HEIGHT GROWTH OF *PINUS RADIATA* IN NEW ZEALAND

OSCAR GARCÍA

Xunta de Galicia,
Centro de Investigacións Forestais de Lourizán,
Apartado 127, 36080 Pontevedra, Spain

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ABSTRACT

Top-height growth components and site index curves, based on permanent sample plot data for New Zealand *Pinus radiata* D. Don, were were used in growth models stratified into eight growth modelling regions. The application of an age adjustment based on seasonal growth patterns helped to reduce variation caused by different measurement dates and the rapid and sustained growth of *P. radiata* throughout the year. Global and local parameters in a stochastic differential equation version of the Bertalanffy-Richards model were estimated simultaneously by maximum-likelihood. Various parameterisations were tried. The procedure has worked reliably, giving satisfactory predictions and making full use of the available data. Comparison of the models obtained showed significant differences across regions, justifying the initial regionalisation and indicating that a broader aggregation would not be advisable.

Keywords: growth models; height growth; site index; height/age curves; *Pinus radiata*.

INTRODUCTION

During the 1980s a series of regional stand growth models for *Pinus radiata*, and one for South Island *Pseudotsuga menziesii* (Mirb.) Franco (Douglas-fir) were developed in New Zealand at the Forest Research Institute, using a common methodology (García 1979, 1988, 1994). The models were embedded in STANDPAK (Gordon 1992), The Forest Master (Tennent 1992), and other software packages, and together with EARLY (West *et al.* (1982), commonly used for silviculture scheduling in young stands) have formed the basis for most* of New Zealand plantation forestry planning (Goulding 1994, 1995).

Only the models for Golden Downs (García 1984) and Auckland Clays (Shula 1989), and preliminary height growth equations for Southland and Kaingaroa (García 1979, 1983), have been published. This article documents the classification into regions, and the top-height growth components (site index equations) of the regional radiata pine models. The data, methods, and results are described and compared.

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* Another model was used by NZFP/Carter-Holt in their Tokoroa forests (Woollons & Hayward 1985).

DATA

Regions and Models

By the early 1980s it was generally recognised that there were distinct regions within New Zealand where the growth patterns seemed to be substantially different (Burkhart & Tennent 1977; Manley & Knowles 1980, for example). In addition, limited computer access and processing power made it necessary to subdivide the large amount of data available into manageable subsets.

A tentative regionalisation was established with the help of geological, soil, and climate maps, and after consultation with experienced foresters and other experts (Fig. 1). The regions reflected qualitative differences in growing conditions, the availability of data, and the economic significance of particular forests or forest clusters. They were not very different from the groupings used by Burkhart & Tennent (1977). Once full model coverage became available, it would be possible to make statistical comparisons between the models and reassess the regional boundaries.

North Island sands

The sand dune forests located on the West coast of the North Island present unique growth development characteristics. Tree form tends to be more slender than elsewhere and the

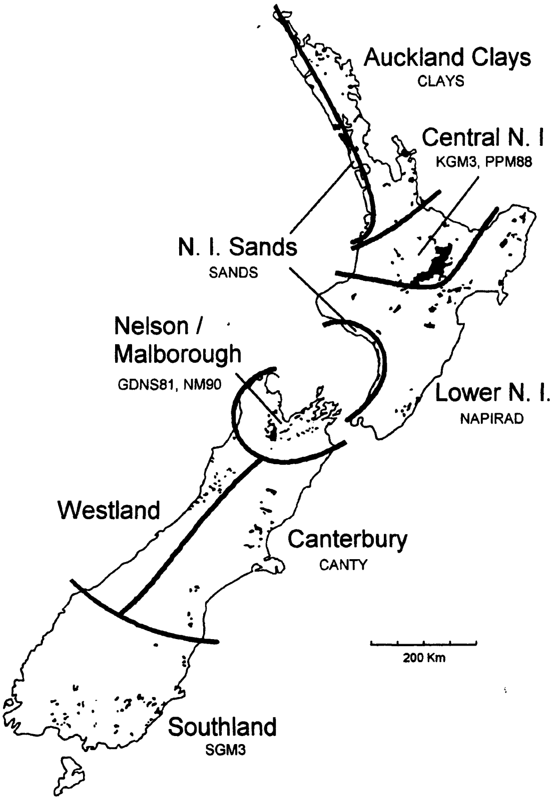

FIG. 1–Growth modelling regions and model identifiers

crowns are light branched and comparatively deep. There are marked differences in growth within small areas due to exposure to salt-laden winds and variations in soil nutrition status. The SANDS growth model was developed for this region, with data from Woodhill, Waiuku, Aupouri, Santoft, Waitarere, Tangimoana, and Mangawhai Forests (A.G.Dunningham, Growth Modelling Co-operative, unpubl. data).

