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Lessons from the global financial meltdown: minimising risk by enhancing value creation in land and water management

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Abstract

"Risk intelligence" is a necessary adjunct to management and planning in land and water resource protection and productivity. Following a brief review of how risk is quantified, packaged and sold in the financial sector, risk management in land and water resource management in New Zealand is assessed in the light of the 2007–2009 global financial meltdown. Conventional planning approaches for scenario planning do not accommodate the wide complexities of risk management because of their inability to adequately quantify and forecast risk for value creation. A case study using a farm in Rotorua, New Zealand, is employed to explore how farming risks in terms of nitrate leaching, phosphorus loss, sedimentation and biodiversity loss, may be minimised in exchange for value creation. Historical variance and trends of simulated variables is estimated through random sampling for future trends using Beta, Triangular and Two-sided Power distributions and Monte Carlo simulations. Despite all three statistical distributions performing relatively well, the choice of the most appropriate one will ultimately be determined by expert judgement.

Keywords: global financial crisis; Monte Carlo simulation; Pareto set; probability density functions; risk management; uncertainty.

Introduction

The 2007–2009 global financial crisis can be considered as occurring because risk was mismanaged, amongst other reasons such as deregulation of banks. Mismanagement was justified through faulty modelling, and the identified opportunities in the form of "credit-default swaps", were in fact a monumental liability that led to a global credit and liquidity crisis. It is important to try and understand how the financial sector, which was supported by one of the world's most sophisticated risk intelligence systems (Dittmar & Kobel, 2008) put together by brilliant minds of ex-physicists failed so miserably to turn financial risk into opportunities. One consequence of the 2007–2009 global financial crisis is a concentration on risk mitigation in a variety of areas that is likely to interfere with the recognition of associated investment opportunities. This paper considers lessons learnt from the recent global financial crisis and applies them to evaluating how risk in land and water management can be turned into opportunities. Land and water are truly "too big to fail" and even if there is a magic bullet to bail society out of any collapse, the disruption to civilisation would be unprecedented and the recovery painfully slow. This work uses an investigative perspective that focuses, in particular, on the modelling approaches and models used to frame risk.

History of risk management in financial systems

Traditionally, financial risk management has taken the form of some type of insurance cover. The lack of alternatives was largely related to difficulty in measuring and managing credit risk in loan portfolios. Although

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some futures and options contracts were written, they were few in number because of unreliability in pricing. Interest-rate risk was low due to the prevailing theory that a firm's value was not affected in most cases by capital or hedging (Modigliani & Miller, 1958), and to Sharpe's (1964) view that risk could be eliminated through portfolio diversification by investors (Buehler et al., 2008a). Black and Scholes (1973) suggested that pricing and mitigation of risk could be made more effective by calculating the value of an option to buy a security. Their model was based on the assumption that the price of a security mimics the random physical process of Brownian motion. In fact, real markets do not exhibit random movements. Their value tends to rise slowly and fall rapidly (Overbye, 2009).

The Black–Scholes model of 1973 was unique in that it placed value on managerial flexibility. Traditional, straightforward calculations of net present value (NPV) had been based previously on an "all-or-nothing" attitude towards an investment or project. Since then, all instruments, capital structures and business portfolios contain options that can expire, be exercised or be sold (Buehler et al., 2008b). With the advent of personal computers, and of VisiCalc (the first relevant computer spread-sheet) in 1979, the pricing and trading of options became easier.

By the beginning of the 1990s, contracts covered a wide range of risk, including change in currencies, equities, interest rates, energy, metals and other commodities. Regulation of the ability to hedge or transfer credit risk decreased, and careless use of credit default swaps (CDSs) based on intangibles, rather than physical assets, brought about banking failures.

Pitfalls resulting from a lack of understanding of risk management that culminated in the 2007–2009 global financial crisis can be summarised as follows:

- Short-term (e.g. 10-year) data led to erroneous forecasts and overconfidence or under-confidence in investment;
- Risk–scenario planning was limited and was based on most-likely or average events. Less-likely and extreme events were ignored;
- Transfer of risk amounted to "kicking the can down the road" but did not eliminate it;
- There was no systems–analysis thinking so that the effects of externalities were not taken into account in order to improve understanding of unintended consequences;
- Only certain stakeholders (the banks but not the regulators), designed the scenarios for risk assessment. There was a complete disregard of decentralised decision-making, which although difficult to coalesce to a consensus, produces a more robust outcome; and
- Remuneration for executives was based on short-term performance.

Lessons from the 2008 global financial crisis

Mohan (2008) defined risk management as a combination of: rigorous modelling; embracing precepts of good management and regulation, true democratic consultation of many in a firm; involvement of the board; and remuneration to executives tied to long-term performance and not short-term gains. We concentrate here on the central problem of a lack of understanding of risk management.

The financial sector used risk models that showed a loss of liquidity and ensuing credit squeeze could happen only once in the lifetime of the universe, that is once every 13.7 billion years (Cohan, 2009). People forgot that models are only as good as the data in them. The data inputted into risk-management financial models generally covered a short period of the previous decade, which was a period of growth and euphoria. Had the models been fitted more appropriately to historic periods of economic stress, the world's financial systems could have been in a far better state today (Nocera, 2009). This assessment is not universally accepted. An alternative viewpoint is that the correct risk management models were used and there was plenty of talent, but that those life-and-death decisions about risk were concentrated in the hands of a few people at the top of just a few organisations (Mohan, 2008).

