# INDIVIDUAL-TREE GROWTH MODEL FOR PINUS RADIATA

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ABSTRACT

This individual-tree distance-dependent growth model predicts the diameter, height, and crown height development of a simulated plot of **Pinus radiata** D. Don trees. The model provides estimates of stand development within approximately ± 5% but fails to predict mortality of small suppressed trees adequately. Further refinement of the mortality and diameter increment function of the model will provide more accurate diameter and height distribution data.

## INTRODUCTION

Individual-tree models have received considerable attention in recent years (Newnham 1964; Mitchell 1969; Arney 1972; Goulding 1972; Stage 1973; Daniels & Burkhart 1975). Such models predict the growth of the individual trees in a plot, often based on the use of competition indexes such as those of Newnham (1964), Opie (1968), Gerrard (1969), Arney (1972), and Hegyi (1974), whose work developed from Staebler (1951). A suitably robust individual-tree model would be of great use to research workers in New Zealand.

There are two stand growth models (Beekhuis 1966; Elliott & Goulding 1976) for predicting the yield of *Pinus radiata* stands in Kaingaroa State Forest. This paper describes a distance-dependent individual-tree simulation growth model (Munro 1974) based on Kaingaroa data.

## DATA AVAILABLE FOR MODEL DEVELOPMENT

Thirty-three plots evenly distributed over site and age were selected from New Zealand Forest Service research plots located in Kaingaroa State Forest. Within age and site index classes, spacing was varied when possible. Plot size varied from 0.04 to 0.2 ha. Each plot was mapped to show tree locations, and the diameter at breast height of each tree was measured during winter in 1977, 1978, and 1979. For between one and five normal well-formed trees in the centre of each plot, height and height to the green crown were also measured. Each sample tree was selected only if there were enough surrounding trees in the plot to enable an estimate of competition to be made. With two increment periods for each tree and a total of 150 sample trees, 300 growth observations were available. The data, summarised in Table 1, were screened to remove such obvious errors as negative or excessive increments.

TABLE 1—Summary of data available

| Variable | Mean | Maximum | Minimum |
|---|---|---|---|
| D.b.h. (cm) | 38.2 | 72.3 | 10.1 |
| Height (m) | 28.8 | 49.0 | 10.1 |
| Crown ht (m) | 13.9 | 31.9 | 1.6 |
| Stocking (stems/ha) | 252 | 4236 | 40 |
| Site index* (m) | 30 | 37 | 25 |
| Age (yr) | 21 | 39 | 10 |

* Site index is defined as the predominant mean height at age 20, as predicted by Burkhart & Tennent (1977)

Data from 17 unmapped Forest Service plots were also made available for this study. These plots could not be used for the estimation of model coefficients and were intended to test the model when it had been developed.

The diameters of trees in some plots could be back-dated several years from the past measurements available for most of the 50 plots. These measurements also provided some data on mortality, as the position of dead trees had been noted in the mapped plots.

## MODEL STRUCTURE

The model developed simulates the growth of a plot of trees in a rectangularly planted stand. To improve computational efficiency the planting positions are assumed to be on a perfect grid. Initial diameters are generated at age 6 years from the Weibull distribution of Bailey (1974). Heights are assigned from a height-diameter regression, with planting mortality assigned by Bernoulli trial, and crowns extending to the base of the tree are assumed. Annual growth is estimated from regression equations developed from the data, with a Bernoulli trial determining survival based on the value of Hegyi's (1974) competition index. Thinning and pruning operations may be simulated annually. The model uses the translation technique of Mitchell (1969) to counter edge effects, as recommended by Monserud & Ek (1974). Detailed descriptions of Bailey's (1974) Weibull distribution and the functions developed for use in the model are given in the Appendix.

### Competition Index

A competition index was required to estimate the effects of tree size and spatial distribution. An initial screening was carried out on the indexes of Staebler (1951), Bella (1970), Gerrard (1969), Hegyi (1974), and Arney (1972). The value of each index was calculated for the sample trees for two increment periods. A saturated quadratic linear regression model containing diameter at breast height and competition was calculated for each index. The contribution of each index was examined as the additional variance explained over the restricted model, containing diameter and diameter-squared only, and was significant at the 0.001 level for all. There was little difference in contribution between the indexes, and those of Gerrard (1969) and Hegyi

(1974) were selected for further examination. As suggested by Daniels & Burkhart (1975), an angle count methodology was used to determine potential competitors for Hegyi's index. This is analogous to the radius factor of Gerrard's competition quotient, and Tennent (1975) showed the importance of correctly choosing such a radius factor.

To examine further the properties of the two indexes a response surface analysis was conducted for each index, with the radius factor as one variable and an imposed maximum distance to a competitor as the other variable. The coefficient of multiple determination of the following diameter prediction regression was used as the response variable (the variables are defined in the Appendix).

$$\Delta g = \beta_1\, d + \beta_2\, c.d^{\frac{1}{2}} + \beta_3\, (d/c)^{\frac{1}{2}}$$

Values of competition were calculated for 16 combinations of distance and radius factor, and the corresponding uncorrected $R^2$ values were also calculated. The response surface was plotted and proved to be ridge-shaped for each competition index, with increased radius factor being required for increased maximum distances. To enable all developmental plots to be fully searched, a maximum distance of 18 m was selected. The corresponding radius factor was estimated from the response surface for future use.