Auckland clays

Another clearly distinct region is formed by the brown clay soils of Northland. Although heterogeneous, the soils are deficient in phosphorus and differ markedly from those in the rest of the country. The use of forest fertilisers is standard practice in the region and the CLAYS model was based on plots with adequate fertiliser (R.G.Shula, Growth Modelling Co-operative, unpubl. data). A modification to predict the effects of phosphorus deficiency was added later (García 1989; Shula 1989). The forests represented in the database were those at Whangapoua, Glenbervie, Maramarua, Riverhead, Tairua, and Waitangi.

Central North Island

This is the most important forestry region in New Zealand. The KGM3 model used data exclusively from the 140 000 ha Kaingaroa State Forest and from nearby Tarawera Forest (A.G.Dunningham, M.E.Lawrence, Growth Modelling Co-operative, unpubl. data). Both forests are located on free-draining volcanic ash or pumice soils laid down by periodic eruptions. These porous soils allow trees ready access to nutrients and moisture and have proved ideal for forestry. PPM88 used the same height growth submodel as KGM3, but modelled more precisely the thinning and pruning responses at the cost of additional stand information (García 1989, 1990). The applicability of these models to other central North Island forests outside the Pumice Plateau is uncertain. There are indications that Rotoehu Forest, for example, behaves differently, although this forest appears to be located in a nitrogen deficiency pocket (Hunter *et al.* 1991). There are also significant differences with predictions from the growth model developed by Woollons & Hayward (1985) for the NZFP/Carter-Holt forests in Tokoroa (I.J.Hayward, D.New, Growth Modelling Co-operative, unpubl. data). For reasons as yet unclear, both qualitative and quantitative differences were apparent in the central North Island between the data from the earliest planted stands ("old crop") and the more recent plantings ("new crop"). Klitscher (1987) found significantly higher natural mortality in the old crop data, and García (1983, Fig.1) suggested some discontinuity in height growth trends. In addition, the *Sirex noctilio* Fabricius wood wasp epidemic of the late 1940s and early 1950s caused heavy mortality and growth reductions, altering the normal development patterns. There also appear to be differences in basal area growth patterns between the two "crops", with the "new crop" having noticeably lower net basal area estimates at older ages. For these reasons, only data from stands planted since 1955 were used in both KGM3 and PPM88.

Hawke's Bay

The NAPIRAD model was based on data from forests in the Hawke's Bay area, centred around the city of Napier. Whereas forests on the main ranges (Kaweka Forest, for example) are sited on older greywacke and argillite soils, the bulk of the data comes from the coastal hill forests, which are a complex mixture of marine sandstones and siltstones, broken by bands of upstanding limestone. The region is known for drought periods over the summer months. Limited testing indicated that the Hawke's Bay model might also perform well for the Wairarapa area further south, and possibly on the East Cape to the north, although the forests in this last area were young and data scarce at the time of development. The data came from the Esk, Gwavas, Mohaka, Wharerata, Patunamu and Kaweka Forests.

Nelson/Marlborough

The northern part of the South Island possesses both characteristic climate and soils. Although many of the forest plantations are located on clay-bound (Moutere) gravel soils, the western fringes are broken by granites similar to those of Fiordland and thrust northward along the alpine fault. Further to the east in Marlborough, the soils are more like those previously described in the Hawke's Bay and Wairarapa districts of the North Island. This region includes the Golden Downs Forest, the second-largest production forest in New Zealand (after Kaingaroa), for which the GDNS81 growth model was produced (García 1984). Subsequently, the NM90 model incorporated additional data from Golden Downs, and also from another 25 forests in the area: Wairau, Motueka, Hira, Dovedale, Rai Valley, Mahana, Greenhill, Trass, Sth Pigeon, Lee, Waiwhero, Hoults, Harakeke, Sunrise, Nth Pigeon, Sunset, Pece, Richmond, Mariri, Tutaki, Waimea West, Orinoco, Tasman, Riwa, and Redwoods (K.R.N.Law, Growth Modelling Co-operative, unpubl. data). The height growth component of GDNS81 is likely to be less reliable than that of NM90 and will not be described here. However, because of some anomalies in the basal area and mortality data used for NM90, GDNS81 may still be preferable for predicting those variables under some circumstances*.

