



Validation of an individual-tree volume equation for *Nothofagus menziesii* (Hook f.) Oerst in Southland, New Zealand

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Abstract

A tree volume equation for New Zealand's *Nothofagus* species was validated using sectional measurements from *Nothofagus menziesii* (Hook f.) Oerst trees at Alton Valley in Southland. Upper stem diameters of 60 trees were measured using a Tele-Relaskop within three plots that had been thinned to stockings of 150, 1500 and 8228 stems ha⁻¹. Bark thickness measurements were obtained from a range of heights up tree stems. Merchantable under-bark volumes of stems calculated from sectional measurement were compared with merchantable volumes predicted by a model developed by Ellis (1979). The model was found to predict volumes with minimal bias even though errors were kurtotic and a little outside the range predicted. The correlation between predicted and estimated volumes was 0.97.

Keywords: Beech; stem volume; stocking; volume equation.

Introduction

Individual tree volume equations are fundamental for forest mensuration (Husch et al., 1972) because they are used to calculate under-bark volume; usually the most sought-after variable following forest inventory or modelling of future forest production. Volume equations vary with forest species and sometimes with site conditions and silvicultural treatments (Ellis, 1979; Gordon et al., 1999; Goulding & Murray, 1976).

The aim of this study was to estimate stem volume of silver beech (*Nothofagus menziesii* (Hook f.) Oerst) in an experiment (Franklin, 1981) in Southland, New Zealand in order to carry out an evaluation of the influence of thinning regimes on returns from timber production and carbon sequestration. Ellis (1979) created a series of volume equations for many forest species indigenous to New Zealand, including one for mature silver, red (*Nothofagus fusca* (Hook f.)

Oerst), hard (*Nothofagus truncata* (Col.) Ckn.) and mountain (*Nothofagus solandri* var *cliffortioides* (Hook f.) Poole) beech (Equation 1). Merchantable volume is denoted by "v" in Equation 1, and is predicted in cubic decametres. The equation was created from a study of 1316 trees that ranged in diameter at breast height (1.4 m above ground level) outside bark (*dbhob*) from 26.4 cm to 218.4 cm and in merchantable height (*h*) from 6.1 m to 25.6 m. Ninety-five percent of predicted individual tree volumes were within 21% of actual volumes during the fitting of the equation. While these estimates may be imprecise for individual trees, measurements of stand volume derived from them can be expected to be relatively free from bias and much more precise.

$$v = e^{1.983718\text{Log}(\text{dbhob})+0.752863\text{Log}(h)-2.263767} \quad [1]$$

Ellis' equation was derived for four species with data from Wellington, Nelson, Westland and Southland

regions, and so it was necessary to check that it would fit data from silver beech at the current study site. This was not a comprehensive validation of the original equation, but it provides an indication of the extent to which the general equation might apply to particular species or sites.

The objective of the study reported here was to compare volumes predicted using Equation 1 with actual volumes measured through sectional measurements of naturally grown silver beech trees in Southland thinned to a variety of stocking levels.

Materials and Methods

Site

All measurements were conducted in an experiment in the Alton Valley (46° 2' S, 167° 35' E), Southland, New Zealand. The site was clear-felled in 1951, with the exception of a few scattered seed trees. Successful natural regeneration resulted in initial stockings of at least 10000 stems ha⁻¹ (Dudley Franklin, personal communication). However, the regeneration was very patchy with thickets of saplings up to 50000 stems ha⁻¹ at 20 years of age, and gaps in between from logging tracks, debris from felled trees and waterfern/kiwakiwa (*Blechnum fluviatile* (R.Br.) Salomon). A thinning experiment was established in 1971 with 15 contiguous 0.2-ha plots. A second thinning was applied in 1980 with residual stockings that varied from 150 stems ha⁻¹ to 8228 stems ha⁻¹. Stands were thinned 'from below', which means that dominant trees were maintained as crop trees. The trial is described in more detail by Baker and Benecke (2001).

Measurements

Twenty trees from each of three thinning treatments (150 stems ha⁻¹, 1500 stems ha⁻¹ and 8228 stems ha⁻¹) were selected for the study. Trees ranged in *dbhob* from 9.5 to 44.8 cm.

Diameters outside bark were measured at 0.2 m (stump height, above which volumes were calculated) and 1.4 m above ground, as well as at approximately 1.5 m intervals using a Tele-Relaskop (Bitterlich, 1978) up to a merchantable height. The intervals were approximate because the nature of Tele-Relaskop measurements means that the exact measurement height was not known until after each measurement was taken and also because small swellings and defects in the stems were avoided. Merchantable height was generally defined by the occurrence of a fork in the stem, but in the absence of defects was defined as height to a 7 cm diameter.

Bark thickness was estimated from diameter using an equation developed by Herbert (1973) and also reported by Wardle (1984). The units reported by Herbert were

cm for diameter and mm for bark thickness, but this resulted in minute bark thicknesses uncharacteristic of silver beech. Wardle reported the same equation, but with the units both in cm. Discussions with both authors revealed that bark units were in cm. The equation was:

$$b = 0.01 + 0.011d \quad [2]$$

where *b* = bark thickness at breast height (1.4 m above the ground) and *d* = diameter and so it was necessary to assume that the relationship between diameter and bark thickness higher up the stem was the same as between bark thickness and *dbhob*.