Another problem with the risk-management models used prior to the 2007 financial crisis was the sole reliance on one measure (daily Value at Risk (VaR)) rather than a raft of integrated measures that would help in assessing what is obviously a multi-faceted problem. Value at Risk essentially measures the maximum amount of money you might lose at a given probability level. For instance, a VaR of $100 million at the 1% level means that you have only a 1% chance of losing more than the amount over the next day. A bank assigns an upper limit to the VaR it is willing to accept and declares, on a quarterly basis, the number of times in the previous quarter that the profit and loss statement showed a loss higher than the daily VaR (Stulz, 2009). The problem with VaR is that (Stulz, 2009):

- A firm using daily VaR to protect itself from losses may have insufficient capital to support the risks it is taking;
- Daily VaR does not capture catastrophic losses that have a small probability of occurring; and
- Daily VaR measures assume that assets can be sold quickly or hedged, so a firm can limit its losses within a day, although experience shows that a dramatic withdrawal of liquidity from the markets can leave firms exposed for weeks or months on positions from which they cannot easily unwind.

However, Taleb (2007), argues that it is not the scenario that falls within 99% probability that matters most but

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rather what happens in the 1% probability zone. Long-term historic data that cover periods of distress shows that extreme events take place more frequently than most humans are willing to contemplate (Nocera, 2009). Rare and unpredictably large deviations, such as the collapse of Enron's stock price in 2001 or the spectacular rise of Cisco in the 1990s, have a greater dramatic impact on long-term returns than "variance", which disregards the big market moves (Mandelbrot & Taleb, 2005). Taleb considered that all current financial risk models (i.e. standard deviation, the Sharpe ratio, variance, correlation, alpha, VaR, Black–Scholes option-pricing model and so on) are flawed. This is because they are based on the principle of the statistical bell curve, which focuses on average values and ignore big departures from the mean.

Mandelbrot and Taleb (2005) advocated a different approach to modelling risk – "fractal theory of risk". In a fractal market, variance and volatility are unlimited. This means that risks are much larger than would be expected by standard theory. The theory on chaos and fractal geometry is beyond the scope of this paper and interested readers should refer to Mandelbrot and Hudson (2004). Unfortunately, this theory has not yet been adopted in educational curricula for Masters of Business Administration, from where many financial analysts get their training.

Lessons in risk management for land and water managers

Although the pitfalls listed in the previous section may seem to be unique to the financial sector, there are some similarities in how risk in land and water management is treated, i.e. short term gains are a major driving force in risk management for land and water. Risk management here is driven by the short-term agendas of successive governments. With limited funding, research organisations and some private companies avoid investigating risk management but instead opt for conventional risk avoidance science (which is affordable though less effective). It is this kind of an environment that can spur catastrophes if it goes unchecked.

Risk avoidance is based on two perceptions. One is to rely on insurance in the event that the unlikely happens. This approach is expensive so some companies just choose not to insure at all and hope that nothing goes terribly wrong. The other is to be careful enough not to make mistakes. This approach involves meticulously planning through a large reliance on simulation modelling to investigate different possible scenarios. In scenario planning, three alternatives are considered, plus or minus some percentage of the standard deviation from the expected. However, this approach does not capture or reflect the possibility of big departures from the mean occurring that have a low probability. Although unlikely, such extreme events have a big impact that dramatically changes the system and are felt for a long period of time. Risk analysis in land and water management is also based on the principle of the statistical bell curve, which focuses on the mean and the small departures from it, ignoring the bigger departures. This is a second similarity with the financial sector. As noted earlier, despite the financial sector having a good grasp of risk management, misuse/inability to use models, led to erroneous quantification of risk.

Tools and markets for risk transfer can enable companies to identify, value, and trade much of the risk that they would otherwise be forced to absorb. Risks that cannot be traded can often be contracted out to third parties or consolidated in business units and sold. Meanwhile, the company can focus on management or even acquisition of specific risks in which they have some competitive advantage. In land and water management, quantifying risk has to go beyond scenario planning or "business-as-usual", and that requires strategic research investment supported through the government and private sector. This research should focus on the ability to quantify risk robustly (as opposed to accurately), which involves adapting conventional models to take account of uncertainty then combining it with expert knowledge/judgement (Leskinen & Kangas, 2000).

The process of combining simulated outcomes with expert evaluation captured via probability distributions and Monte Carlo simulations to achieve reliable risk quantification is a relatively new concept in management of New Zealand's land and water resources. Supporting, resourcing and implementing this concept is a nontrivial task that will certainly benefit from a strategic visionary championing by leading primary industry governance. It will also avoid pitfalls from the "Flaw of Averages"[1] (Savage, 2003) thereby determining robust forecasting. It is our belief that such an approach to quantifying risk will guard against misuse of models and may give better insight into packaging risk. However, for land and water management, packaging helps to focus innovation in the creation of value from risk, unlike the financial sector that sells risk. Note that many fundamental operational, financial, marketing, and strategic choices involve rigorous quantification of risk.

The New Zealand context

New Zealand's economy is based on agricultural exports, mostly meat, dairy products, forest products, fruits and vegetables, fish, and wool. Until the 1930s,

[1] Savage (2003) summarised the underestimation of risk by assuming average conditions (which is similar to bell-curve assumptions) as the Flaw of Averages. The Flaw of Averages states that plans based on the assumption that average conditions will occur are usually wrong.