Canterbury

The rest of the South Island is clearly subdivided by the Southern Alps, with the Canterbury region to the East. The Alps exert a major influence on the climate, which is generally drier than in Nelson or Southland. Soils on the plains consist of stony gravels covered by a fine loess which can be severely effected by drought. These differ from the downlands of North Canterbury which have dense grey soils, and the coastal hill soils which are a complex mixture of sediments. Lawrence (Growth Modelling Co-operative, unpubl. data) analysed data from the foothills and from the plains and concluded that although there were differences in site index, a common growth model was applicable to both groups of forests. The model, CANTY, was based on data from Eyrewell, Balmoral, Ashley, Omihi, Geraldine, Burnham, Bottle Lake, Hanmer, and Waimate Forests.

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* Although somewhat outside the main topic here, a referee has suggested that further information might be of interest. Users of GDNS81 in other forests of the Nelson region had expressed concern about a possible overprediction of mortality at low stockings, and its effect on estimated piece sizes. Extensive analysis showed no appreciable bias with the model data, but indicated a possibility of lower mortality in more recent observations. As already mentioned, a similar reduction in natural mortality over time had been detected in the central North Island. NM90 was fitted including the newer data from Golden Downs, as well as plots from the other forests in Nelson/Marlborough. Unfortunately, under pressure from key users much data that documented mortality was subjectively excluded, possibly causing bias in the opposite direction and other side effects (O.García, Growth Modelling Co-operative, unpubl. data; M.E.Lawrence, Growth Modelling Co-operative, unpubl. data).

Otago/Southland

The southernmost part of the country, roughly south of the Waitaki River, contains soils of much older origin than those in Canterbury. Lowland forests in the east are generally on dense grey schist loess soils while further west on the ranges the soils are derived from greywacke and basalt. The growth pattern of *P. radiata* is also noticeably different in this region, with higher diameter growth relative to height growth than in other parts of New Zealand (J.Beekhuis, pers. comm.). A preliminary growth model was developed with a small data subset (García 1979). New height relationships were later fitted with a much extended set of height-age data, re-estimating also the other growth model parameters to obtain the SGM3 growth model (Law, Growth Modelling Co-operative, unpubl. data). The data came from Berwick, Otago Coast, Longwood, Hokonui, Blue Mountain, Blackmount, Silver Peaks, Herbert, Otago Coast, Taringatua, Rowallan, and Stewart Island.

The exact location of some of the boundaries between regions is somewhat arbitrary and occasionally there might be doubts in the classification of new plantations or of borderline forests. In practice, existing forests are clustered in a way that caused few doubts about the appropriate groupings for model development and the regions reflect distinct and discrete units rather than smooth gradients in growing conditions.

Seasonal Growth

Pinus radiata has a long growing season. Maximum growth occurs in spring and summer, although there may be some growth throughout the year (Barnett 1971). Ideally, measurements should be made on a fixed date, preferably within the period of dormancy or close to the time of minimum growth. In practice, a large proportion of the data used in modelling has been obtained at times of appreciable growth rates, and some adjustment for seasonal growth effects is therefore desirable.

García (1979) developed a fractional age adjustment, based on a "physiological time" corresponding to uniform size increments. The curves in Fig. 2 (derived from Tennent 1986) represent the accumulated height and diameter at breast height (dbh) increments as a fraction of the annual totals. The origin was shifted to the middle of July (close to the growth

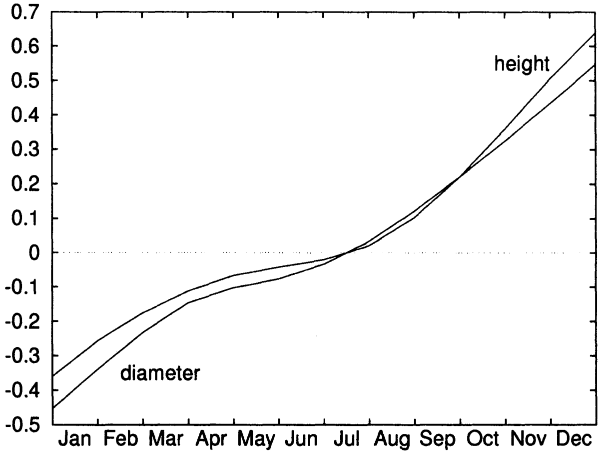

FIG. 2–Relative seasonal growth

minimum), which was taken as the reference date. Fractional values for each month read from the curves were added to the nominal age as a seasonal growth correction (Table 1).

