DEVELOPMENT OF A COMPOSITE TAPER EQUATION TO PREDICT OVER- AND UNDER-BARK DIAMETER AND VOLUME OF *EUCALYPTUS SALIGNA* IN NEW ZEALAND

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(Received for publication 12 December 1995; revision 9 August 1996)

ABSTRACT

Data from 240 trees sampled throughout the climatic range of *Eucalyptus saligna* Sm. in New Zealand were used to develop and validate equations from which taper and bark-thickness could be estimated. Tree breast-height diameter over bark (*dbh*) and tree height were used as the predictor variables. Conditioning guaranteed that the taper equation would predict *dbh* as over-bark diameter at breast height. Combination of the equations produced a composite under-bark taper equation which can be integrated to derive under-bark volume estimates for any stem section, ensuring compatibility of taper and volume.

Keywords: taper; tree volume; *Eucalyptus saligna*.

INTRODUCTION

Eucalyptus saligna has been planted in New Zealand on a wide range of sites. It is most suited to the warm temperate areas of the northern half of the North Island, from Northland to Taranaki/Wanganui and Bay of Plenty/Hawke's Bay (Hay 1995).

Tree volume and taper equations are used to determine under-bark stem volume of whole trees from predictor variables such as breast-height over-bark diameter (*dbh*) and tree height (*H*). They can also predict volume, diameters, and taper of stem sections. These equations are basic components of stand inventory, growth and yield prediction, forest planning, and product simulation systems.

Currently the only tree volume equation applicable to *E. saligna*, derived from New Zealand-grown trees, is an unpublished, multi-species, merchantable volume table derived in 1961, intended for use in the National Exotic Forest Survey for estimates of merchantable volume of stands of mixed eucalypt species. This table includes only 10 *E. saligna* trees and under-estimates the sample volume of these trees by 12.3%. It does not cater for varying merchantability standards and does not incorporate any mechanism for predicting stem taper. Tree sectional measurement data have been collected from various locations in the North Island to derive a new taper equation specifically for *E. saligna*.

To calculate basic yield information, only estimates of total stem volume under bark (*vub*) are required, but modern yield and pre-harvest assessment systems such as MARVL (method for the assessment of recoverable volume by log type) (Deadman & Goulding 1978; Deadman 1989) require equations that can predict the volume and dimensions of any stem section from ground level to the tip of the tree. This functionality can be provided by combination of a total stem volume equation with an equation to predict a proportion of this volume as a function of top diameter (Clutter 1980; Shiver & Brister 1992). However, this type of solution does not allow for variation in stump height and is intended primarily for the prediction of volume of the whole extracted piece above stump height to a cut-off top diameter.

An alternative solution is the use of compatible volume and taper equations which have been successfully applied in New Zealand to a variety of species (Goulding & Murray 1976; Gordon 1983a; Katz *et al.* 1984; Hayward 1987). Compatible equations were initially developed by Demaerschalk (1972) as a means of producing consistent results when retro-fitting a taper equation to an existing local or regional volume equation, but in practice the compatibility constraints may compromise the predictive ability of the taper equation (Candy 1989).

The approach used here was to develop a composite taper equation which meets the requirements of both general growth and yield systems and the more detailed pre-harvest assessment systems, by predicting the volume and dimensions of any stem section. Tree diameter and height were used as the predictor variables. Although improvements in precision of volume prediction are possible by including additional stem diameter measurements as predictors (Bi 1994), the additional cost is difficult to justify in normal growth, yield, and pre-harvest assessments.

Tree diameter, height, and level above ground to any point on the stem (*h*) were used in an equation that described the over-bark profile of *E. saligna* stems, and also in an equation that predicted the ratio of under-bark to over-bark sectional area and hence the under-bark diameter. The combination of these two equations was then analytically integrated to produce an expression which gives under-bark volume estimates for any stem section. This composite approach allows a number of logical constraints to be incorporated into the model without compromising the simplicity and flexibility of the component equations.

DATA

Existing mensurational data from 12 locations were collated with data collected from three additional locations to give representative coverage of the climatic range of *E. saligna* in New Zealand. Sample trees were selected to cover the range of *dbh* at each location (Table 1).

The trees were measured for *dbh* (at breast height 1.4) and *H*, then over-bark diameters (*dob*) were measured at approximately 3-m intervals up the stem starting at 3 m above ground. Either one or two diameters were measured below breast height—at 0.7 m above ground, or at 0.15 and 0.7 m above ground.