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New Zealand's dairy export quota was around 35% to 45% of its total exports. However, the country lost its preferential trading position with the UK in 1973 that led to a decline in commodity prices for these exports (O'Connor et al., 2007). Consequently, New Zealand's adjusted gross domestic product per capita fell from about 115% of the Organization of the Economic Cooperation and Development (OECD) average, to 80% in 1990 (EconomyWatch, 2011). New Zealand primary-sector commodity prices decreased between 1974 and 2002 (as shown in Figure 1) and since then have remained well below those of a century ago (Hendy et al., 2007). During the same period, the price of manufactured goods declined by about one quarter. The risk of further price decreases in the commodity market could be seen as an opportunity for expanded investment in manufacturing, particularly in high-tech commercialisation, as a means of increasing value (Callaghan, 2009). Recent governments have developed initiatives to develop a higher technology economy, with mixed success.

As with many other economies of the world, there is no silver bullet to spurring growth and sustaining a healthy economy for New Zealand, rather silver buckshot that simultaneously meets economic growth and environmental vitality. The multiple silver buckshot will include creating value from minimising risk for the primary industry, in addition to creating a higher technology economy. This approach may generate higher returns on equity than the operation of traditional risk portfolios (Buehler et al., 2008b).

Below is a list of some examples of realised and potential value creation from minimising risk for New Zealand land and water management:

- The brush-tailed possum (*Trichosurus vulpecula* Kerr), introduced to New Zealand in 1837 is a major pest in New Zealand. It is currently destroying native forest, birds and their eggs, fungi, land snails and fruits (King, 2005). Using possum fur to make a range of garments and hide for making golf gloves has put a value on this resource;
- The habitat of the endemic yellow-eyed penguin (*Megadyptes antipodes* Hombron & Jacquinot) has been threatened in the past but has been successfully rehabilitated and protected (Boessenkool et al., 2009), contributing substantially to ecotourism (Wright, 1998);
- A small company in the Gisborne area is recycling used cooking oil, a pollution risk and converting it to biodiesel (Hale et al., 2006);
- Gorse (*Ulex europaeus* L.), an introduced plant that is now a serious weed in New Zealand, is well-adapted to poor quality soils of low fertility and low rainfall areas (Poteet, 2006). It has a significant amount of oil in its woody branches and stems, making it highly flammable. Although no documentation exists on the amount of oils that could be extracted from harvesting the foliage of the gorse plant, there is a potential of gorse as a biofuel raw material (*The Economist*,

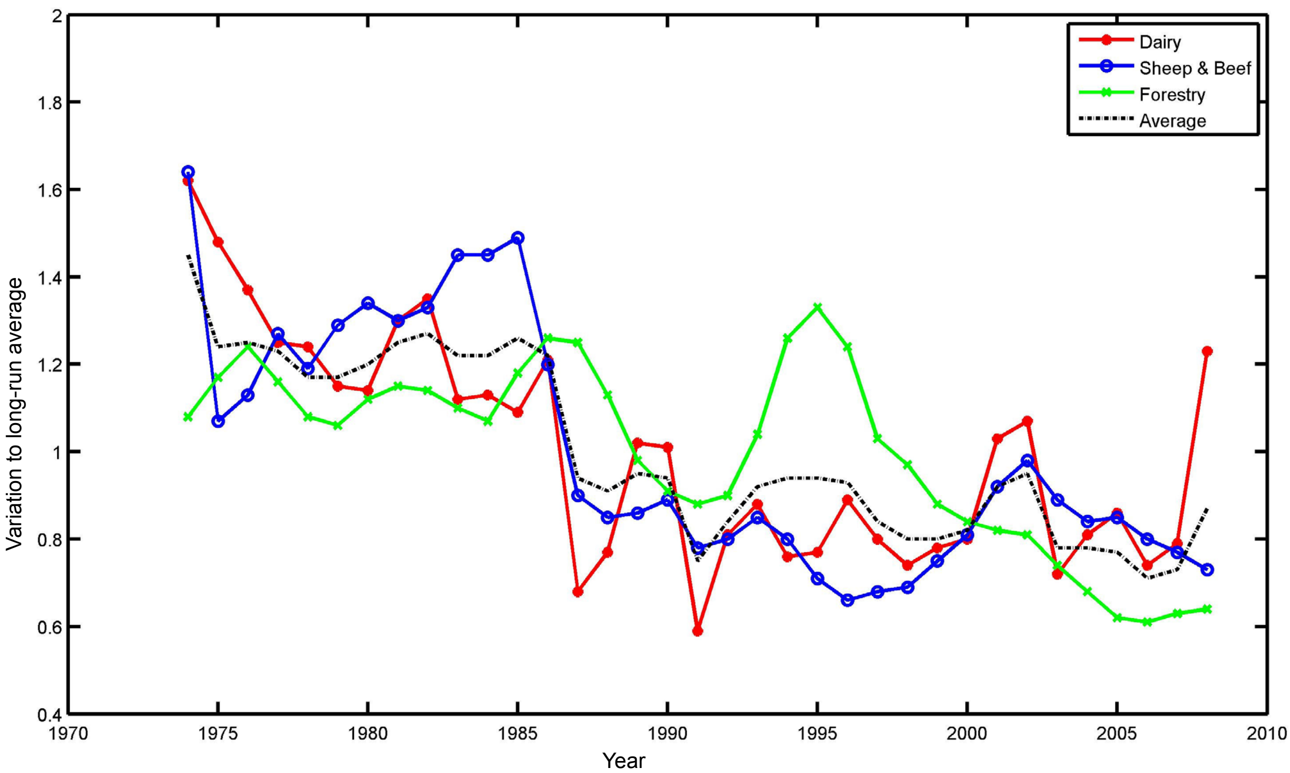

FIGURE 1: Trends in New Zealand primary sector commodity prices from 1974–2002 using data from Hendy et al., (2007) and extended to 2008 under a Motu project (Zhang, pers. comm. 2011).