Analysis

Volumes of trees up to a merchantable height were calculated using Smalian's formula (Husch, et al., 1972) for each section between diameter measurements, and with diameter measurements adjusted by the bark thickness model in Equation 2 so that they were inside-bark. These volumes were then compared with volumes predicted by Equation 1. Residuals were defined as measured volume minus predicted volume, and they were plotted against predicted values and tested for differences between stocking treatments. They were also tested for normality using the Shapiro-Wilk criterion.

A regression was constructed between predicted stem volume and measured stem volume with no intercept, and then we tested for significantly different regressions between stocking levels using stocking level as a class variable.

Results and Discussion

Volume predicted by Equation 1 was strongly correlated with measured volume ($r = 0.97$, $P < 0.0001$), and predicted volumes exhibited very little bias with respect to stem *dbhob* and predicted value (Figure 1). The mean residual was -0.0007 m^3 . Residuals were not normally distributed ($W = 0.9306$, $P = 0.002103$), but instead were strongly kurtotic (kurtosis = 1.63). Ten of sixty residuals were beyond the confidence limits suggested by Ellis (1979) of $\pm 21\%$ for single trees, and errors were heteroscedastic (Figure 2). Ellis used data obtained from destructive stem analysis, while this study employed diameter measurements made remotely from an optical device and in only one plane. This difference in methodology may explain the additional kurtosis observed in the current dataset, because indirect optical estimates, while unbiased, may have been more subject to errors than direct physical measurements made with callipers or tapes on felled trees. Regardless of methodology, the resulting calculated tree stem volume would be expected to produce kurtotic residuals in models, because it is a cubic quantity with a very high range of values. Heteroscedasticity and kurtosis could have

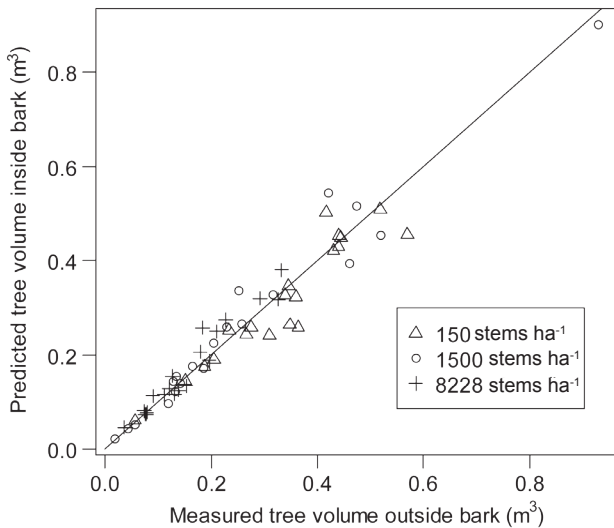


FIGURE 1: Predicted volumes from Ellis' (1979) equation versus measured volumes of individual silver beech trees at Alton Valley in Southland. The line goes through the origin and (1,1).

been avoided by using scaled power transformations during model construction, in which case when testing the model we would have employed any transformations specified by Ellis and also found less heteroscedasticity and kurtosis.

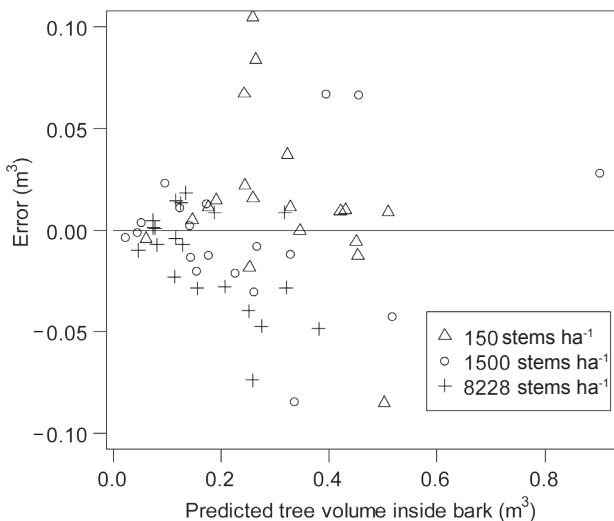


FIGURE 2: Errors (symbols) versus predicted (solid line) volume for individual silver beech trees at Alton Valley in Southland.

The regression between observed and predicted stem volume was of course highly significant, but this relationship was not quite significantly affected by stand stocking ($P < 0.066$). Adding stocking level

to the analysis as a class variable explained only 1% more variation than a model with only predicted stem volume, despite a hint of bias with respect to stocking in Figures 1 and 2. The probability of a type II error was therefore high, and Ellis' (1979) formula may be slightly biased with extremes of stocking even though we did not find a statistically significant bias. The slope of the regression was 0.986 and this value was not significantly different from 1.0 (standard error = 0.0174). A value of 1.0 for this coefficient would indicate no overall bias for Equation 1.

One large outlier had potential to exert large leverage, and so we repeated the regression without this datum and arrived at almost exactly the same result, with the slope = 0.989 (standard error = 0.0192).

Conclusions

Ellis' (1979) model of individual beech tree volume was found to have minimal bias and performed as expected for silver beech in Southland. Although residuals were kurtotic with more than expected outside the range given in reports of model fitting, the current authors concluded that they could proceed with a study of wood production and carbon sequestration without significant bias in estimates of merchantable volume.

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