GRADING RANDOM-WIDTH LUMBER BY COMPUTER

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ABSTRACT

A computer program, FLGRADE, has been developed that can grade random-width factory lumber to the Western Lumber Grading Rules. FLGRADE uses data obtained from digitised boards. Board profile (including wane) is represented using a series of line segments which combine to form a polygon. All other defect data are represented using diagonal co-ordinates representing the smallest encompassing rectangle. The algorithm creates a list of cuttings by considering areas of placement. Using dynamic programming, an initial two-stage solution is generated based on the rip-first cutting procedure. Alternative solutions which allow more flexible cutting sequences (i.e., either rip first or cross-cut first) and which do not restrict the number of stages are then generated to determine whether a higher percentage of cuttings, and hence a higher grade, can be obtained.

Grades generated by FLGRADE for a set of boards for which defect data had previously been digitised, resulted in 80% receiving the same grade as had been manually assigned. Of the remaining boards, approximately 6% received a higher grade and 14% a lower grade. All computer assignments agreed with manual assignments within one factory lumber grade.

Keywords: computer grading; WWPA factory lumber grades; ripping; cross-cutting; remanufacturing; dynamic programming; heuristic method.

INTRODUCTION

Sawn lumber exports from New Zealand (NZ) to the United States (US) market have increased markedly over the last few years. Substantial volumes, intended for subsequent remanufacturing, are sold as random width lumber, graded to the US Western Lumber Grading Rules. As both random-width sawing, and the grading of random-width lumber are relatively new practices to New Zealand, producers of random widths must acquire skills in these areas to compete effectively in the market. To this end, the Ministry of Forestry and the New Zealand Forest Research Institute have provided Random-width Sawing Workshops, in addition to producing a series of publications directly relating to these practices. These are intended to facilitate the learning of a set of rules which are substantially more intricate than their New Zealand softwood grading rule counterparts.

Random-width factory lumber is intended for remanufacturing into door and window frames, moulding, and millwork items. With the Western Lumber Grading Rules (WWPA

1991, 1993), factory lumber grades are defined by the percentage of cuttings which can be recovered from boards. Individual cuttings must satisfy both dimension and quality criteria. Alternative combinations of cuttings can be obtained from many boards, with some combinations yielding a higher grade than others. Realisation of the full grade potential of a board is thus dependent on the ability of the individual grader. The lack of repeatability and inconsistencies which can be attributed to manual grading practices can be removed by generating a solution by computer analysis of the problem.

Although computer programs which apply the US National Hardwood Lumber Association grading rules for lumber grading have been available for some time (Hallock & Galiger 1971; Klinkhachorn *et al.* 1988) it was not until recently that the Western Lumber Grading Rules received attention. A program which applies the WWPA grading rules for dimension, boards, and selects and finish categories has been developed using an expert system language (Zeng *et al.* 1996). Rules for these categories differ substantially from the factory lumber grading rules which assign lumber grade on the basis of the percentage of cuttings that could be recovered by ripping and cross-cutting.

Yield cutting programs which consider the rip-first and cross-cut first problems are well documented—RIPYLD (Stern & McDonald 1978), MULRIP (Hallock & Giese 1980), OPTYLD (Giese & McDonald 1982), CROMAX (Giese & Danielson 1983), CORY (Brunner *et al.* 1989), the ALPS yield optimisation cutting program (Klinkhachorn *et al.* 1989), and ROMI-RIP (Thomas 1995) are some examples. CROMAX uses a cross-cut first cutting strategy whereas RIPYLD, MULRIP, OPTYLD, and ROMI-RIP use a rip-first cutting strategy, and CORY considers both strategies. Several versions of CORY, each differing in the number and/or order of sawing stages (changes from cross-cut to rip or vice versa) have been developed, and subsequently applied to investigate the effect of stages on dimension yields. Anderson *et al.* (1992) noted significant yield improvements when the number of sawing stages was increased from two to three. Lesser yield improvements were obtained when a further stage was added.