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2009) or for use as feedstock for producing electricity (Poteet, 2006);

- Biosolids waste has a potential for ethanol production (Wang et al., 2006), burning to generate energy or as a fertiliser for planted forests (Wang et al., 2008);
- The competition between forestry and agriculture could be alleviated by offering plantation programmes to farmers who have less productive land on their farms;
- Reducing the risk of landslides in steep country by planting (high-value) forestry trees, managed through selective harvesting, which in turn create value in the form of a new market for high-value timber and protection against more landslides (Hamilton, 2005);
- Converting livestock manure into a domestic renewable fuel source (biogas) that could be used to help meet renewable portfolio standard requirements (Cuéllar & Webber, 2008); and
- Unlocking much needed investment capital for Maori in Maori-owned land by generating electricity power from geothermal potential or wind farms (New Zealand Geothermal Association, 2009).

Value creation beyond monetary gains

The above examples minimise risk through monetary gains in the form of new industries. Such initiatives can take a long time to establish as they require multiple components to combine at the appropriate time such as, government support for infrastructure of the new industries, private investment from interested parties, and secured market(s) for the new industries. However, when value is in the form of accumulated environmental services, such as clean water, water quantity, rich biodiversity and topsoil sustainability, risk minimisation to create such non-monetary value takes on a whole different dimension, because it is a nontrivial task. The main reasons for this non-triviality are:

- A lack of coherent paradigms to compare and contrast incommensurable environmental services in order to determine optimal “trade-offs” between conflicting demands, in particular, economics and environmental vitality;
- Efforts to measure environmental services with a common denominator, i.e. in monetary terms, so that they are substitutable (e.g. losing $100 billion-worth of pasture land and gaining $100 billion-worth of manufacturing infrastructure without being worse off than before) have been met with nervousness because of the difficulty, in practice, to price these services accurately and appropriately (The Economist, 2012); and
- Limited empirical data.

Despite these limitations, several attempts have been made to use models for defining non-monetary value of environmental services in an attempt to quantify risk. This, in turn can be used to enhance or create more value, which is a significant and important part of land and water value creation.

Risk management for land and water managers

A growing emphasis on mathematical modelling makes much risk management and risk research incomprehensible to most people. Land and water managers must deal with economic, environmental and social factors, and it is difficult to find a balance among conflicting objectives. Consequently, there is a need to quantify risk in terms of trade-offs and also by creating value. Therefore, benefits are a combination of economic and non-economic values.

Traditional valuation methods use discounted cash flow and “real options”, i.e. to make, abandon, expand, or contract a capital investment (Ferreira et al., 2009). This approach does not incorporate the effects of demand, price volatility, social responsibility and environmental vitality necessary for land and water management. Another common problem in the evaluation of social and environmental knowledge is the lack of readily usable information and/or simulation models. In some situations, such information might be descriptive in nature and scattered in numerous different case studies making it hard to collate and exploit. Even where good quality time-series data or simulation models do exist for a particular scenario, it is still not necessarily optimal to rely solely on trends or outputs for forecasting. Often additional information is needed to project realistic scenarios into the future (Leskinen & Kangas, 2000).

External information may come from experts and this raises other issues, such as who are the experts for the task at hand, how are they to present their views and what is the quality of the expert judgements applied (Wright et al., 1996)? Kangas and Leskinen (2005) responded to the first question that there is no rigorous way to assess the competence of experts in a planning situation. As for the presentation of views, the time-tested Delphi technique (Linstone & Turoff, 1975) is a common and simple way of achieving as reliable and consistent judgements as possible within an expert group by means of consecutive questionnaires and controlled feedback. Means and medians of the judgements may be used in the process as a way of capturing the breath and inconsistencies of the judgements. The quality of expert judgements may come down to the statistical distribution and Monte Carlo simuations used to describe the experts’ views.

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With the quantification of risk satisfied, we propose that risk management for land and water may then proceed as follows:

- Forecast risk and identify opportunities to create value from it;
- Aim for a wide margin of profit to allow for potential changes, even disasters, i.e. recognising real-world risk and uncertainty; and
- Insure against loss and price fluctuations, i.e. protect for "worst case" and acknowledge the inconceivable.

Method used for quantifying risk

Expert judgement is at the crux of risk quantification (Raymond, 1999). Quantification of expert judgement was severely compromised in the 2007–2009 financial downturn which led to model outputs being misinterpreted. Quantification of risk involves providing a systematic approach to assessing expert judgements expressed as statements and combining it in a pragmatic and consistent fashion into the mathematical expressions required for use in an analytical model.