Bark Subsample

In order to determine an accurate relationship between over- and under-bark diameters, a random selection of sample trees from Taheke, Kawerau, and Warkworth (43 trees

TABLE 1–Descriptive statistics of the sectionally measured sample trees by location.

Location	No. of trees measured*	Age (years)	Stocking (stems/ha)	Min. dbh (cm)	Mean dbh (cm)	Max. dbh) (cm)	Mean H (m)
Athenree A	6(1)	33	200	52	61	72	38
Athenree B	10	28	200	46	56	67	34
Athenree C	9	20	1500	8	18	26	21
Frankton	10	5	6666	5	9	15	13
Hawkes Bay	48	3	6000	3	6	9	7
Kawerau	34	9	1200†	4	16	37	16
Rotoehu A	11	14	1700	15	23	33	27
Rotoehu B	5(1)	14	1700	24	27	30	29
Rotoehu C	5	11	2240	9	13	16	17
Rotoehu D	9	23	1400	27	40	52	41
Rotoehu E	19(1)	11	1400	8	19	27	21
Silverdale A	8	25	400	31	42	67	29
Silverdale B	9(1)	25	400	30	44	67	28
Tairua	9(1)	18	1000	14	23	33	23
Taheke	39(1)	17	800	8	26	46	25
Warkworth	14	13	1000	29	35	40	26
Total	245(5)						

* The numbers of trees in parentheses were not used in the main data set.
† Kawerau sample trees were taken from a spacing trial: this value represents the average stocking rate.

altogether) was measured more intensively. Throughout this paper this group is referred to as the "bark subsample" as at each sectional point on the stem, diameter was measured before and after the bark was peeled off. This provided 359 observations of *dob* and under-bark diameter (*dub*).

Data Editing

All sectional measurements were run through a comprehensive set of computer edits to screen out possible measurement and recording errors. Trees with extreme or inconsistent measurements were removed. Graphical displays of tree profiles were compared with sample averages to select outliers and atypical trees for more detailed checking. A total of 240 trees was considered suitable for inclusion in the main data set. The range of *dbh* and *H* is shown in Table 2.

ANALYSIS

The analysis proceeded in two stages. First the bark subsample was examined to develop an equation to predict *dub* from *dob*, tree size, and the position on the stem. This equation

TABLE 2–Range covered by tree variables for the main data set and the bark subsample.

Variable		Minimum	Mean	Maximum
dbh (cm)	Subsample	4.000	22.000	46.000
	Main data set	3.000	26.000	72.000
H (m)	Subsample	7.000	22.000	34.000
	Main data set	4.000	27.000	47.000
Sectional *vub* (m3)*	Subsample	0.004	0.301	1.945
	Main data set	0.002	0.500	6.399

* *vub* = stem volume under bark

had to produce logical predictions of *dub* at any point on a stem over the range of tree sizes in the data set, and also had to be suitable for combining with a taper equation to predict the volume of arbitrary stem sections. The bark subsample used to derive this equation included only data where *dub* was measured directly (after removing the bark), to ensure that the error often associated with indirect measurement via bark gauges (Carron & McIntyre 1959; von Althen 1964; Gordon 1983b) was not incorporated in the equation.

The main data set was then used to derive a taper equation for predicting *dob* from the position on the stem and tree size.

Bark Equation

The ratio of the sectional area under-bark to the sectional area over-bark, $(dub/dob)^2$, varied little with tree size but showed a clear relationship with the height above ground. A variable L was defined as $H - h$, that is, the length between the tip of the stem and the measurement point. The under-bark to over-bark sectional area ratio was plotted over L/H, which represents the length from the tip as a proportion of tree height and ranges from 0 at the tip to 1 at ground level. The sectional area ratio increased slowly from $L/H = 0$ to approximately half-height, remained fairly constant until $L/H = 0.7$ then decreased at an increasing rate to ground level. A polynomial in L/H, restricted to two terms, was found to follow this pattern of change in the under-bark to over-bark sectional area ratio from tree tip to ground level. Residual analysis showed no trends with predicted values, predictor variables, or tree size variables.

When this polynomial was fitted by location, two groups of data emerged. The measurements from Taheke showed slightly thinner bark than those from Warkworth and Kawerau. Analysis of the residual error from fitting the polynomial as two curves and as a single curve (combined data), indicated that the differences were statistically significant (Table 3).