For the above remanufacturing models, solution techniques take the form of either complete enumerative or heuristic procedures. These methods have two severe disadvantages—namely, a rapid increase in computation time with an increase in problem size, and questionable solution quality, respectively. These problems can be overcome by using dynamic programming (DP) techniques which, although commonly used in other forestry applications such as stand management (Brodie & Kao 1979), and the problem of cutting trees into logs (Pnevmaticos & Mann 1972; Briggs 1980; Maness & Adams 1991), have received little attention in the remanufacturing industry. Only one published application could be found where DP had been employed. In their paper, Carnieri *et al.* (1994) considered both rip-first and cross-cut first cutting of clear lumber and particleboards. Prior to this Carnieri *et al.* (1993) considered the problem of cutting lumber containing one defect, with the solution being generated by a heuristic method.

The object of many remanufacturing models is to compute an efficient cutting strategy based on a remanufacturer's cutting bill (a customer order for specific dimension parts). In contrast, the object of the model presented in this paper is to compute the potential recovery of clear cuttings which can be obtained between defects, by combinations of ripping and cross-cutting, in order to determine a grade for the lumber.

The algorithm presented here applies the WWPA grading rules for factory lumber. These stipulate not only the dimensions and percentages of cuttings required, but also the cutting strategy. For some grades the rules stipulate that "pieces are to be ripped full length before the ripped stock may be cross-cut". Yet for other grades exceptions to the rule, allowing pieces to be cross-cut first, are also provided. Two-stage cutting patterns, corresponding to the rip-first strategy, are generated using the principles of dynamic programming whilst general cutting patterns, which do not restrict the number of cutting stages, are generated using a heuristic procedure. As the requirements of the factory lumber grades, the concepts of cutting stages, and the principles of dynamic programming form the essence of the proposed algorithm, a brief description of each is given below. The remainder of the paper develops the proposed algorithm which is illustrated with an example, summarises the results of a comparison between computer-generated and manually assigned grades for 35 digitised boards, and discusses factors which may affect the accuracy of predictions.

Since this paper deals with WWPA grading rules, measurements quoted are imperial rather than metric.

Factory Lumber Grades

Factory lumber grade requirements are based on the percentage of potential cuttings which can be obtained from a board. Cuttings in turn are based on door and sash components, and consist primarily of stiles, rails, and muntins (Fig. 1). Lumber grades considered in this paper are: Mouldings, Factory Select, No. 1 Shop, No. 2 Shop, No. 3 Shop, and Finger Joint Common Shop. A summary of the main grade requirements for 5/4" and thicker boards (extracted from WWPA 1991) is given in Table 1.

No. 1 cuttings are clear on both sides, whereas No. 2 cuttings may admit minor imperfections. Cutting dimensions are illustrated in Table 2.

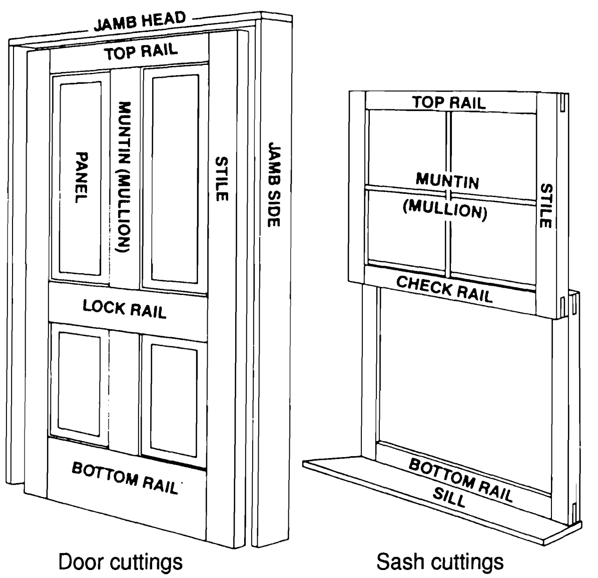

FIG. 1–Door and sash cuttings (reproduced with permission from WWPA 1991)