The model is developed by ascribing an event to a probability density function representing its likelihood of occurrence, and applying Monte Carlo simulation techniques to the model to forecast the entire range of possible outcomes. Risk is treated as a distribution of random values, rather than as a "business-as-usual" single, fixed or deterministic value. One difficulty is the selection of a valid probability distribution that will reflect the behaviour of historical data. Standard distribution patterns often fail to represent real-world processes in which unusual events are common. The parameters of the selected distribution are often difficult to estimate from sample data or subjective information (professional judgement). Fine-tuning or editing the shape of the fitted distribution is also difficult, partly because a limited number of parameters is available, and partly because no effective mechanism exists for direct manipulation of the shape of the fitted distribution while updating corresponding parameter estimates.

The experts are individually asked to provide a best-case estimate of an outcome in the model. They are also asked to provide an assessment of risk (his or her unsureness) associated with the estimate using risk adjectives of "low", "moderate", and "high". Professional judgement is used to determine the distribution that gives the best representation of this information (Fente et al., 1999). In an era of "better, faster, cheaper" programme management, planning is based on "best-case scenarios" in land and water management. However, many decision makers fail to concede that the probability of achieving the "best-case" goal is by its very nature zero[2] and that faster and cheaper is also riskier.

Sometimes the distribution is selected from subjective information about the mode, specific percentiles or low-order moments (Kuhl et al., 2006). The most common ones are the: Beta distribution model (Gupta & Nadarajah, 2004); Triangular distribution model (Dorp & Kotz, 2003); and Two-sided Power (TSP) distribution model (Dorp & Kotz, 2001) because the parameters defining them are easily determined from the minimum, the most likely, and the maximum value. Details of each of these models are given in Appendix 1.

Examples exist in the literature of where expert judgement and historical data were uitilised for forecasting, such as Galway's (2007) approach. This was based on the work of Morgan and Henrion (1990), Chaloner (1996), and Meyer and Booker (2001) and involved:

- Asking a number of experts to contribute their views on possible risk, taking historical data into account;
- Asking experts and clients to provide upper, lower and most-likely values;
- Fitting a Beta, Triangular or TSP distribution to the data, depending on the information received; and
- Using the "reverse transformation method" (Appendix 2) that employs Monte Carlo simulations for the chosen probability density function, a position was determined and presented back to the experts and clients.

Risk strategy proposition for New Zealand

Changing the face of New Zealand's primary sector through a revamped approach of risk quantification and value identification will benefit small and medium-sized enterprises (SMEs)[3] by increasing market security for the sector and increasing potential GDP growth. Approximately 60 000 farms are classified as SMEs and they have a huge impact on water quantity and quality, greenhouse gas emissions, biodiversity loss, topsoil sustainability, phosphorus loss and nitrate

[2] In general any estimates in land and water management are made up of many independent elements. If each element is planned as best case, say with a probability of achievement of 10%, then the probability of achieving best case for a two-element estimate is 1%; for three elements, .01%; and for many elements, infinitesimal. In effect, it is zero (Raymond, 1999).

[3] SMEs in New Zealand are companies with 19 or fewer employees (O'Reilly et al., 2011), which accounts for 97% of all businesses and approximately 42% of the total economic output.

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leaching. These problems can be viewed as inevitable consequences of food and forest production necessary for the needs of a growing global population. Foley (2010) referred to the intersection of land use, food and fibre production, and maintaining environmental vitality as the “other inconvenient truth”. One approach to dealing with this issue is: (a) to freeze the footprint for agriculture and forestry and set aside the remaining land for existing biodiversity; and (b) to farm smarter producing multiple outputs that will include a range of environmental services.

To achieve such an outcome, involves input at national, regional and local levels. Nationally, coherent, strategic government policies are required to spur creation of value from the risk of high concentration of nutrients and sediments in waterways, depletion of biodiversity, topsoil losses and increases in greenhouse gas emissions. Such policies need to encourage value creation from risk by finding a balance between land use, food and fibre production, and environmental vitality. Getting the balance right is difficult due to competing interests from different stakeholders. Environmental policy is often fragmented into individual initiatives that seem divorced from each other. Multiple policy initiatives encourage disparate solutions that have no sense of connectedness and in some cases conflict with each other. This can result in miscalculation of risks that will lead to sub-optimal value creation, if any, and limited risk minimisation.

Regional strategies should be a mosaic of protocols (i.e. the silver buckshot) specific to the needs of different landowners in any one catchment. Their purpose should be to support decentralised decision-making of landowners in a way that will not compromise regional and national strategies and achieve the desired value creation from risk.

The land and water management of individual farms will be guided by the above regional and national policies. This is a nontrivial task, although there is an ambitious long-term research programme that is currently underway designed specifically to deal with this task (BEACON, 2012). It is still in its infancy and so far the farm level value creation is showing good progress (Chikumbo et al., 2012). Given how critical the farm level plays in ultimately determining the smart regional/national policies, we illustrate it with a case study farm in the Rotorua region.

Of course, local, regional and national strategies are all interrelated. Globally, more attention should be given to decentralised land use decision-making and the paradigms and political doctrines that shape these decisions (Menarchik, 1993).

Case Study: Land use management alternatives for Wharenui Farm, Rotorua, New Zealand

In 2006, a case study was carried out to determine alternatives in land use and management options over a 50-year period that would balance productivity of food and fibre production, profitability, and environmental vitality. The study used Wharenui Farm, owned by Ngati Whakaue Tribal Lands Inc. The farm covered an area of 1492 ha and of this area, 20% (299.2 ha) was allocated to pasture for dairy cattle, 11% (163 ha) to pasture for beef cattle, 36% (533.4 ha) to pasture for sheep and 19% (286 ha) to radiata pine (*Pinus radiata* D.Don) forest. The remaining 14% was occupied by protected forest, roading and other infrastructure (Chikumbo et al., 2011).