Combined results from Aupouri (North Island Sands), Whakarewarewa (central North Island), Mohaka (Hawke's Bay), and Otago Coast (Southland) Forests are given in Fig. 2. There were significant differences between forests (Tennent 1986) and, despite the variation in seasonal growth patterns from year to year, the timing of the growth in height and in diameter was clearly different. A compromise adjustment, based on a limited amount of older data, was given by García (1979), and used in preliminary models for Southland and Kaingaroa (García 1983), in Golden Downs (GDNS81), and in Hawke's Bay (NAPIRAD). A modified version incorporated into the New Zealand Permanent Sample Plot System by McEwen (1979) was used in the other models (Table 1). Because of potential errors due to the variation and sensitivity to measurement dates at the peak of the growing season, and to the differences in height and diameter growth, only measurements taken between February and October (inclusive) were used for growth modelling.

TABLE 1–Age adjustments for seasonal growth (years)

	Tennent (1986) height	Tennent (1986) dbh	García (1979)	McEwen (1979)
Jan	–0.31	–0.40	–	–0.3
Feb	–0.22	–0.29	–0.2	–0.2
Mar	–0.14	–0.19	–0.2	–0.1
Apr	–0.09	–0.12	–0.1	0.0
May	–0.05	–0.09	–0.1	0.0
Jun	–0.03	–0.05	0.0	0.0
Jul	0.00	0.00	0.0	0.0
Aug	0.06	0.08	0.1	0.1
Sep	0.16	0.17	0.2	0.2
Oct	0.29	0.27	0.3	0.3
Nov	0.43	0.38	–	0.5
Dec	0.57	0.49	–	0.6

Measurements

Data were obtained from the New Zealand Permanent Sample Plot System (McEwen 1979; Pilaar & Dunlop 1990) and screened according to a number of criteria. Measurements had to be from planted stands (not natural regeneration); with a sufficient number of height sample trees; taken between February and October; and not affected by excessive windthrow, artificial fertilisers (except in CLAYS), poison thinning, or other atypical treatments. All the data were graphed and examined for gross errors. Plots with at least three measurements, preferably at intervals of at least 1 year, were used for the height growth models.

Ages were adjusted for seasonal growth as described above. Top height was chosen as the measure of height, defined as the height of the 100 largest trees per hectare. In practice, it is computed by fitting the Petterson height/dbh equation to the sub-sample of the plot trees measured for total height (McEwen 1979); the estimated height for the quadratic mean dbh of the proportion of the largest-diameter (at 1.4 m) trees in the plot corresponding to 100 per hectare is then considered the top height.

It is generally accepted that top height is little affected by stand density over a wide range of treatments, although some growth reduction may be expected for the more extreme thinning/pruning combinations (Maclaren *et al.* 1995). Another measure of dominant height used in New Zealand is predominant height, the average height of the tallest trees in 0.01-ha subplots. The difference between top height and predominant height is usually small and can often be ignored in practice. Conversion equations have been given by Burkhart & Tennent (1977) and West *et al.* (1982), among others. Note that slight biases may arise from differences in plot size (García 1998).

Characteristics of the data are summarised in Table 2. Site indices (index age 20 years) are derived from model estimates for each plot, and their extremes may be somewhat affected by outliers.

TABLE 2–Height data summary

Model	Number of plots	Number of measurements	Age (years)		Height (m)		Site index (m)		
			min	max	min	max	min	mean	max
SANDS	356	1494	4	46	3	42	13	25.0	36
CLAYS	114	535	5	32	4	44	24	30.0	37
KGM3	284	2551	3	40	3	52	22	31.5	39
NAPIRAD	167	723	4	29	4	47	21	29.8	38
NM90	385	2023	3	48	2	49	19	28.5	36
CANTY	120	590	5	35	3	38	15	22.4	31
SGM3	258	1199	1	57	0	57	15	24.5	33

METHODS

Models

The adopted methodology assumes that the height increment of a stand at any time is a function of its current height, following the Bertalanffy-Richards equation

$$dH/dt = \eta H^m - \kappa H, \qquad (1)$$

where H is top height and t is age. The parameters η, κ, and m may vary among different stands. This equation was used for animals by von Bertalanffy (1938, 1949), and studied in detail and popularised in plant growth modelling by Richards (1959). Since its introduction into forestry by Pienaar & Turnbull (1973) it has been one of the most commonly used for tree height growth.