The analysis showed that both indexes reached similar maxima, although the radius factors and maximum distances differed. As similar maxima were obtained, both indexes were calculated for all subject trees in the 33 plots, and during the development of prediction equations for the annual growth of the trees both indexes were included. However, competition quotient did not show any advantage in predictive ability over Hegyi's index and was dropped because of the greater complexity of its calculation.

### Tending Operations

Two of the most important tending operations carried out in Kaingaroa Forest are thinning and pruning, the trees being selected partly on the basis of their quality and partly to maintain an even spacing. A heavy selection intensity usually leads to a more random selection, as some small trees must be pruned or large trees felled to achieve the stated targets.

A heuristic selection procedure is used in the model as there were insufficient data available to construct a tending subroutine analytically. When less than 50% of stems are to be selected the selection is made by Bernoulli trial with the probability of selection proportional to the diameter of the tree. Any trees in excess of 50% are selected with uniform probability, regardless of size.

Pruning is simulated in the model by raising the green crown level to the specified pruning height. The effect is felt through the diameter and crown increment functions. No pruning shock is simulated. Only trees pruned on all previous occasions are considered in future prunings. A row thinning operation was also included in the model, where every $n^{th}$ row may be removed.

## VALIDATION

The validation of the model consisted of statistical validation of each function developed, with associated examination of residual patterns, validation of the model's over-all structure, and comparison between predicted and observed development of the 17 validation plots.

The model's over-all structure was examined by repeated running at a wide variety of initial conditions. This disclosed several program errors which were corrected, and one functional deficiency. The height increment function produced predominant mean height growths which varied greatly with stocking and thinning. The function was amended to ensure plot height development was closer to the site index curves of Burkhart & Tennent (1977). The function is:

$$\Delta h = \frac{dH}{dt} e^{.139(C-c)}$$

Further examination of the model showed a general tendency for predicted predominant mean height development to follow the site index curves, with reduced height growth at very low stockings.

The model was next compared with the validation plots. Each validation plot's growth was simulated six times, to obtain the mean effect of the several stochastic elements with a standard error of less than 1%. As the model simulates growth starting at age 6, only the nine plots with ages near 6 years could be simulated; plots with their first measurement later had undergone indeterminable treatment which could not be simulated. These remaining eight plots did provide information up to age 40 which indicated that the model's structure was consistent with these older plots.

Examination of the nine plots available for direct comparison showed the procedure over-estimated mortality. As there were no further data available, the coefficients of the probit line were adjusted iteratively until predicted mortality was close to observed mortality of the 33 developmental plots. The modified probit line is:

$$p = 1.6 + 0.18c$$

The model was then rerun to simulate the development of the nine validation plots.

The comparison showed that the model provides realistic and consistent simulations of the validation plots. There was no consistent bias. Table 2 shows the number of plots which were under-estimated (within 5%) and over-estimated at the oldest observed age for each plot. Basal area is most variable; however, seven of the nine plots have a predicted final basal area within 10% of the observed. The stand data provided by the model appear reliable.

The model was next compared with one specific plot to examine the simulated diameter distributions. The plot was chosen because it included a thinning operation

TABLE 2—Summary of plot deviations

| Variable | Low | Within 5% | High |
|---|---|---|---|
| Stems/ha | 2 | 6 | 1 |
| Stand basal area | 3 | 3 | 3 |
| Mean d.b.h. | 1 | 7 | 1 |

and because the simulated growth was close to the observed growth. The distributions differed slightly, but showed the same general shape. The mode and range of the distributions was similar in both the observed and simulated distributions. However, the simulated distributions show greater negative skewness at higher ages. This is because the stochastic mortality function occasionally results in the survival of small diameter trees. Under high competition some trees have no predicted diameter growth, but survive long after an actual tree would have died. The actual volume and basal area involved are negligible, but this indicates that the mortality function and the diameter increment function need to be considered jointly.

## CONCLUSION

The individual-tree simulation growth model developed provides estimates of stand development close to the plots observed. However, the mortality function in conjunction with the diameter increment function have led to a tendency to over-estimate the number of small-diameter trees. This has little effect on the stand values predicted.

With further refinement of the mortality and diameter increment function the model will provide more accurate diameter and height distribution data.