The bias and residual variation in the predicted value of *dub* with respect to the two groups of *dub* data were then calculated by re-arranging the polynomial fitted to the combined data (Table 4). The residual standard deviation was not large in the combined fitting and the bias proved to be too small to justify separate equations.

TABLE 3—Analysis of residual sums of squares for testing for differences in bark thickness by location.

Source	d.f.	Sum of squares	Mean square
Residuals about hypothesis model (single curve)	356	1.153 75	0.003 24
Residuals about maximum model (two curves)			
Taheke	183	0.370 93	
Warkworth / Kawerau	170	0.436 16	
	353	0.807 09	0.002 29
Difference	3	0.346 66	0.115 55

$F_{(3,353)} = 0.115\ 55 / 0.002\ 29 = 50.5$

TABLE 4—Residuals from Equation 1 for subdivided and complete *dub* data sets.

Group	No. of observations	Residual mean (cm)	Residual s.d. (cm)
Warkworth, Kawerau	173	–0.24	0.62
Taheke	186	0.33	0.59
Combined	359	0.05	0.69

The equation fitted to predict *dub* from *dob* is:

$$dub = \sqrt{dob^2(\alpha_0 + \alpha_1\, L/H + \alpha_2\, (L/H)^8)} \qquad \ldots 1$$

where $\alpha_0 = 0.8161$ (ese. 0.0091)
$\alpha_1 = 0.09528$ (ese. 0.01829)
$\alpha_2 = -0.2312$ (ese. 0.01524)

The coefficients were estimated by a multiple linear regression of $(dub/dob)^2$ on L/H and $(L/H)^8$.

Taper Equation

Individual observations of *dob* and *L* made on the 240 sample trees selected for the main dataset (Table 1), together with associated tree, stand, and location variables, were allocated at random to two data subsets. A Development Subset containing 943 observations was used to develop the taper equation and make initial coefficient estimates, and a Validation Subset of 942 observations was used to independently evaluate the form of the developed equation.

As expected, plots of sectional area relative to breast-height sectional area, $(dob/dbh)^2$, over L/H showed that the sectional area ratio increased monotonically from zero at the tip of the tree to a value of approximately 1.2 at ground level. However, there was a considerable amount of variation not directly related to L/H. Tree size appeared to be an associated factor when the observations were labelled by location. The data from Hawke's Bay which included the smallest trees (a mean *dbh* of 6 cm) were grouped at one edge of the band of points, with other locations following approximately in order of average tree size.

The coefficients of a simple polynomial function in L/H fitted to $(dob/dbh)^2$ were estimated for each tree and examined for relationships with tree size and total tree taper, dbh/H. Some of the coefficients were associated with *dbh* and *H*, none with dbh/H. These relationships provided the basis for constructing a new, more general equation based around a polynomial in L/H which was fitted to the data and refined and simplified. This equation was conditioned (using the coefficient of the L/H term) to ensure that the taper curve passed through *dbh* at breast height.

The errors from the equation for both the Development and Validation Subsets are shown in Table 5. The mean errors when the equation was applied to the Validation set were also calculated by proportion of tree height, together with approximate confidence intervals around each mean. Precision in the predictions of *dob* decreased towards the top of the tree (Fig. 1); more of the diameters were over-predicted in this section of the stem, although the bias was only marginally significant.

The validation results were considered satisfactory, as the model appeared to be free from systematic bias and produced similar levels of precision when applied to both the Development

TABLE 5–Residuals from the taper equation for the development and validation subsets.

Data set	No. of observations	Residual mean (cm)	Residual s.d. (cm)
Development set to which the model was fitted	943	–0.0400	1.9074
Validation set used to evaluate the model	942	–0.0702	1.8663