TABLE 1–Summary of grade requirements for 5/4" and thicker factory lumber

Lumber grade	Grade requirements *Each board must contain at least …*
Mouldings	2/3 of the surface area in clean Moulding Rips Up to 10% of a consignment may contain Short Moulding Rips
Factory Select	70% of No. 1 door cuttings. Subject to a maximum of 2 muntins; no board may contain muntins only.
No. 1 Shop	50% of No. 1 door cuttings. Subject to a maximum of 2 muntins
No. 2 Shop	25% of No. 1 door cuttings, or 33.3% of No. 1 and No. 2 door cuttings, or 40% of No. 2 door cuttings
No. 3 Shop	30% of any combination of No. 1, No. 2 door cuttings, sash, jamb and sill cuttings, and moulding rips.
Finger Joint Common Shop	50% of finger-joint cuttings

TABLE 2–Dimensions of factory lumber cuttings

Cutting	Length	Width
Moulding rips	10' and longer	1" and wider
Short moulding rips	6' to 9'	1" and wider
Stiles	80" to 90"	5" or 6"
Bottom rails	28" to 36"	9" or 10"
Muntins	42" to 48"	5" or 6"
Top rails	28" to 36"	5" or 6"
Sash	28" and longer	$2^1/_2$", $3^1/_2$", $4^1/_2$" and wider
Jamb and Sill	36" and longer	5" and wider
Finger-joint	9" and longer	$2^1/_2$" and wider

Each cutting is given a tally (board measure). This is calculated by multiplying the thickness (in inches) by the width (in inches) by the length (in feet), and dividing by 12. Board footage for the whole board is calculated in the same way, except that only whole board feet are measured. Board footage may also be adjusted when wane is present, by subtracting the tally associated with those unusable portions of a board. This is commonly referred to as "scaling off". For Moulding grade lumber, wane exceeding 10% of the area of the board is scaled off. For Shop grade lumber, wane exceeding 5% is scaled off. Boards requiring more than 50% scale-off are not permitted in either grade.

Cutting Stages

Staged cutting refers to guillotine cuts made in successive stages. Examples of two-stage and three-stage guillotine cuts respectively are shown in Fig. 2(a) and (b), whilst a general guillotine example is given in Fig. 2(c). As early as 1966, Gilmore & Gomory showed how cutting patterns could be generated for staged guillotine cuts using dynamic programming recursions.

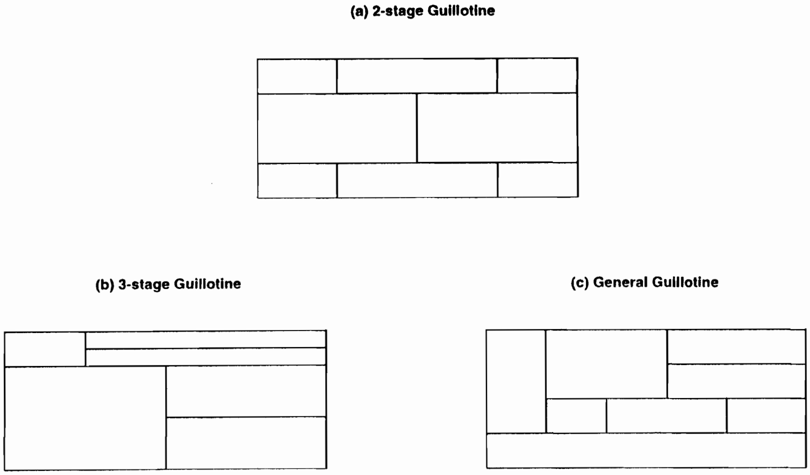

FIG. 2–Guillotine cutting patterns

Dynamic Programming

Dynamic programming is a technique that can be applied to solve many optimisation problems, and applications can be reduced to finding the shortest, or longest, path joining two points in a given acyclic network. For large networks, the computational efficiency of evaluating the shortest (longest) path far exceeds that of explicitly enumerating all possible routes. DP overcomes the disadvantages associated with exhaustive enumerative and heuristic approaches beacause it shortens the computation time required for finding guaranteed optimal solutions. There are many textbooks devoted to DP to which the interested reader should refer. Examples include those of Dreyfus & Law (1977), Dernado (1982), Nemhauser (1966), and Winston (1991).

