COMPOSITE TAPER EQUATIONS TO PREDICT OVER- AND UNDER-BARK DIAMETER AND VOLUME OF *EUCALYPTUS PILULARIS, E. GLOBOIDEA,* AND *E. MUELLERIANA* IN NEW ZEALAND

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ABSTRACT

Composite tree taper equations were fitted to data from sectionally measured, New Zealand-grown *Eucalyptus pilularis* Sm., and to combined data from *E. muelleriana* Howitt and *E. globoidea* Blakely. Although the sample size was small, the equation for *E. pilularis* fitted well. The combined *E. muelleriana, E. globoidea* equation was less satisfactory and should be used only as an interim solution.

Keywords: tree volume; taper; *Eucalyptus pilularis*; *Eucalyptus globoidea*; *Eucalyptus muelleriana.*

INTRODUCTION

Eucalypts of the stringybark group have been planted in small scattered blocks throughout much of the warm, temperate areas of New Zealand. The most common species included in the stringybark group are *E. muelleriana* and *E. globoidea*. Although botanically different from the true stringybarks, *E. pilularis* and *E. microcorys* F. Muell. are often included in this group owing to their similar wood properties, i.e., good strength and durability ratings (Haslett 1990). All these species are suited to a wide range of sites in New Zealand, from Northland to the Bay of Plenty/Hawke's Bay and Taranaki/Manawatu in the North Island, and areas of Nelson/Marlborough and Banks Peninsula in the South Island. Much of the older resource of these species is currently being milled and used for flooring, decking, and decorative uses such as panelling.

Tree volume and taper equations are used to determine diameters and volumes (under- and over-bark) of whole tree stems and stem sections. These equations require measurements of tree breast height over-bark diameter (*dbh* cm) and tree height (*H* m), and are incorporated as basic components of stand inventory, growth and yield prediction, forest planning, and product simulation systems. The development of an equation for *E. pilularis*, and an interim equation for *E. muelleriana* and *E. globoidea*, is described in this paper.

Many formulations of volume and taper equations, with different properties and strengths, have been used to predict tree volumes and diameters (Goulding & Murray 1975; Max & Burkhart 1976; McClure & Czaplewski 1986; Kozak 1988; Candy 1989; Shiver & Brister

1992; Bailey 1994). Composite equations (Gordon *et al.* 1995) have been used successfully for *E. saligna* Sm. grown in New Zealand and were applied here to data from New Zealand-grown stringybarks. Composite equations combine an over-bark taper equation with an equation to predict the proportion of under-bark to over-bark cross-sectional area, resulting in an under-bark taper equation which is integrable to calculate the wood volume and diameters of any stem section. The equations are structured to predict zero over-bark (*dob* cm) and under-bark (*dub* cm) diameters at the tip of the tree, and to predict decreasing diameters from ground-level to tree tip. A constraint incorporated in the equation ensures that *dbh* will be predicted by the over-bark taper equation at breast-height.

DATA

Tree sectional measurement data for *E. muelleriana, E. globoidea, E. pilularis*, and *E. microcorys* had been collected from stands in Northland, Auckland, Coromandel, and the Bay of Plenty from as early as 1959. These data were collated and assembled into four data sets. Conversion from imperial to metric measure was required for some of the measurements. Most trees had been measured for *dbh* (at 1.4 m) and *H*, with *dob* and bark thickness measured at approximately 3-m intervals up the stem, starting at 3 m above ground. Either one or two diameters, with bark thicknesses, were measured below breast height: at 0.7 m above ground, or at 0.15 and 0.7 m above ground. The locations at which trees were sampled are listed in Table 1.

After they were screened for extreme or inconsistent measurements, the data were grouped by species. Only the *E. pilularis* data set contained an adequate number of trees. The range of *dbh*, *H*, under-bark sectional volume (*Vub*), and form-factor (*FF*) is shown in Table 2. Form-factor was calculated as the ratio of *Vub* to the volume of a cylinder based on *dbh* and *H*.

RESULTS AND DISCUSSION

Comparisons were made to determine whether the differences between species precluded fitting one taper equation to the combined data. A multiple range test of mean form-factor by species (Table 3) indicated that only *E. muelleriana* and *E. globoidea* had similar form. One equation was fitted to the *E. pilularis* data set and one to the combined *E. muelleriana* and *E. globoidea* sets. There were insufficient measurements of *E. microcorys* to derive an equation.