A multi-objective optimisation problem was formulated with 11 objectives, which were increased to 14 with the inclusion of three spatial constraints (Chikumbo et al., 2011). Environmental objectives were maintenance or improved water quantity, and reduction in sedimentation, phosphorus loss and nitrate leaching. Profitability objectives were included cost reductions, and increase in revenue, Earnings Before Interest and Taxes (EBIT), and Earnings Before Interest, Taxes, and Depreciation (EBITD). Fibre production was explicitly defined with sawlog and pulpwood production. Food production was not explicit but included in the profitability accounting. A heuristic search algorithm, Metropolis Algorithm (Press et al., 1992), was used to solve the problem. The outcome of this analysis was a recommendation that Ngati Whakaue Tribal Lands Inc maintains diversification, but increases the land area under beef cattle and forestry at the expense of sheep operations. In addition, the current dairy farming operations at Wharenui should be replaced either with a new dairy operation incorporating nitrification (N) inhibitor technology or by beef cattle and plantation forestry.

Heuristic search algorithms are convenient for solving large and computationally demanding problems with a guarantee for good solutions that may be optimal or near-optimal. The longer the algorithm is run (i.e. more computational time), the greater the likelihood to attain optimality.

Although a useful result was obtained for the optimisation problem using the Metropolis Algorithm, the authors acknowledged the weaknesses of the results and i.e. the Flaw of Averages (Chikumbo et al., 2011) and also an inability to find a range of possible solutions (i.e. Pareto set) as opposed to a single solution. Accounting for uncertainty in the optimisation

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inputs by using empirical data and/or time-series trends from simulation models would have provided more insight into the result, giving decision-makers greater capacity to innovate and create value from the farming risks. We now examine how some of these simulation inputs may or should have been evaluated and quantified to avoid the Flaw of Averages.

Returns/costs risk analysis

Forecasting the costs and returns over the 50-year planning period for new and untried management options is even more difficult than for existing options. We attempted, therefore, to demonstrate how subjective opinion could be used to improve forecasting quality of two key components of the original analysis sawlog production and nitrate leaching by using historical data and expert judgement for costs and returns. In the Wharenui model, costs and returns were based on deterministic values adjusted for time value of money through a fixed discount rate over the 50-year period.

For example, a 2003–2006 data-set of prices for high-quality pruned logs, compiled by the New Zealand Ministry of Agriculture and Forestry, was used to demonstrate forecasting trends that could be used in the formulation of the multi-objective formulation by Chikumbo et al., (2011) in an effort to find a robust solution. A clear decline in log prices was observable between 2003 and 2006 (Figure 2). The moving minimum, median and maximum values were, therefore, tracked using exponential functions estimated at R^2 values above 0.9. Maximum likelihood estimation was used to define the two shape parameters, θ_1 θ_2 and required for each of the models in Appendix 1.

Results of simulations based on Beta, Triangular and TSP distributions were estimated with results as shown in Figure 2. Trends derived from the Beta and TSP distributions were similar to those apparent in the empirical data, due largely to flexibility in representation of different forms. Trends from the Triangular distribution captured the effect of the moving minimum, median and maximum but tended to underestimate results. Provided that sufficient historical information for costs/returns is available, the Beta distribution (where shape parameters have

