

NOTE

THEORETICAL RATIO BETWEEN "ONE-SIDED" AND TOTAL SURFACE AREA FOR PINE NEEDLES

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Both the interception of solar radiant energy and photosynthesis depend on the surface area of foliage. "One-sided" foliage surface area is used in the calculation of intercepted radiant energy, whereas total foliage surface area is commonly used in the calculation of photosynthesis for amphistomatic foliage. In experiments, either "one-sided" or total surface area can be measured; hence, there is a need to be able to convert from "one-sided" to total surface area and vice-versa.

The length and width of foliage of broadleaved species are large in comparison to their thickness. If such foliage is placed on a horizontal surface so that the foliage thickness is perpendicular to the surface, then the "one-sided" surface area is the vertical projection of the foliage on to the horizontal surface. Total surface area is assumed to be twice the "one-sided" surface area. Problems arise when one attempts to define "one-sided" surface area for pine needles as the needle width approaches the needle thickness and there is no obvious top or bottom to the needle. Bennett & Rook (1978) defined the "one-sided" surface area for *Pinus radiata* D. Don needles (usually three needles per fascicle) to be the chord width, *ab*, (Fig. 1). Measurements (Bennett & Rook 1978) indicated that the total surface area was 2.4 times the "one-sided" surface area. Johnson (1984) measured both "one-sided" and total surface area for several species of pine with two to five needles per fascicle. These data indicated that the ratio of total to "one-sided" surface area varied between 2.85 and 3.49. Sinclair & Knoerr (1982) stated (without showing the derivation) that if the cross-section of a fascicle is a circle, then, by geometry, the ratio of "one-sided" to total surface area for a needle in a three-needled fascicle is 0.3. This ratio is similar to those obtained by Johnson (1984) but different from that obtained by Bennett & Rook (1978).

The object of this note is to define "one-sided" surface area for pine needles, and to derive, mathematically, the ratio between "one-sided" and total surface area for pine needles, based on the assumption that a pine fascicle is circular in cross-section and that each needle cross-section is a sector of this circle.

I started by considering how "one-sided" surface area is measured for broadleaved species. A common way is to use an optical planimeter. For broadleaves, the leaf is placed flat on the planimeter. The question is, how should pine needles be placed on a

planimeter? Some people have assumed that needles must be placed in a fixed orientation. For example, Bennett & Rook's (1978) definition of "one-sided" surface area implies that a needle is placed so that the chord, *ab*, (Fig. 1) is parallel to the planimeter surface. However, needles are arranged in fascicles. Within a pine fascicle the curved surface of a needle is in at least two different positions (Fig. 2) depending on the number of needles in the fascicle. Since a fascicle has no top or bottom surface, and there is no obvious reason why needles should have a preferred orientation within the fascicle, the

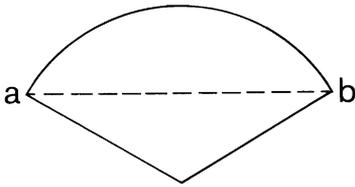


FIG. 1—Chord, *ab*, measured by Bennett & Rook (1978).

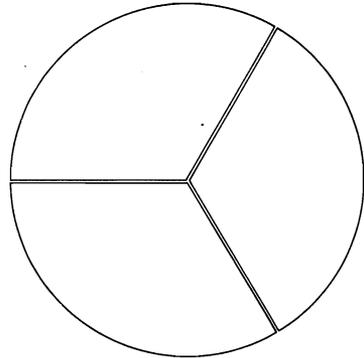


FIG. 2—Cross-section of a fascicle, for a three-needled fascicle.

above indicates that needles should not be placed on a planimeter in any fixed manner and that, mathematically, the "one-sided" surface area should be obtained by averaging over all needle orientations, i.e., the "one-sided" surface area is given by:

$$\int_0^{2\pi} (\text{vertical projection of needle cross-section on to a horizontal plane}) d\theta \dots\dots\dots (1)$$

The ratio of "one-sided" to total surface area is given by:

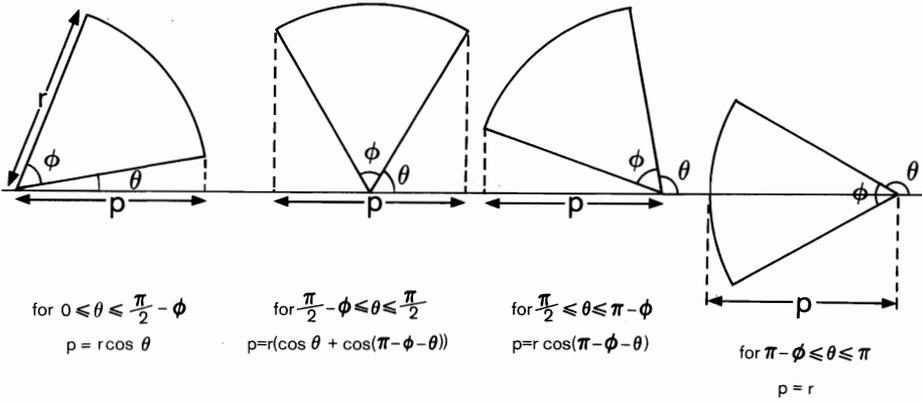
$$\frac{\int_0^{2\pi} (\text{vertical projection of needle cross-section on to a horizontal plane}) d\theta}{\int_0^{2\pi} (\text{surface area of needle cross-section}) d\theta} \dots\dots\dots (2)$$

where θ is the angle between one side of the needle cross-section and the horizontal (see Fig. 3).

Integrating (see Appendix) gives the result that the ratio of "one-sided" to total surface area is $1/\pi$.