Bark Equations

The sectional area ratio $(dub/dob)^2$ was plotted over proportional height for *E. muelleriana* and *E. globoidea*. This showed the need for separate bark equations as the bark in *E. globoidea* is considerably thicker (Fig. 1). A quadratic equation relating the sectional area ratio to proportional height was fitted to the *E. pilularis* sample and linear equations were fitted to the *E. globoidea* and the *E. muelleriana* samples.

The bark equation is:

$$\frac{dub^2}{dob^2} = a_0 + a_1 \frac{L}{H} + a_2 \left[\frac{L}{H}\right]^2 \qquad (1)$$

TABLE 1–Descriptive statistics of the sectionally measured sample trees by location

Species	Location	Age (years)	Mean *dbh* (cm)	Mean *H* (m)	Number of trees	Stand detail
E. pilularis	Rotoehu, Bay of Plenty	11	18.9	16.8	18	Planted 1960, thinned 1971
		12	14.3	16.2	3	Planted 1960, thinned 1971
	Tairua, Coromandel	18	22.6	20.8	8	Planted 1960, thinned 1978
		16	21.7	21.8	9	Planted 1962, thinned 1978
		18	22.6	19.4	7	Planted 1960, thinned 1978
	Riverhead, Auckland	33	31.6	26.3	25	Planted 1926
		33	32.2	22.4	9	Planted 1926
	Clevedon, Auckland	37	36.8	25.9	8	Planted 1926
E.muelleriana	Athenree, Coromandel	11	21.9	17.2	13	Planted 1961, thinned 1972
	Tairua, Coromandel	19	22.9	20.2	8	Planted 1960, thinned 1978
	Riverhead, Auckland	31	49.6	30.7	7	Planted 1928
E. globoidea	Tairua, Coromandel	18	19.1	18.1	8	Planted 1960, thinned 1978
	Rotoehu, Bay of Plenty	17	26.8	22.0	4	Planted 1960, thinned 1976
E. microcorys	Tairua, Coromandel	18	17.8	15.9	9	Planted 1960, thinned 1978

where L = length between the tip of the stem and the measurement point (m).

Coefficients with standard errors are listed in Table 4.

Predictions of *dub* were calculated from Eqn 1 and compared with actual values. Mean and standard deviation of the *dub* residuals are given in Table 5.

Taper Equations

The taper equation fitted was:

$$\left[\frac{dob}{dbh}\,\frac{H-1.4}{H}\right]^2 = \beta_c \left[\frac{L}{H}\right]^{\beta_1} + \beta_2 \left[\frac{L}{H}\right]^{\beta_3} \tag{2}$$

TABLE 2–Range covered by tree variables by species.

Species	Variable	No.	Minimum	Maximum	Mean	Std.dev.
E. pilularis	*dbh* (cm)	74	5.6	47.5	26.2	10.0
	H (m)	74	9.0	35.1	22.2	6.0
	Vub (m^3)	74	0.008	1.796	0.475	0.419
	FF	74	0.189	0.439	0.303	0.045
E. muelleriana	*dbh* (cm)	28	5.9	59.4	29.1	14.5
	H (m)	28	9.0	34.4	21.5	6.8
	Vub (m^3)	28	0.008	2.131	0.548	0.599
	FF	28	0.208	0.321	0.261	0.027
E. globoidea	*dbh* (cm)	11	13.5	28.2	21.6	5.0
	H (m)	11	15.2	23.7	19.4	2.6
	Vub (m^3)	11	0.048	0.361	0.181	0.092
	FF	11	0.183	0.292	0.235	0.030
E. microcorys	*dbh* (cm)	7	10.7	23.5	18.0	4.5
	H (m)	7	10.9	19.8	16.3	3.2
	Vub (m^3)	7	0.046	0.295	0.171	0.094
	FF	7	0.336	0.471	0.378	0.047

TABLE 3–Comparison of mean form-factor by species

Data set	Number of trees	Mean form-factor	Duncan* grouping
E. microcorys	7	0.378	A
E. pilularis	74	0.303	B
E. muelleriana	28	0.261	C
E. globoidea	11	0.235	C

* Means with the same letter (Duncan grouping) are not significantly different

TABLE 4–Coefficients and coefficient standard errors of bark equation.