It is mathematically more convenient to write (1) as a linear differential equation on a power transformation of H,

$$dH^c/dt = b(a^c - H^c). \qquad (2)$$

By calculating the derivative on the left-hand side and rearranging, it is easy to verify that this is the same as (1), with $m = 1 - c$, $\eta = a^c b/c$, and $\kappa = b/c$ (García 1979, 1983).

Given an age-height pair (t_0, H_0), solving the differential equation (2) gives the height at any other age t

$$H = a[1 - (1 - H_0^c/a^c)e^{-b(t-t_0)}]^{1/c}. \qquad (3)$$

The parameter a is the asymptotic height approached as $t \to \infty$, while b can be seen as a time-scale factor that modifies the rate of growth (*see also* (2)). The "shape" of the height-age curve is determined by c; the inflection point occurs at a fraction $(1 - c)^{1/c}$ of the final height a.

In site index equations the curves are commonly forced to pass through the origin by setting $t_0 = H_0 = 0$ in (3). For added flexibility, an initial non-zero t_0 or H_0^c has been allowed as an additional parameter, accepting that the model may not be applicable at the seedling stage.

The concept of site index is based on representing the expected height development by a one-parameter family of curves. The curves are conventionally indexed by the height at some fixed index age (e.g., 20 years for *P. radiata* in New Zealand), this height being the site index. A family of site index curves can be obtained from (3) by taking one of the parameters as site-dependent, varying across stands ("local"), while the other parameters are common to all the stands ("global"). More generally, a parameter transformation (reparametrisation) can be performed to use any real-valued function of the original parameters as the site-dependent quantity (García 1983, 1994). The parameters can be related to the customary site index by substituting site index and index age for H and t in (3).

For each region, three site parametrisation alternatives were tested:

(i) Parameter a local. This produces so-called anamorphic curves, where the curves differ in scaling along the height axis. The asymptotes a are proportional to the site index.

(ii) Parameter b local. The curves have a common asymptote, differing in scaling along the time axis. Site affects height growth rates by a constant factor (*see* equations (1) and (2)).

(iii) Reparametrisation equivalent to Burkhart & Tennent (1977). This was achieved by substituting

$$a = S/(1 - e^{20\mathrm{k}S})^{1/c}, \qquad b = kS,$$

and making the new parameters k global and S (the site index) local.

A fourth parametrisation,

$$a = \alpha + \beta S, \qquad b = -\frac{1}{20} \ln\left[1 - (S/a)^c\right],$$

with α and β global and S local (García 1979), was tested on a few data sets. It was not significantly better than (i) or (ii), which coincide with the special cases $\beta = 0$ and $\alpha = 0$, respectively.

In addition, models with and without the t_0 or H_0^c parameters were tried.

Estimation

Any rational estimation procedure must assume some model, however rough, for the variability in the data. The impact of environmental perturbations was represented by a random process added to the right-hand side of (2). In addition, the H^c were assumed to be affected by normal, independently distributed, measurement errors. Both the environmental and the measurement variances are additional parameters to be estimated from the data.

Given this stochastic model it is possible to compute the likelihood function—that is, the probability of obtaining the observed data—as a function of the parameters. The maximum-likelihood estimates are those values of the parameters for which the likelihood reaches a maximum. All the parameters, global and local, are estimated simultaneously by maximum-likelihood, using a specialised numerical optimisation algorithm (for details *see* García 1983). Seber & Wild (1989, Section 7.5.3) also described some aspects of this approach.

We experimented with including a separate environmental variance for the early growth (the parameter σ_0 of García (1983)), and with various assumptions of global, local, or fixed environmental and measurement variances. Generally, the best results were obtained with one environmental and one measurement variance, both global.

"Best" models were selected according to Akaike's criterion. This is equivalent to maximising the log-likelihood adjusted by substracting the number of parameters (Atkinson 1980; García 1979, 1983). Final results were always evaluated successfully through residual computations and graphical validation.

Usage

Height growth models are used for estimating site index and predicting future heights.