METHOD

Potential grades are influenced by both the location of defects and the cutting pattern applied to that board. For a computer algorithm, the representation of board and defects must also be considered. Many remanufacturing models mathematically describe board defects as rectangles (denoted by pairs of diagonal co-ordinates) enclosing the periphery of that defect. This representation is also adopted in this paper for all defects other than wane. Wane, or lack of it, is represented using a series of line segments which combine to form a polygon. This representation allows a more accurate representation of the board profile. This is important as wane influences "scaling off" and hence the board footage calculation upon which percentage cuttings are based. Defect positions are recorded on both faces and a "glass" board, with defects of both faces visible, is created (Fig. 3).

Potential lumber grades are tested sequentially, starting from the highest quality factory lumber grade (Mouldings) and working downwards. An outline of the grade assignment procedure is shown below.

Step 0: Lumber grade ← unassigned

Step 1: Is lumber of Mouldings grade potential?
If true then assign grade and go to step 4.

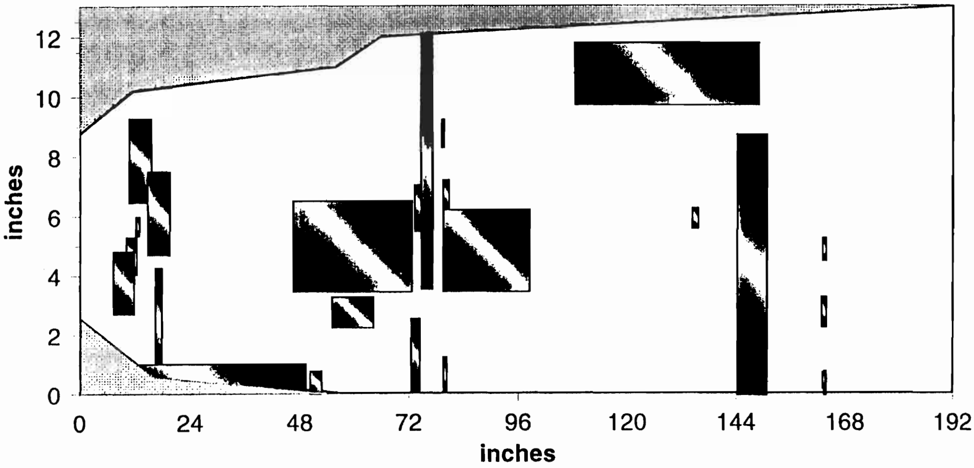

FIG. 3–Board showing defects

Step 2: Is lumber of Shop grade potential?
If true then assign grade and go to step 4.

Step 3: Is lumber of Finger Joint grade potential?
If true then assign grade and go to step 4.

Step 4: Stop.

Moulding stock is obtained by ripping the lumber into strips 1" and wider, 10' and longer, with areas of wane and defects being removed by cross-cutting the strips. This process is depicted by the two-stage guillotine cutting patterns of Fig. 2. Shop grade stock and finger-jointing stock may also be obtained in this manner. In addition, for shop grade stock, provision is made (WWPA 1991) for cross-cutting first when a higher grade can be obtained. This is depicted by the general guillotine cuts of Fig. 2. In short, moulding and finger-jointing stock may be obtained only by a rip-first methodology whereas shop grade stock may be obtained by either rip-first or cross-cut first methodologies.