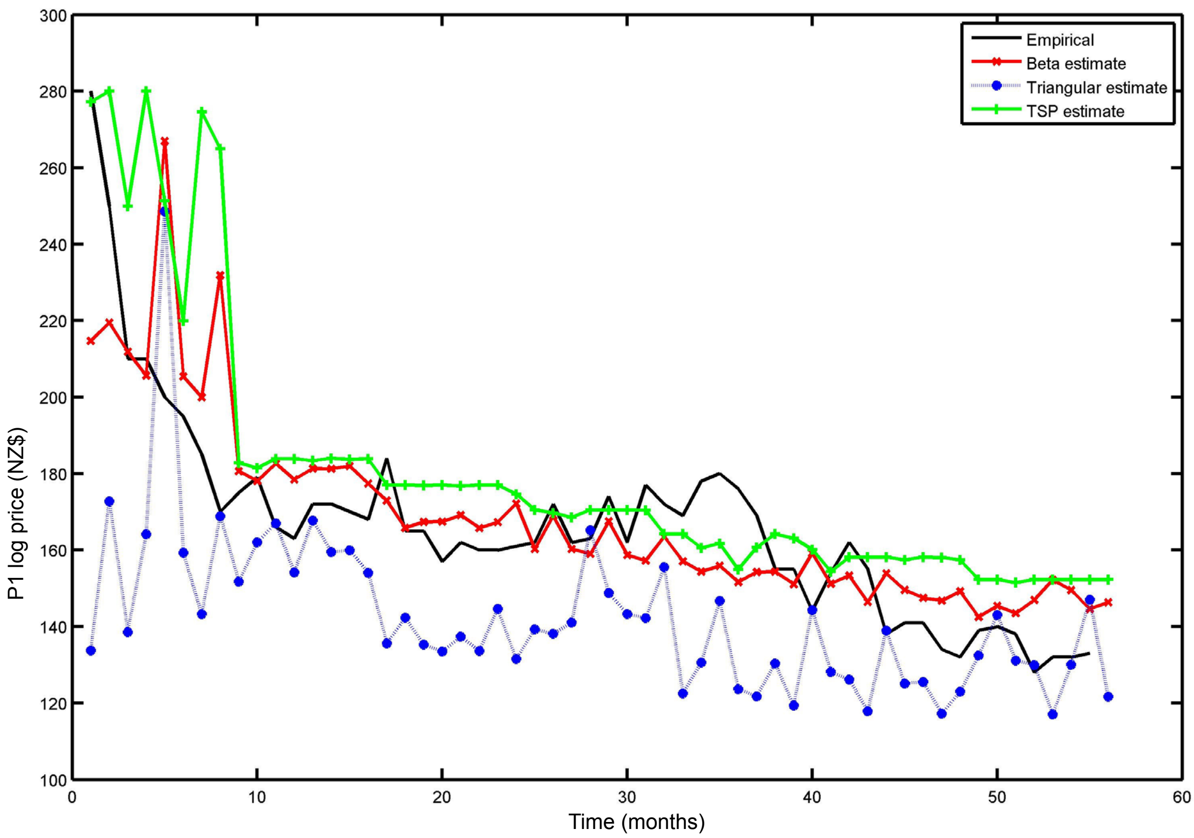

FIGURE 2: Comparison of empirical 2003–2006 log price data with estimates derived from Beta, Triangular and Two-sided Power (TSP) distributions.

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to be estimated first) appeared to be the most useful, on a par with the TSP. However, the TSP would be preferable because of its ease to calculate its shape with a single parameter. A clear downward trend in prices is evident and should have been incorporated in the optimisation formulation. How to do this still remains an active area of research because including the estimates as shown in Figure 2, would certainly result in a noisy search space, which may pose some serious challenges in searching for a good solution (Di Pietro et al., 2004). However, Beyer and Sendhoff (2007) believe that the noisy search space lead to more robust solutions although the optimisation technique of choice becomes limited to direct (randomised) search methods including evolutionary algorithms.

Biophysical trend risk analysis

We also investigated how biophysical variables would be impacted by avoiding the Flaw of Averages. These were nitrate leaching, water quantity (as in runoff), phosphorus loss and sedimentation associated with different land uses. Here we considered nitrate leaching associated with dairy farming because of the massive amounts of fertiliser applied to dairy pasture and urine passed by these farm animals. This was estimated to average 43 kg nitrate ha^{-1} year^{-1} using a model called OVERSEER (Thomas et al., 2005). This value was assumed to remain constant over the 50-year planning period. No time-series data were available, so only the Triangular and TSP distributions could be used. The required minimum and maximum values were derived from results of a trial on a dairy farm in the Rotorua District where monitoring had been carried out over a period of four years. Farm practice, soil type and rainfall conditions were similar to those on the Wharenui property (Burgess, 2003). Minimum (9 kg ha^{-1} year^{-1}) and maximum (121 kg ha^{-1} year^{-1}) values for nitrate leaching were used to simulate a most-likely scenario. Estimates derived from Triangular and TSP distributions are shown in Figure 3.

Triangular distribution estimates were more conservative, less reflective of the observed minimum and maximum values, and showed fewer perturbations than those derived from the TSP distribution. The TSP was, therefore, regarded as the better candidate for representation of nitrate leaching from dairy farming at Wharenui. Note that the TSP estimates were difficult to determine because of the requirement for a shape parameter to be randomly selected within the range [0,1].