Given an observed height-age pair, the corresponding local parameter representing site effects can be obtained by solving equation (3). If a is local, we get

$$a = \left[\frac{H^c - H_0^c e^{-b(t-t_0)}}{1 - e^{-b(t-t_0)}} \right]^{1/c} , \tag{4}$$

and for b local,

$$b = -\frac{1}{t - t_0} \ln \frac{a^c - H^c}{a^c - H_0^c} \tag{5}$$

(the Burkhart-Tennent parametrisation was rejected in all regions). A site index can be computed by substituting these values and the index age in (3):

$$S = a[1 - (1 - H_0^c/a^c)e^{-b(20-t_0)}]^{1/c} . \tag{6}$$

Explicit expressions in terms of the observation (H, t), obtained by substituting (4) or (5) into (6), can be used if so desired:

$$\mathrm{S = H} \left[\frac{1 - e^{-b(20-t_0)}}{1 - e^{-b(t-t_0)}} \right]^{1/c} ,$$

or

$$S = a[1 - \{1 - (H/a)^c\}^{\frac{20-t_0}{t-t_0}}]^{1/c} ,$$

assuming $H_0^c = 0$.

When there are several height-age observations, a number of schemes may be used to estimate site index. The simple procedure of averaging the site indices obtained as above for each observation is probably adequate, and has been adopted in the Permanent Sample Plot System. In the development of the regional growth models, the maximum-likelihood estimates of the local height growth parameters were used to derive site index.

If it is known that early growth has been atypical (for example, due to establishment techniques different from those represented in the PSP data) it may be preferable to use a "floating" site index curve, ignoring the curve origin parameters. Pairs of measurements may be substituted for t_0 , H_0 , t, and H in (4) or (5) to estimate the local parameter.

To predict future heights, an estimate of the stand site index (or, equivalently, of the local parameter) is needed. This can be calculated from previous measurements as shown above, or obtained by other means. Given the site index, the local parameter value is computed by (4) or (5), with the site index and index age substituted for H and t.

If predictions are required for a future or hypothetical stand, or if no reliable measurements are available, heights may be estimated using equation (3). On the other hand, if a good recent measurement is available, it may be used as a starting point in place of H_0 and t_0 in (3). This would ensure that errors due to past deviations from the expected trend are reduced, and that predictions are consistent with the current height. The underlying principle is that growth is affected by the current state of the stand, and not directly by the time elapsed since planting. Again, more elaborate methods for making use of all the available measurements are possible, but using just the latest observation is likely to be sufficient.

Starting (3) from a known height and age will in general cause the height predicted at the index age to differ from the site index. This may disturb those used to the traditional application of site index curves. However, site quality is a property of the site, not of the stand. Site index should therefore be more properly defined as the expected or average height at the index age for hypothetical stands growing on that site; specific stands will vary.

RESULTS

The Models

Results for the best models in each region are given in Table 3.

With one exception, the best site parametrisation was with b as local. That is, the site index curves share a common asymptote and differ in time-scale factor. For SANDS, however, the best results were obtained by making a local, so that the site effect is modelled by a scaling of the height axis. This is consistent with observations suggesting that poor soil conditions and possibly coastal wind salt spray limit the height that trees can attain.

TABLE 3–Model parameters

Model	a	b	c	t_0	H_0^c
SANDS	local	0.05547	0.8590	1.038	–
CLAYS	60.49	local	0.9152	–	–2.726
KGM3	64.60	local	0.7140	–	–
NAPIRAD	64.13	local	0.6973	–	–
NM90	60.39	local	0.7808	1.225	–
CANTY	53.70	local	0.7402	1.275	–
SGM3	57.40	local	0.5944	–	–

In several of the models, curves not passing through the origin provided no significant or practical improvement, while in others a right-shift of about 1 year produced a better fit. The negative sign of H_0^c in CLAYS precludes interpreting this as an initial height; in fact, the curves cut the time axis at 1.5 years for the average site. (These origins represent averages for the data; the effects of different establishment techniques might be modelled by altering t_0.)

Regional Comparisons

Having completed model coverage of the main regions, it is interesting to compare them and assess any differences. (The SANDS model clearly has a different structure and is excluded from the comparison.)