In this paper, a solution to the rip-first (two-stage guillotine cutting pattern) problem is obtained using DP techniques. The DP procedure serves two purposes as it not only determines an optimal solution to the rip-first problem but also creates a list of all clear cuttings. This list is then used in conjunction with a best-fit decreasing heuristic procedure to generate general guillotine cutting patterns, and hence alternative solutions for shop grade stock. The highest grade obtained is assigned to the board. Both methods are described below.

Rip-First Solution Procedure

The rip-first solution procedure generates two-stage guillotine cutting patterns and is solved using DP. The DP procedure is divided into three main parts. Firstly, potential ripping positions are superimposed, at equidistant intervals, on the board. Then, at each position the location and tally of all clear cuttings which are of the required dimensions (*see* Table 2) are determined and added to a list. Cuttings within the list may overlap one-another or even be completely contained within another cutting. Finally, an optimal solution is obtained using recursion.

In DP nomenclature, the equidistant intervals are equivalent to stages. For this problem, there is a one-to-one correspondence existing between stage and state. The problem of determining the combination of rips and cross-cuts (in that order) that will yield the greatest tally corresponds to finding the longest path from the first level to the last. This may be determined as follows:

Suppose that the board is divided (lengthwise) into n levels, each corresponding to a potential ripping position, and suppose that cuttings may be any of m widths $w_1, w_2, \ldots, w_m$. Further, let c_{ij} be the total tally of all feasible clear cuttings of width w_i ($i = 1, 2, \ldots m$), ripped such that the lower edge of the cutting is allocated at level j ($j = 1, 2, \ldots n$).

Then the longest path can be found by applying the following recursive relationship:

$$f(j) = \max_{i=0\ldots m} \{ c_{ij} + f(j+w_i) \} \qquad 0 \leq j+w_i \leq n$$

Here, f(j) is the optimal policy function at level j. The initial condition for the problem is

$$f(n) = 0$$

To enable gaps between consecutively ripped pieces, an artificial width w_0, equal to the discretisation interval, with a zero valued tally, is introduced. The solution procedure moves backwards stage by stage, each time finding the optimal policy for that stage, until it finds the optimal policy starting at the initial stage. Total tallies are calculated for each of the cutting types (e.g., No. 1 door cuttings, No. 2 door cuttings) and simultaneously optimised.

Alternative Solution Procedure

The list of clear cuttings generated in the course of the DP procedure can be used to generate general guillotine cutting patterns. These may be of either the cross-cut first type, or the rip-first type. Here, rip-first solutions may differ from those generated using the above DP procedure as the number of stages is no longer restricted to two. Alternative solutions are generated using a best-fit decreasing procedure. This is a heuristic procedure which places feasible cuttings within the board in order of decreasing weight (tally). Because this process will return a local solution for the problem, which may differ from the global solution, the problem is repeated using a number of different starting solutions. The procedure is outlined below.

Best-fit Decreasing Procedure

Step 0: {Initialisation}
bestgrade ← unassigned
maxsearch ← No. of different starting solutions
i ← 1

Step 1: Mark all cuttings in list as unscanned

Step 2: Select cutting with i^{th} highest tally

Step 3: Repeat
Mark all overlapping cuttings as scanned
Select cutting with highest tally
Until all cuttings are scanned

Step 4: Determine grade using selected cuttings

Step 5: if grade better than bestgrade then bestgrade ← grade

Step 6: i ← i + 1

Step 7: if i <= maxsearch then go to Step 1
else go to Step 8

Step 8: Stop

Example

The board of Fig. 3 scales 16 board feet. The algorithm first determines whether the board is of Mouldings grade potential. As no moulding rips can be obtained, the grade cannot be achieved, thus potential Shop grade is then considered. Application of the DP recursion to the two-stage cutting pattern yields the following solution (Fig. 4) to the rip-first problem:

- 2 × top rails (yielding a total of 2.19 board feet)
- 4 × sashes (yielding a total of 3.27 board feet)