The ratio of "one-sided" to total surface area obtained by Sinclair & Knoerr (1982) and Johnson (1984) approximates $1/\pi$. Although Johnson (1984) suggested that the ratio of "one-sided" to total surface area changes with species, my calculation indicates that the ratio is constant regardless of the number of needles in a fascicle. Deviations

a/ $0 < \phi < \frac{\pi}{2}$



b/ $\frac{\pi}{2} \leq \phi \leq \pi$

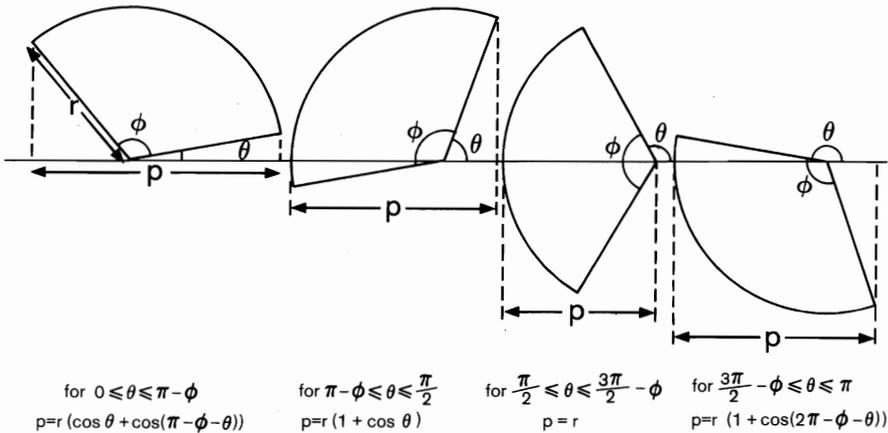


FIG. 3—Projection, p, of a needle cross-section for different included angles, ϕ , and for different orientations of the cross-section.

from the theoretical value of $1/\pi$ would arise if the assumption of a circular fascicle cross-section was incorrect or if the needles passed through the planimeter in a preferred orientation. If needles pass through the planimeter in a preferred orientation, the ratio of "one-sided" to total surface area depends on both the orientation of the needle and the number of needles in a fascicle (Fig. 3). For example, if $\theta = 0$, then the ratio of "one-sided" to total surface area is (from Fig. 3 and Appendix):

$1 / (2 + \phi)$ if $0 < \phi < \pi/2$

and

$(1 + \cos(\pi - \phi)) / (2 + \phi)$ if $\pi/2 \leq \phi \leq \pi$

It is concluded that, for all pine species where a needle cross-section can be assumed to be a sector of a circle, the ratio $1/\pi$ should be used to convert from total to "one-sided" surface area. The ratio π should be used to convert from "one-sided" to total surface area provided that needles have not been arranged in any preferred manner when measuring "one-sided" surface area. In order to overcome confusion it is necessary that the method of measuring leaf area, and whether values of leaf surface area are "one-sided" or total, be clearly stated.

REFERENCES

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APPENDIX

The ratio of "one-sided" to total surface area is given by:

$$\frac{\int_0^{2\pi} (\text{projection of needle cross-section}) d\theta}{\int_0^{2\pi} (\text{surface area of needle cross-section}) d\theta} \quad (1)$$

It is assumed that the cross-section of a needle is represented by a sector of a circle. Owing to symmetry, Eqn 1 is equivalent to:

$$\frac{\int_0^{\pi} (\text{projection of needle cross-section}) d\theta}{\int_0^{\pi} (\text{surface area of needle cross-section}) d\theta} \quad (2)$$

$$\int_0^{\pi} (\text{surface area of needle cross-section}) d\theta = \pi r (2 + \phi) \quad (3)$$

The projection of the needle cross-section depends on the included angle ϕ , and the orientation of the needle (Fig. 3). For $0 < \phi < \pi/2$:

$$\begin{aligned} & \int_0^{\pi} (\text{projection of needle cross-section}) d\theta \\ &= \int_0^{\pi/2 - \phi} r \cos\theta d\theta \\ & \quad + \int_{\pi/2 - \phi}^{\pi/2} r (\cos\theta + \cos(\pi - \phi - \theta)) d\theta \\ & \quad + \int_{\pi/2}^{\pi - \phi} r \cos(\pi - \phi - \theta) d\theta \\ & \quad + \int_{\pi - \phi}^{\pi} r d\theta \\ &= r \sin(\pi/2 - \phi) \\ & \quad + 2r (1 - \sin(\pi/2 - \phi)) \\ & \quad + r \sin(\pi/2 - \phi) \\ & \quad + r\phi \\ &= r (2 + \phi) \end{aligned}$$

For $\pi/2 \leq \phi \leq \pi$

$$\begin{aligned} & \int_0^{\pi} (\text{projection of needle cross-section}) d\theta \\ &= \int_0^{\pi - \phi} r (\cos\theta + \cos(\pi - \phi - \theta)) d\theta \\ & \quad + \int_{\pi - \phi}^{\pi/2} r (1 + \cos\theta) d\theta \\ & \quad + \int_{\pi/2}^{3\pi/2 - \phi} r d\theta \\ & \quad + \int_{3\pi/2 - \phi}^{\pi} r (1 + \cos(2\pi - \phi - \theta)) d\theta \\ &= 2r \sin(\pi - \phi) \\ & \quad + (1 + \phi - \pi/2 - \sin(\pi - \phi)) \\ & \quad + r (\pi - \phi) \\ & \quad + r (1 + \phi - \pi/2 - \sin(\pi - \phi)) \\ &= r (2 + \phi) \end{aligned}$$

Hence, regardless of the value of ϕ :

$$\int_0^{\pi} (\text{projection of needle cross-section}) d\theta = r (2 + \phi)$$

Hence, the ratio of "one-sided" to total surface area is $1/\pi$.