Site index curves for the various models are shown together in Fig. 3. They have been made to pass through a common height of 13 m at age 10. The differences appear somewhat larger than those for curves with a common site index (height at age 20), but are perhaps more relevant to typical applications where site index is estimated at an early age. Heights at 20, 30, and 40 years are listed in Table 4. Note, however, that in most instances there are few or no data for the older ages (Table 2).

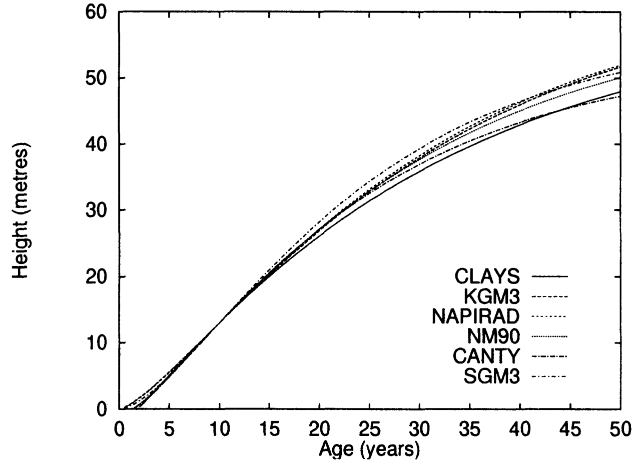

FIG. 3–Site index curve comparison. The curves pass through 13 m at age 10.

TABLE 4–Predicted heights with common height of 13 m at 10 years (Fig. 3)

Model	20 years	30 years	40 years
CLAYS	26.1	35.8	42.9
KGM3	26.9	37.9	45.9
NAPIRAD	27.2	38.2	46.3
NM90	27.2	37.7	45.0
CANTY	27.1	36.9	43.3
SGM3	28.3	39.3	46.5

It is possible to test statistically for model differences by using the likelihood values and estimated standard errors. To facilitate the analysis, a common model form was fitted to each of the data sets, and to the pooled data from all the regions. This model is similar to those previously fitted, but it includes the t_0 term. The parameter estimates and computed log-likelihoods are shown in Table 5. In some instances the estimates in Table 5 differ substantially from those in Table 3, suggesting a degree of over-parametrisation when t_0 is included.

The hypothesis of a common over-all model can be checked with a likelihood-ratio test. Let λ be the maximised likelihood assuming separate models (i.e., the product of the likelihoods over all the models) divided by the one obtained for the pooled data model. Under the null hypothesis, $-2 \ln \lambda$ is asymptotically distributed as χ^2 with the degrees-of-freedom equal to the difference in the number of parameters for the two cases (Wilks 1962; Seber & Wild 1989).

TABLE 5–Parameters and likelihoods for common model form

Data set	*a*	*c*	t_0	log-lik.
CLAYS	56.06	0.800	0.857	524.7
KGM3	64.63	0.712	0.009	3 908.3
NAPIRAD	70.34	0.795	0.682	1 028.3
NM90	60.39	0.781	1.225	2 738.2
CANTY	53.70	0.740	1.275	921.4
SGM3	60.38	0.653	0.562	1 760.0
Pooled	62.19	0.742	0.634	10 235.7

In terms of the likelihood logarithms (log-likelihoods), the statistic is

$$-2 \ln \lambda = -2[\ \Sigma(\text{individual log-likelihoods}) - (\text{pooled log-likelihood})] = 1290.4\ .$$

The degrees-of-freedom is 25, counting the parameters in Table 5 plus the two variances. The probability of a $\chi^2_{25} \geq 1290.4$ is extremely small (the probability of $\chi^2_{25} \geq 44$ is 0.01), so that the hypothesis of a common model can be rejected. A similar result would have been obtained with the models of Table 3.

Accepting that a common national model is untenable, it is still interesting to examine differences between individual models. Some information on this is provided in Fig. 3 and Table 4. Indications of model differences could also be obtained by computing likelihood ratios for various data set groupings, although no valid significance tests exist when a grouping is based on knowledge of the data.

