordinary least squares compounds the problem by producing a biased estimate for the coefficient of log D which is not statistically obvious in significance tests unless the identity is taken into account.

There are now a number of techniques available for estimating over-identified equations. Two-stage least squares is one of the simplest of these and provides consistent and less-biased estimates for an over-identified equation (Goldberger, 1964). Equation (10) was therefore re-estimated using this technique. The estimation was carried out in two distinct stages, although computer programs are now available which will carry out the estimation in one pass. The first stage involved the estimation of the "reduced form equation", using the logarithm of diameter as the dependent variables and *all* the exogenous variables as independent variables.† The predicted values of this variable were then used in the place of the actual values in re-estimating equation (10):—

$$\log N = 3.261 - 2.091 \log D - 5.101/A + .059S - .00064S^2 + .295 \log E \quad (11)$$

(R<sup>2</sup> = .95)      (.128)      (.666)      (.005)      (.00008)      (.048)

Dhrymes' (1970: p. 274) test of significance, though not exact for a system with a lagged endogenous variable ( $\log N(t)$ ), clearly shows that the coefficient of the diameter variable in equation (11) is *not* significantly different from -2.0. The values for the other coefficients are, as one would expect, close to those in equation (3).

We conclude that Reineke's stand density index, or any modification of it, should not be used as a measure of stand density because it is likely to produce biased estimates. The identification problem may also warrant consideration in other growth and yield studies, since it is common practice to estimate volume as a function of basal area, the

† Because of the constraints on the coefficients in equation (6), the method of indirect least squares (Goldberger, 1964) could also have been used to derive equation (11). The results would be identical. Two-stage least squares has a wider range of application and it therefore seemed preferable to illustrate its use here.

two being jointly dependent variables. Complete specification of the system and of the equation concerned is required, however, before identification can be gauged.

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