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## APPENDIX 1

### FUNCTIONS USED IN THE MODEL

The following variables are used in the paper and this Appendix.

| | | |
|---|---|---|
| $b$ | = | height to base of green crown (m) |
| $c$ | = | competition of an invidual tree |
| $C$ | = | predominant mean competition |
| $d$ | = | diameter at 1.4 m (cm) |
| $\Delta$ | = | first difference function |
| $g$ | = | basal area per tree ($cm^2$) |
| $G$ | = | stand basal area ($m^2$/ha) |
| $h$ | = | tree height (m) |
| $H$ | = | predominant mean height (m) |
| $S$ | = | stocking (stems/ha) |
| $v$ | = | total stem volume inside bark ($m^3$) |
| $Y_{24}$ | = | twenty-fourth percentile of d.b.h. distribution |
| $Y_{93}$ | = | ninety-third percentile of d.b.h. distribution |

The functions developed for the model reflect the limitations of the data. There were many inaccurately measured heights and crowns, which resulted in excessive and negative increments. The need to maintain objectivity resulted in data sets with high variances, and low precision of parameter estimates. The development of the functions showed the need for a more accurately measured data set. Each function is discussed briefly below.

*Initial diameter distribution*

The initial stand is generated using Bailey's 1974 Weibull distribution. Diameters at age 6 years are assigned randomly by use of the inverse probability transformation technique. Bailey's distribution is calculated from the following system of metric unit functions.

$$\hat{Y}_{24} = -2.2857 + \frac{1828.36}{S} + 4.497\ ln_{10}\ (H)$$

$$\hat{Y}_{93} = -5.0717 + \frac{1611.11}{S} + 11.65\ ln_{10}\ (H)$$

$$\hat{\gamma} = \frac{2.2711}{ln_e \left[ \dfrac{\hat{Y}_{93}}{\hat{Y}_{24}} \right]}$$

$$\hat{\theta} = 3.6438\ \hat{Y}_{24}\hat{\gamma}$$

These parameter estimates lead to the following cumulative Weibull distribution.

$$F(x) = 1 - e^{-\hat{\theta} x^{\hat{\gamma}}}$$

*Initial height*

The initial height of each tree is assigned from the following height-diameter function. The function was conditioned to predict a height of 1.4 m for trees of zero diameter, and to ensure maximum height was close to predominant mean height. The parameters were estimated by non-linear least squares from 145 observations of height and diameter at age 6 extracted from the sample plot records.

$$h = 1.4 + 1.04\ (H - 1.4)\ (1 - e^{-0.242d})^{1.64}$$

The asymptotic $R^2$ value is 0.859.

*Mortality*

Only 14 trees could be identified as having died through competition. The competition index of each tree and its neighbours was calculated. The data were divided into five competition classes and used as the basis of a probit analysis, as described by Monserud (1976). Although there were very few mortality data available, the probit analysis resulted in a probit line with a non-significant $\chi^2$ (5) statistic between observed and predicted mortality of 1.39. The probit line is given below.

$$p = 1.539 + 0.593c$$

A Bernoulli trial based on $p$ establishes mortality in the model. The parameter estimates were re-estimated during the validation process, as described in the text.

*Diameter increment*

A multiple linear regression, weighted by the square root of the ratio of green crown length to total height, was used to estimate diameter increment. The regression was developed after examination of the variables available by factor analysis and subsequent regression analysis. Examination of residuals showed no distortion of fit. However the high variance in the data resulted in a low $R^2$ value. The regression estimated is shown below.

$$\Delta d = 2.614 + 1.38\ \frac{(h - b)}{h} - 0.527c - 0.0217d$$

$$R^2 = 0.536$$

$$n = 285$$

*Height increment*

The height increment data contained many errors. Some excessive growth and negative increments were omitted, but the resulting 102 observation data set contained a great variability. Examination of data plottings showed a strong relationship between height increment and competition, but no relationship between height increment and other variables. The variable chosen for predictions was the ratio of observed height increment to the differential of the height age function of Burkhart & Tennent (1977). An exponential decay model was indicated. However the negative increments prevented the model being fitted in a linearised form, and resulted in instability of parameter estimates in a non-linear form. As a result the data were divided into four competition classes and a linearised regression was calculated for the class means. When this regression was used for residual analysis there was little distortion of residuals but some evidence of non-homogeneity of variance. The parameter estimates were retained. The regression calculated was:

$y = 0.5419 - 0.139c$

$R^2 = 0.997$

$n = 4$

where $y = ln \left\{ \dfrac{\Delta h}{\frac{dH}{dt}} \right\}$

This function was replaced in the validation process, as described in the text.

*Crown increment*

Errors in height and crown height measurement resulted in a restricted 39 observation data set for estimation of crown increment. Various theoretical models were examined to predict crown height. However, the variability of the data resulted in unsatisfactory parameter estimates for such models. Finally stepwise regression was used to produce a variety of functions. After inspection for biological consistency, the following regression function was chosen.

$\Delta c = 0.0577\ (h - b) + 0.1768\ \Delta h.c$

$R^2 = 0.694$

When corrected for the mean, $R^2 = 0.124$

*Volume function*

The model predicts diameter and height growth only. Volume is estimated using an equation developed by Goulding & Murray (1976) for young-crop radiata:

$v = (0.000025934d^2 + 0.0013407)h$