Sample	a_0	a_1	a_2
E. pilularis	0.828 (se. 0.014)	0.166 (se. 0.051)	–0.339 (se. 0.042)
E. muelleriana	0.891 (se. 0.008)	–0.298 (se. 0.012)	0.000
E. globoidea	0.691 (se. 0.020)	–0.152 (se. 0.028)	0.000

TABLE 5–Mean and standard deviation of *dub* residuals.

Sample	Mean (cm)	Standard deviation (cm)	No. of observations
E. pilularis	0.030	0.915	666
E. muelleriana	–0.043	0.797	267
E. globoidea	0.031	0.708	78

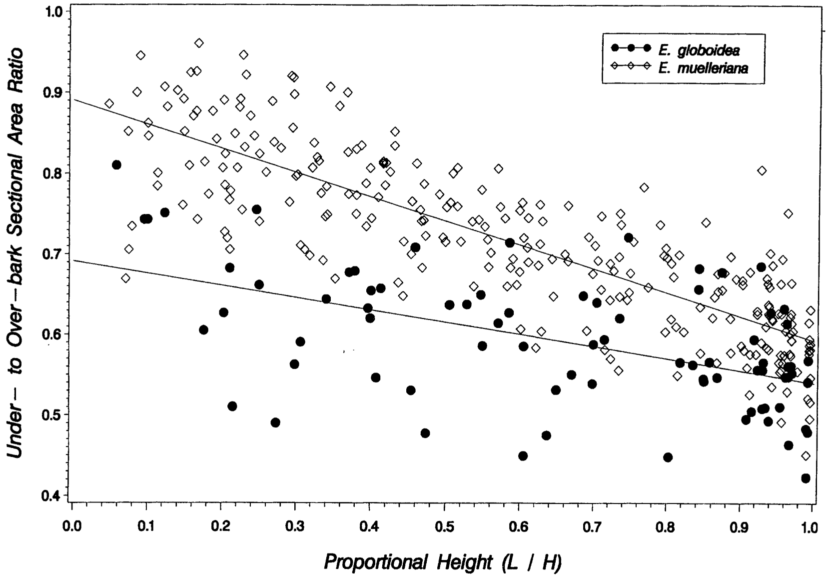

FIG. 1–Under- and over-bark sectional measurements used to show the sectional area ratio $(dub/dob)^2$ decreasing with proportional length from the tree tip ($L/H = 0$) to ground level ($L/H = 1$). The relationship differs in both rate and level for *E. muelleriana* and *E. globoidea*, *E. globoidea* having thicker bark.

β_c was used to condition the equation to predict the actual diameter at breast height and is given by:

$$\beta_c = \frac{(1-\frac{1.4}{H})^2 - \beta_2\,(1-\frac{1.4}{H})^{\beta_3}}{(1-\frac{1.4}{H})^{\beta_1}} \tag{3}$$

Coefficients with standard errors are listed in Table 6.

TABLE 6–Coefficients and coefficient standard errors of taper equation.

Sample	β_1	β_2	β_3
E. pilularis	1.487 (se. 0.045)	0.617 (se. 0.019)	21.836 (se. 1.129)
E. muelleriana and *E. globoidea*	1.719 (se. 0.044)	0.414 (se. 0.019)	18.981 (se. 1.438)

A rearrangement of Eqn 2 predicts diameter over-bark as:

$$dob = \sqrt{\left[\frac{dbh\,H}{H-1.4}\right]^2 \left[\beta_c\left(\frac{L}{H}\right)^{\beta_1} + \beta_2\left(\frac{L}{H}\right)^{\beta_3}\right]} \tag{4}$$

Examination of the *dob* residuals showed the *E. pilularis* over-bark diameters to be well predicted. Some indication of different stem shape was seen when the *E. globoidea* and *E. muelleriana* residuals were compared. The equation appeared to under-estimate the *dob* of the *E. muelleriana* sample over the middle section of the stem. For the *E. globoidea* sample, *dob* was under-estimated over the section of the stem from approximately 60% to

80% of tree height. Mean and standard deviation of over-bark diameter residuals for the two fitted equations are given in Table 7.

The different taper curves by species, with over- and under-bark diameters, are illustrated in Fig. 2.

TABLE 7–Mean and standard deviation of *dob* residual.