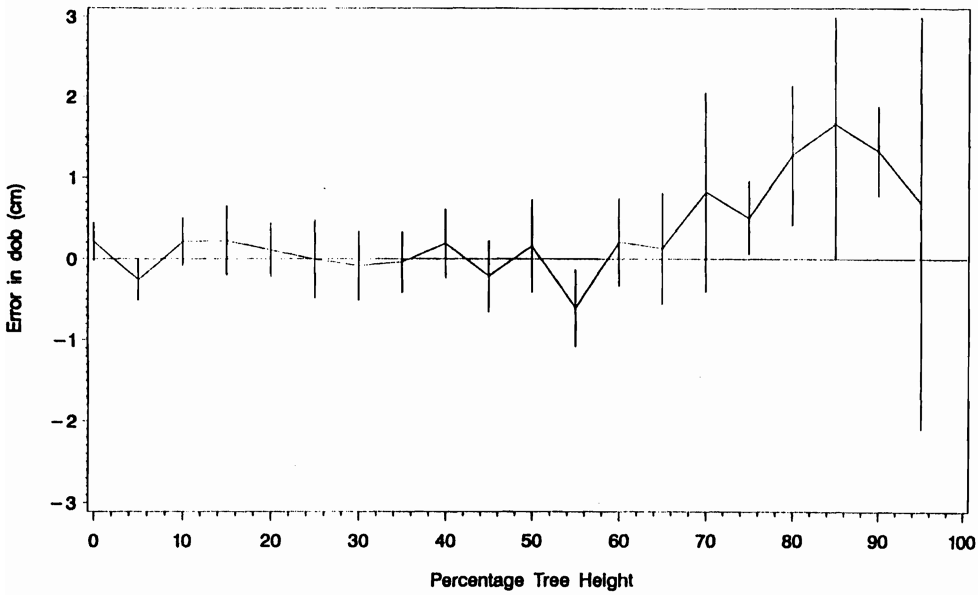

FIG. 1–The mean prediction error for *dob* in the validation subset. Mean values are connected by straight lines. Bars indicate two standard errors either side of each mean.

Subset and the Validation Subset. To refine the coefficient estimates the two sets were combined and the coefficients re-estimated from the whole data set.

The taper equation is:

$$dob = \sqrt{dbh^2 \left[\beta_1 \left(\frac{L}{H}\right)^{\left[\frac{\gamma_1}{H^{0.2}}\right]} + \frac{\beta_2}{(dbhH)^{0.3}} \left(\frac{L}{H}\right)^{\gamma_2} \right]} \qquad \ldots 2$$

where

$$\beta_2 = 4.298 \quad \text{(ese. 0.061)}$$
$$\gamma_1 = 2.610 \quad \text{(ese. 0.122)}$$
$$\gamma_2 = 30.72 \quad \text{(ese. 3.09)}$$

$$\beta_1 = \frac{1 - \dfrac{\beta_2}{(dbhH)^{0.3}} \left(1 - \dfrac{1.4}{H}\right)^{\gamma_2}}{\left(1 - \dfrac{1.4}{H}\right)^{\frac{\gamma_1}{H^{0.2}}}}$$

The coefficients were estimated using non-linear regression (Freund & Littell 1991) with $(dob/dbh)^2$ as the independent variable.

Further analysis of the residuals showed no error trends except for a tendency to over-estimate *dob* in six of the larger trees when *h* was greater than 0.6*H*. These trees all came from the Athenree A and Athenree B samples which were drawn from stands at low stocking. It is likely that the small *dob* measurements were due to heavy branching in the crown resulting in rapid diameter reduction in the main leader. The mean of the *dob* residuals was 0.08 cm with a standard deviation of 1.91 cm.

As taper equations are most commonly used for predicting under-bark diameters and volumes, a composite equation from the combination of Equations 1 and 2 was evaluated against the 359 *dub* measurements from the bark subsample. The bias and residual variation when the composite equation was used to predict *dub* are given in Table 6. Although these measurements were not independent, having been used in fitting the bark equation (1), plots of the residual error against estimated values and other variables indicated that the composite equation performed well (Fig. 2).

TABLE 6–*Dub* residuals from composite equation (Equations 1 and 2 combined) when applied to the bark subsample.