Another comparison can be made by plotting confidence regions for the parameters. For large samples, under certain "regularity conditions" maximum likelihood estimators are asymptotically normally distributed, with variances and covariances that can be estimated from the second derivatives of the likelihood function (Wilks 1962; Seber & Wild 1989). An approximate confidence region for an *n*-dimensional parameter vector θ is then given by an ellipsoid

$$(\theta - \hat{\theta})' \mathrm{C}^{-1} (\theta - \hat{\theta}) \leq \mathrm{k},$$

where $\hat{\theta}$ and C are the estimated mean and covariance matrix, and *k* is the value of χ^2 with *n* degrees-of-freedom for the desired confidence level. Therefore, using the estimated standard errors s_a and s_c and correlation coefficient *r*, approximate 95% confidence regions for model parameters *a* and *c* can be computed from

$$\frac{1}{1-\mathrm{r}^2}\left[\frac{(a-\hat{a})^2}{s_a^2} - 2\mathrm{r}\frac{(\mathrm{a}-\hat{a})(\mathrm{c}-\hat{c})}{s_a s_c} + \frac{(\mathrm{c}-\hat{c})^2}{s_c^2}\right] \leq 5.99;. \qquad (7)$$

The confidence ellipsoids obtained for the models in Table 5 are shown in Fig. 4 (the differences in t_0 seem of little consequence).

The regularity conditions invoked above are difficult to verify in this situation, although the regions could alternatively be interpreted from the point of view of likelihood inference (García 1983). In any event, a number of approximations and fine points of interpretation are involved, so that Fig. 4 must be considered only as indicative.

DISCUSSION AND CONCLUSIONS

The estimation procedure has worked reliably, making full use of the information available in the data. In New Zealand, it has been used with other species—Douglas-fir,

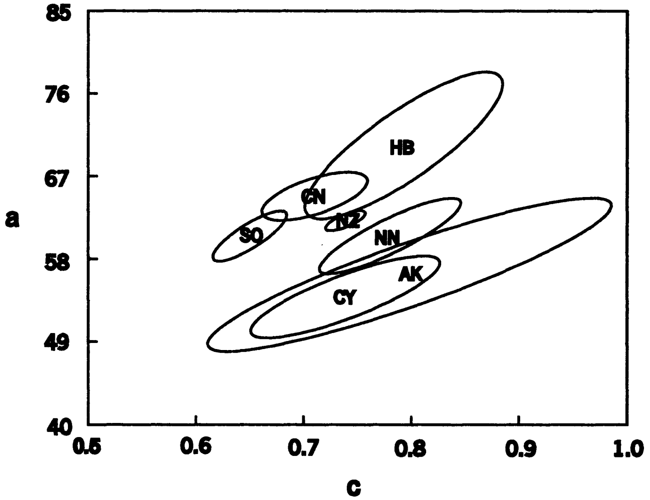

FIG. 4—Confidence regions for parameters a and c of the models in Table 5.
Key for data sets: AK = CLAYS, CN = KGM3, HB = NAPIRAD, NN = NM90, CY = CANTY, SO = SGM3, NZ = pooled data.

Eucalyptus saligna Smith, and *Acacia melanoxylon* R.Br. (Australian blackwood)—in some instances obtaining satisfactory results with very small data sets. The software is freely available and has also been used successfully with *Picea abies* (L.) Karsten (Norway spruce) in Czechoslovakia (M.Černý, pers. comm.), *Pinus sylvestris* L. (Scots pine) (Ortega Z. 1989) and *P. halepensis* Miller (Aleppo pine) (Erviti A. 1991) in Spain, and *Eucalyptus* spp. in Australia (M. Rayner, pers. comm.) and Chile (García 1995).

The results indicate that a common set of site index curves for the whole of New Zealand would not be satisfactory. And despite the height growth patterns in KGM3 and NAPIRAD being very similar, there seem to be important differences in other aspects of stand development. Likewise, the practical relevance of similarities between models for non-adjacent regions, such as CLAYS and CANTY, is probably limited.

The regionalisation used appears natural, and seems to have struck a good balance in the usual modelling conflict between bias and precision. Some refinements of the boundaries could be useful, especially in the central North Island and possibly between the lower South Island and Marlborough areas.

The regions must be seen as a convenient stratification of growth "patterns", represented by global parameters, with local parameters expressing differences in "productivity" varying continuously within them. In the future, more data might indicate an increased number of strata as desirable. Alternatively, models could be localised by increasing the number of local parameters (García 1979, 1994).

In principle, it should be possible to correlate global parameters with continuous environmental variables, instead of stratifying into discrete regions. Some variables, however, are naturally discrete (geology and soils, for example), and strong correlations between others (altitude, temperature, precipitation, topography) would make it difficult to prove causal relationships as opposed to after-the-fact rationalisations. Environmental variables, notably topography, might be more useful for predicting shorter-range variations in productivity.

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