This gives a total of 5.46 board feet in cuttings; thus the board attains a No. 3 Shop grade as it contains more than the required percentage of cuttings (30% of any combination of No. 1, No. 2 door cuttings, sash, jamb and sill cuttings, and moulding rips). Note that this solution is not unique. An example of an alternative solution where the same grade is obtained is shown in Fig. 5. This contains the following cuttings:

- 2 × No. 1 bottom rails (yielding a total of 3.94 board feet)
- 2 × No. 1 sashes (yielding a total of 1.23 board feet)

This gives a total of 5.27 board feet and thus also attains No. 3 Shop grade.

The algorithm then determines whether a better grade can be obtained by applying a general cutting pattern. Application of the best-fit decreasing procedure results in:

- 2 × No. 1 bottom rails (yielding a total of 4.13 board feet)
- 3 × No. 1 sashes (yielding a total of 1.89 board feet)

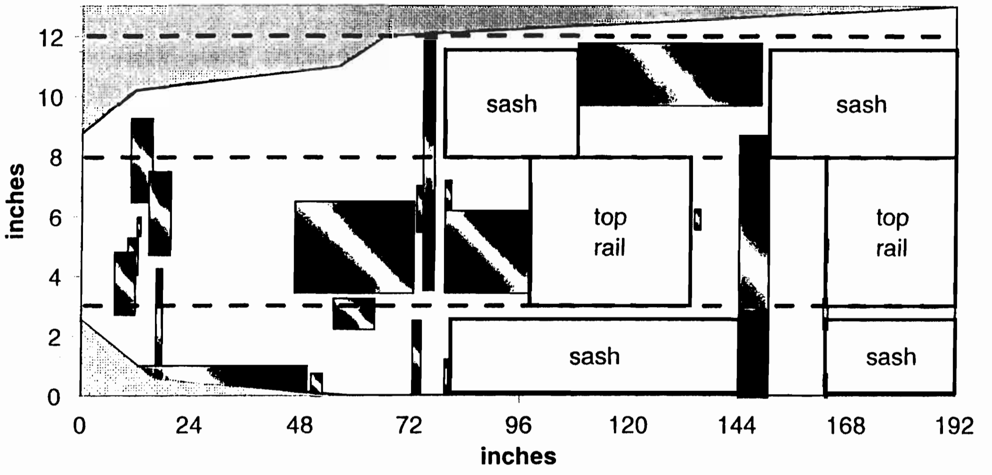

FIG. 4–Solution to rip-first problem

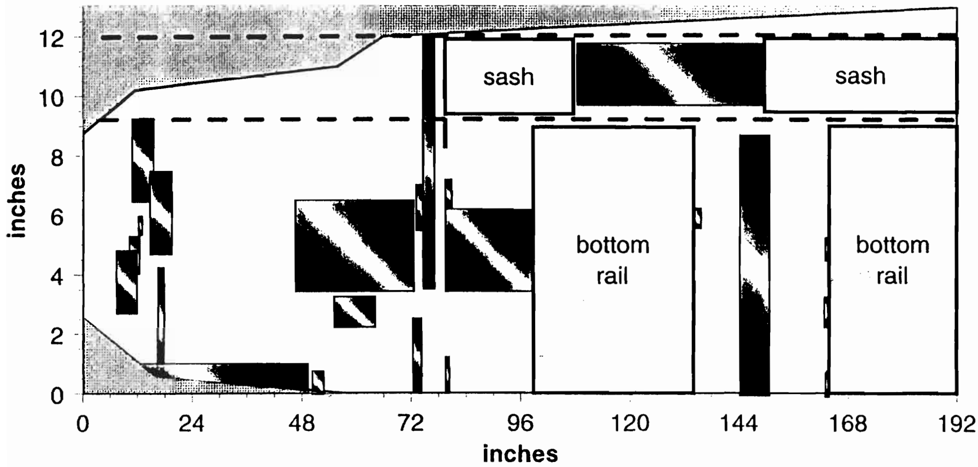

FIG. 5–An alternative solution to rip-first problem

The board thus attains No. 2 Shop grade as more than the required 25% of No. 1 door cuttings has been achieved. The cuttings obtained by this method are shown in Fig. 6. This solution is an example of a three-stage guillotine cutting pattern and is obtained by a cross-cut first operation. This is followed by ripping, and then further cross-cutting. Having assigned a No. 2 Shop grade to the board, the computer program then terminates.