Sample	Standard deviation (cm)	Mean (cm)	No. of Observations
E. pilularis	1.792	0.049	666
E. muelleriana and *E. globoidea*	2.035	1.022	345

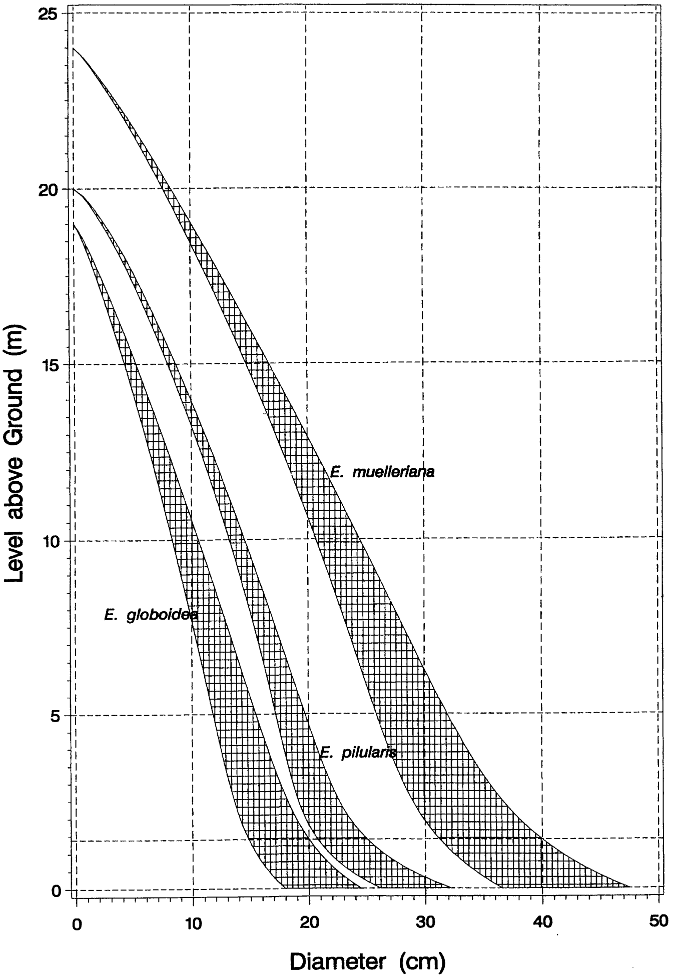

FIG. 2–Over-bark and under-bark taper curves for *E. globoidea* (*dbh* = 20 cm, H = 19 m), *E. pilularis* (*dbh* = 25 cm, H = 20 m), and *E. muelleriana* (*dbh* = 40 cm, H = 24 m). Shaded areas indicate the double bark thickness, boundaries represent *dub* and *dob* predicted from Eqn 1 and 4. Tree *dbh* and H were selected, after ranking the sample means, to illustrate taper curves for all three species.

Calculating Section Volumes

The under-bark volume (m^3) of a section of stem from the tip down to point L_1 can be predicted by integrating the product of the taper equation and the bark equation.

$$Vub_{L_1} = \frac{\pi\, dbh^2\, H^2}{40000(H-1.4)^2}\left[\frac{a_0\,\beta_c\,L^{\beta_1+1}}{H^{\beta_1}(\beta_1+1)}+\frac{a_0\,\beta_2\,L^{\beta_3+1}}{H^{\beta_3}(\beta_3+1)}+\frac{a_1\,\beta_c\,L^{\beta_1+2}}{H^{\beta_1+1}(\beta_1+2)}+\frac{a_1\,\beta_2\,L^{\beta_3+2}}{H^{\beta_3+1}(\beta_3+2)}+\frac{a_2\,\beta_c\,L^{\beta_1+3}}{H^{\beta_1+2}(\beta_1+3)}+\frac{a_2\,\beta_2\,L^{\beta_3+3}}{H^{\beta_3+2}(\beta_3+3)}\right]_0^{L_1} \quad (5)$$

Usage of the Taper Equations

The taper equation for *E. pilularis* appears to fit the data well, showing little bias and having a small standard error. The combined *E. muelleriana, E. globoidea* equation is less satisfactory and the data indicate that individual equations may be warranted. Until further sectional measurements are collected this combined equation should be adequate as an interim solution.

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