No. of observations	Residual mean (cm)	Residual s.d. (cm)
359	0.30	1.52

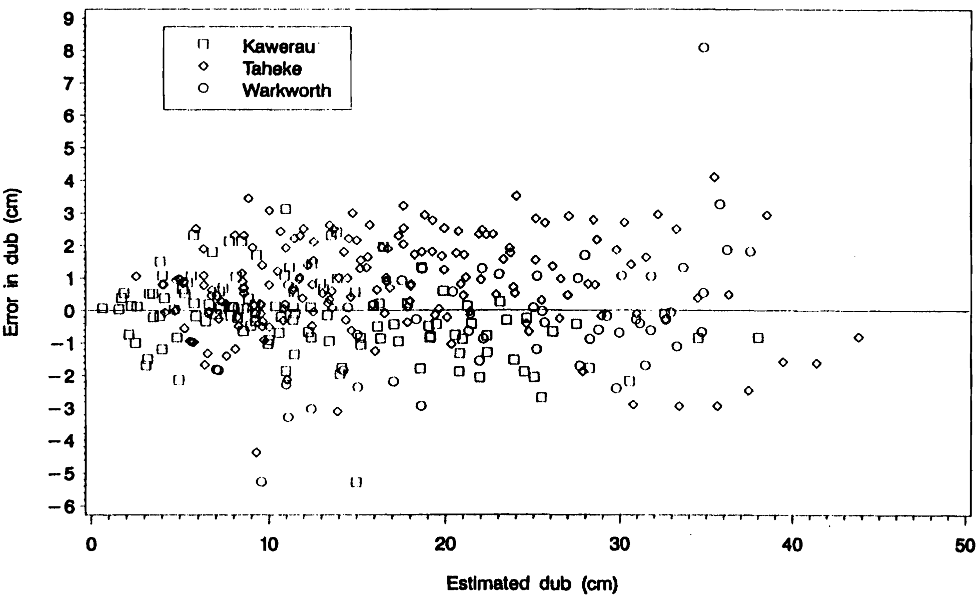

FIG. 2–Residual error in *dub* prediction plotted over estimated *dub* values for bark subsample data. Points are keyed to the sample locations.

Derived Volume Equation

Stem volume under bark from the tip of the tree to a point L_1 metres below the tip is calculated by summing the sectional area from L=0 to L=L_1. Equations 1 and 2 were combined and integrated to give an expression for stem volume in cubic metres.

$$vub_{L_1} = \frac{\pi}{40000}\int_0^{L_1} dub^2 dL$$

$$= \frac{\pi}{40000}\int_0^{L_1}\left[\left[\alpha_0 + \alpha_1\frac{L}{H} + \alpha_2\left(\frac{L}{H}\right)^8\right] dob^2\right] dL$$

from Equation 1

$$= \frac{\pi dbh^2}{40000}\int_0^{L_1}\left[\left[\alpha_0 + \alpha_1\frac{L}{H} + \alpha_2\left(\frac{L}{H}\right)^8\right]\left[\beta_1\left(\frac{L}{H}\right)^{\frac{\gamma_1}{H^{0.2}}} + \frac{\beta_2}{(dbhH)^{0.3}}\left(\frac{L}{H}\right)^{\gamma_2}\right]\right] dL$$

from Equation 2

integrating:

$$= \frac{\pi dbh^2}{40000}\left\{\frac{\alpha_0\beta_1}{H^{\frac{\gamma_1}{H^{0.2}}}\left(\frac{\gamma_1}{H^{0.2}}+1\right)} L_1^{\frac{\gamma_1}{H^{0.2}}+1} + \frac{\alpha_0\beta_2}{(dbhH)^{0.3}\,H^{\gamma_2}(\gamma_2+1)} L_1^{\gamma_2+1}\right.$$

$$+ \frac{\alpha_1\beta_1}{H^{\frac{\gamma_1}{H^{0.2}}+1}\left(\frac{\gamma_1}{H^{0.2}}+2\right)} L_1^{\frac{\gamma_1}{H^{0.2}}+2} + \frac{\alpha_1\beta_2}{(dbhH)^{0.3}\,H^{\gamma_2+1}(\gamma_2+2)} L_1^{\gamma_2+2}$$

$$\left.+ \frac{\alpha_2\beta_1}{H^{\frac{\gamma_1}{H^{0.2}}+8}\left(\frac{\gamma_1}{H^{0.2}}+9\right)} L_1^{\frac{\gamma_1}{H^{0.2}}+9} + \frac{\alpha_2\beta_2}{(dbhH)^{0.3}\,H^{\gamma_2+8}(\gamma_2+9)} L_1^{\gamma_2+2}\right\} \quad \ldots 3$$

The prediction of over-bark volume is simpler since only Equation 2 must be integrated.

Diagrammatic Representation of Equations 1 and 2

Over- and under-bark taper curves for two trees are shown in Fig. 3. The bark thickness increases with tree size (age) and decreases with the level up the stem. Values are always positive and never exceed the *dub*. The diagram illustrates some of the logical features of the taper equation in that *dob* and *dub* have a value of zero at the tip of the tree, diameters decrease monotonically from ground level to tree tip, and a *dob* equal to *dbh* is predicted at breast height. One limitation of the taper equation is that the height above ground of a specific *dub* cannot be determined directly by rearranging Equations 1 and 2 to predict L. However, the simple shape of the taper curve from ground level to tree tip means that L can be determined by numerical methods very quickly.