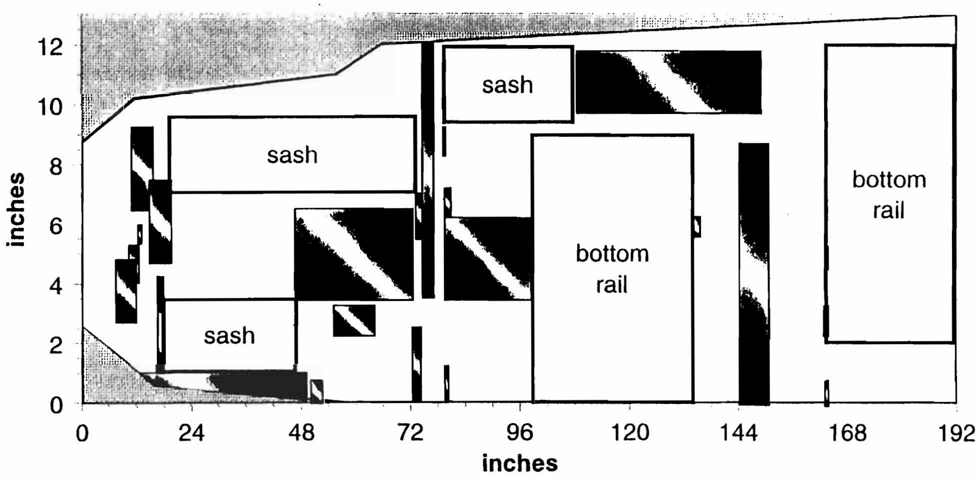

FIG. 6–Solution to the general cutting problem

Grade Comparison

Thirty-five boards which had been manually graded were used as a benchmark for the performance of the computer algorithm. The boards were obtained from five 4.9-m sawlogs, sawn using a cant sawing pattern which allowed 5/4" boards to be produced. Prior to being digitised, the boards were dried, surfaced, and manually graded. The board data files obtained from this process contained a description of all defects on both faces of the board.

This description includes: defect position (represented by a surrounding rectangle), defect type, and face of board on which defect is exposed. Defect types included bark pockets, knots (including spike, loose, intergrown, partially intergrown, and tightly encased knots), pith, sloping grain, resin pockets, and needle fleck. Wane data from the board files, together with sketches of the board wane, were used to reconstruct the board profile.

The computer algorithm was applied to the 35 board descriptions. Using five local searches in the best-fit decreasing procedure, and a 1-inch step interval, the grade distributions of Fig. 7 were generated. A comparison of computer *v.* manual grades on an individual board basis (Table 3) revealed that 28 of the 35 boards recorded the same grade, two recorded a higher grade, and five recorded a lower grade. When the grades differed, the grade shift was by one grade only.