A series of under-bark taper curves for a range of tree sizes which span the data set is given in Fig. 4. There is a noticeable change in shape with tree size, from the fairly conical shape for the small tree to a fuller, more paraboloidal curve for the larger trees. The butt-swell is accurately modelled.

CONCLUSIONS

The taper equations show little bias and reasonable precision considering the range of tree sizes covered. Volume estimates can be derived from the integrated composite Equation (3), thereby ensuring the compatibility of taper and volume. Conditioning guarantees that *dbh* will be predicted by the taper curve over bark at breast height. Equations (1), (2), and (3) should provide reliable predictions of the volume and taper of *E. saligna* in New Zealand over the range of *dbh* and H shown in Table 1.

ACKNOWLEDGMENTS

Data collection and derivation of the *E. saligna* equations was undertaken as a project forming part of the work programme of the New Zealand FRI Management of Eucalypts Co-operative. Thanks to

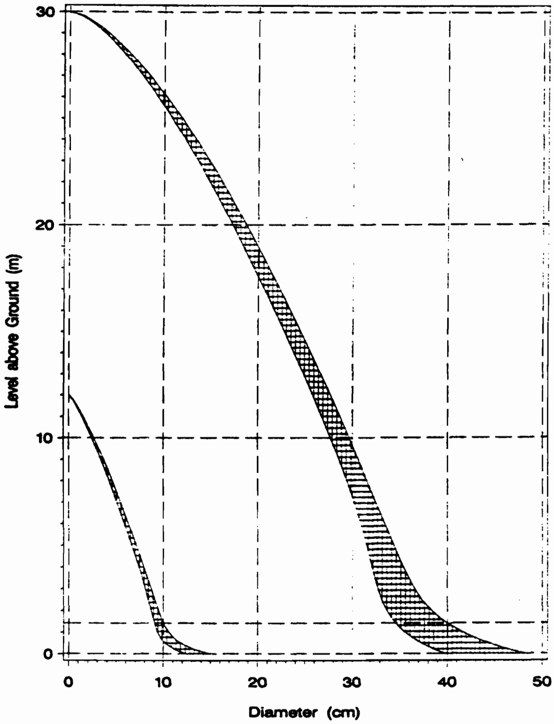

FIG. 3—Over-bark and under-bark taper curves for a small (*dbh* 10 cm, *H* 12 m) and average sized (*dbh* 40 cm, *H* 30 m) tree. Shaded areas indicate bark thickness; boundaries represent *dub* and *dob* predicted from Equations 1 and 2.

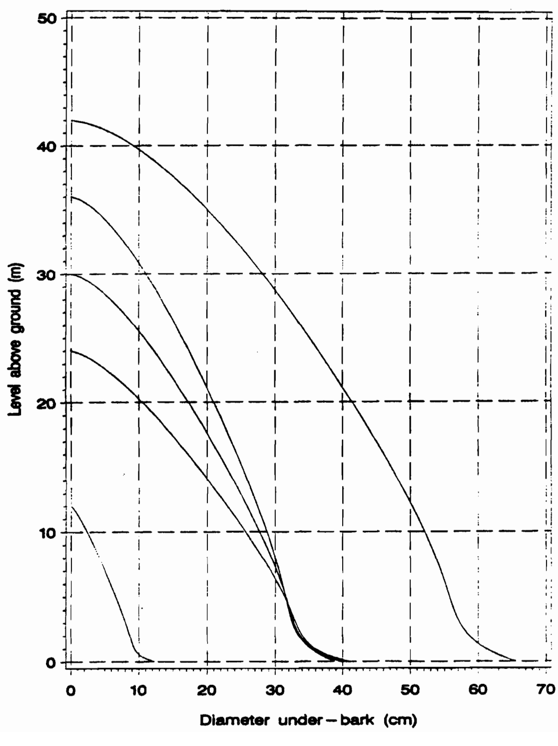

FIG. 4—Under-bark taper curves predicted by Equations 1 and 2 for a range of tree sizes (*dbh* = 10–70 cm, *H* = 12–42 m) spanning the data set.

the personnel from Fletcher Challenge Forests Ltd (formerly Tasman Forestry Ltd) and Carter Holt Harvey Forests Ltd (formerly NZFP Forests Ltd, Whangarei) who assisted at two of the new sites, Taheke and Warkworth. Thanks are also due to G. Oliver, NZ FRI, for providing access to sectional measurement data, and to our reviewers for helpful comments and suggestions.

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