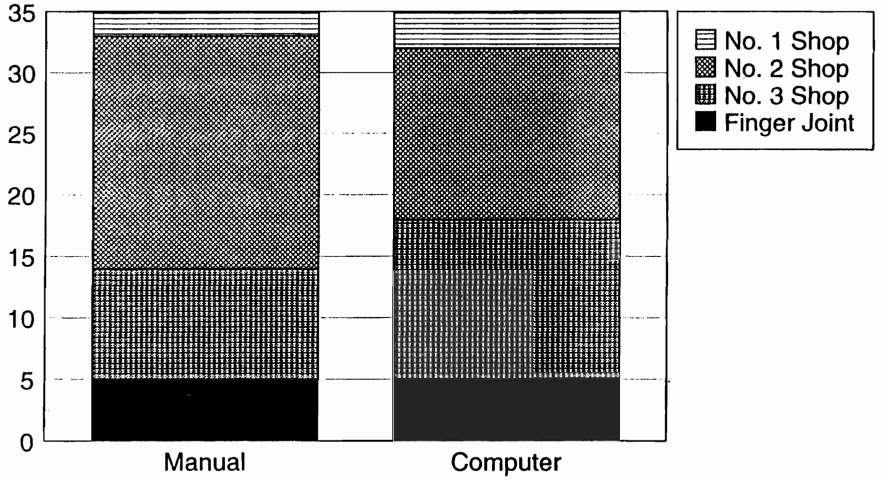

FIG. 7–Grade comparison of 35 boards

TABLE 3–Number of grades obtained by each method

Manual grade	Computer grade				
	No. 1 Shop	No. 2 Shop	No. 3 Shop	Finger Joint	Total
No. 1 Shop	2				2
No. 2 Shop	1	13	5		19
No. 3 Shop		1	8		9
Finger Joint				5	5
Total	3	14	13	5	35

DISCUSSION AND CONCLUSION

Results of this study indicate that the computer program, FLGRADE, can be used as a predictor of WWPA factory lumber grades for random-width boards. The accuracy with which the grades can be predicted is dependent on a number of factors. Those which are thought to be more influential than others are summarised below:

- Manual methods are susceptible to subjective grading.
- Board data are manually digitised.

- Defects (excluding wane) are represented using rectangles enclosing the periphery of the defect.
- The board is divided into equidistant levels, i.e., discrete intervals rather than a continuous function.
- Except for top rails which are counted as No. 2 cuttings, all other cuttings generated by the algorithm were of No. 1 cuttings quality.

Manual grading was done with the utmost care; however, because of the variable human element, there is still the possibility that errors of judgement were made. Errors may occur when the operator is visually assessing areas for placement of cuttings, or when determining the best combination of cuttings, or when determining the percentage of board area occupied by unacceptable characteristics. For example, had the unacceptable area been judged to occupy more than 5%, then scaling-off would have been permitted and a higher (Shop) grade may have been indicated. The fact that the grader can visually access only one surface at a time, and hence may overlook defects or make an incorrect assessment of a defect's location, may also influence the final grade. Another possibility is that some minor blemishes may have been recorded as No. 2 cutting quality, yet these blemishes may not have been digitised—thus the computer would grade the cutting as No. 1 cutting quality.

With the computer algorithm, the element of variability in the decision process is removed. Assignment of cuttings, and hence grades, is nonetheless dependent on both board data and solution methodology. In this study the board data were manually digitised. Complete digitisation, recording all defects, and accurate positioning of the co-ordinate system on the two board faces is required. Defect representation, using rectangles, should also be scrutinised. This simplistic representation may be sufficient for some defects but is less appropriate for others. Spike knots in particular may need an alternative representation. Finally, the computer assignment of a grade was based on a solution methodology which divided the boards into equidistant levels. If the spacing between levels is reduced, then the generation of a higher (or equivalent, but not lesser) grade may result if more clear cuttings are obtained. Note here that all clear cuttings, except top rails, were of No. 1 quality.

A further study, involving almost 400 boards, is planned. This study, in addition to providing greater scope for comparison, will address some of the above issues. Defect representation will also be examined.

It has been shown here that Dynamic Programming recursion can be used to solve the rip-first problem. As defects and clear cuttings can be considered, application of this technique to processing and remanufacturing problems should also be considered. When cross-cut first or rip-first cutting patterns which permit flexible cutting strategies (i.e., number of cutting stages not restricted) are required, the list of clear cuttings generated in the course of the DP procedure can be used to generate general guillotine cutting patterns.

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