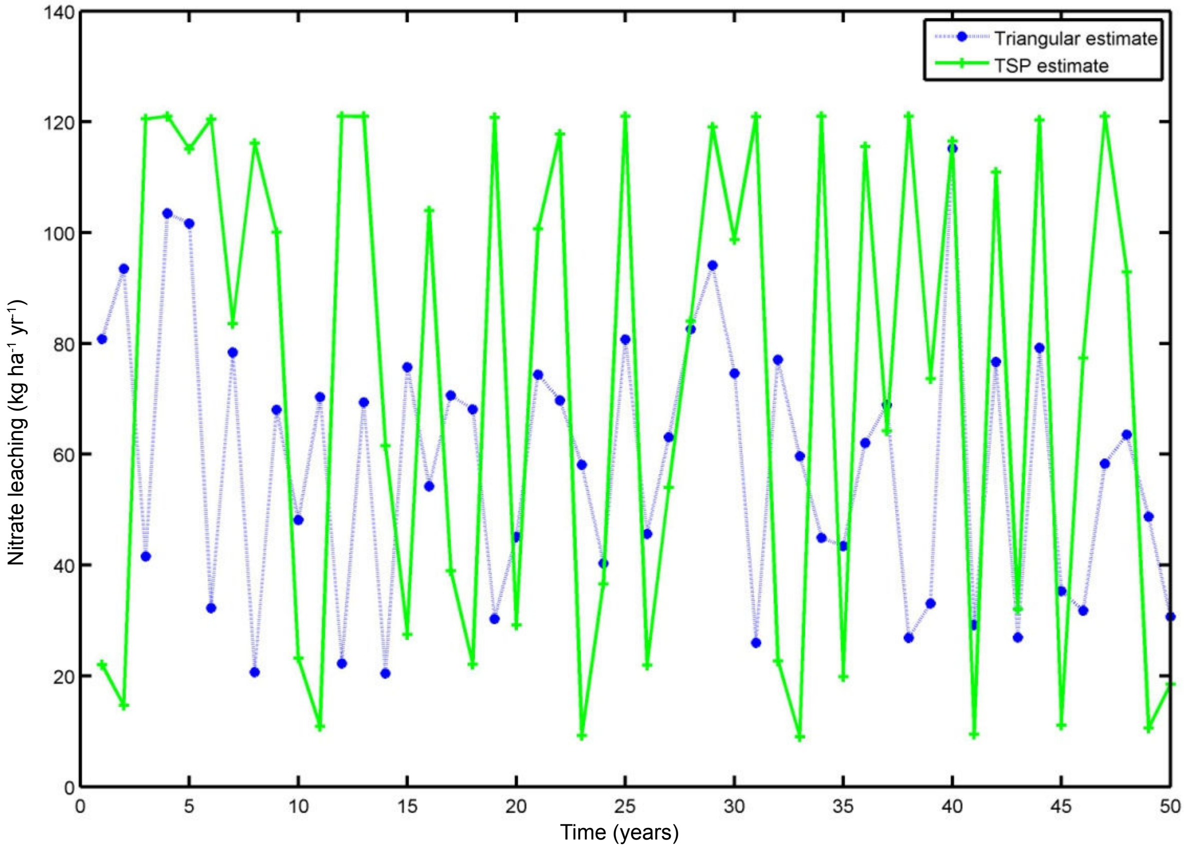

FIGURE 3: Trends in nitrate leaching from dairy farming estimated from Triangular and TSP distributions.

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Discussion

The TSP, despite its relative mathematical simplicity, performed well in all cases, although not as accurate as in some cases as the Beta distribution. However, all simulation inputs to the Wharenui optimisation model (Chikumbo et al., 2011) would have to be individually checked to ascertain which statistical distribution would be the most appropriate one to use, not only for just representing the historical trend and variance, but its adjustability to fit the prevailing socio-economic climate that shape the future, as determined by expert judgement. That means that any of the three statistical distributions may be used depending on which one best represents the historical variance and trend and also captures the expected change in trend as anticipated by the experts. Since this paper was focused on quantification of risk with the view of creating value, we limit our discussion to elicitation of risk based on the Wharenui farm simulation inputs.

Availability of historical data facilitates the simulation of future trends and generates confidence in the inclusion of uncertainties inherent in the real world. On the other hand, reliance on historical information may lead to inadequate exploration/visualisation of innovative options for the future. The simulation approach presented here increases the possibility of finding a robust solution that can accommodate the stochastic dynamics observed in the real world. The use of averages encourages overconfidence in the ability to quantify uncertainty and distorts judgement (Galway, 2007).

Comparisons between results derived from optimisation models using either averages (e.g. Chikumbo et al., 2011) or statistically sampled data as inputs would go a long way towards quantification of risk. Use of a statistically sampled data-set rather than an average is ultimately a significant contribution to risk management in land and water management.

Subjective probability distribution elicitation can improve risk analysis of returns, costs and biophysical-trends in land and water management. Lack of empirical evidence and reluctance of managers to use risk analysis techniques both point to a need for the evaluation of probability techniques, especially in the area of untested management options (Galway, 2004). Currently, risk analysis relies mainly on anecdotal evidence. Land and water managers using risk analysis as a management tool need to be convinced about the usefulness of techniques adapted from those used by the financial sector and this may only be achieved through demonstration of various case studies. We firmly believe that optimisation studies using simulations with statistically-sampled inputs will provide a better foundation for decision-making.

However, a noisy search space would result if forecasted trends as shown in Figures 1 and 2 are used for formulating a multi-objective optimisation problem, such as in Chikumbo et al., (2011), making it hard for any search algorithm to monotonically improve its performance. This is because the noisy landscape will interfere with the evaluation of the search points, causing the search to be noisy according to some probabilistic distribution. That means the “true” search can never be identified as superior points may be eliminated in favour of inferior ones and thereby degrading the performance of the search. Also all search algorithms work on finding the balance between exploitation and exploration, where exploration aims at locating promising zones or niches within the search domain and exploitation locally fine tunes the search within the promising niche (Chikumbo, 2009). With a noisy search space, both exploitation and exploration will be compromised with a heightened chance of pre-convergence. One way would be to systematically sample points using Monte Carlo simulation and calculating the appropriate percentile point at each time but currently this would be too time-consuming (Pietro et al., 2004). Such kinds of computation may become feasible in the future. One way forward currently would be to still design a noisy search space, but with objective functions or fitness functions (as in the case of evolutionary algortihms) that sum up the evaluations, making it possible to monotonically improve the performance. Also, given that it is a multi-objective optimisation there has to be a set of solutions that satisfy all the objectives, i.e. a Pareto set of optimal solutions instead of one, each of which is distinguished by a unique variegation of “trade- offs” amongst all the objectives over the whole planning period (Chikumbo et al., 2012). The solutions show a wide range of trade-offs, giving the experts and stakeholders an opportunity to weigh in most possible outcomes, which will influence the ultimate ranking. Therefore, the inherent concept of combining information and expert judgement still remains intact in this approach, with the added advantage of explicit trade-offs among environmental services that preserves the connectedness of the natural capital, rather than the crude way of putting a dollar value on everything for all kinds of capital substitution. Chikumbo et al., (2012) generated novel land use mixes that will satisfy all objectives, which were not obvious in Chikumbo et al., (2011), raising optimism about determining value creation from risk minimisation in land and water management. However, there still remains much to do at the farm level before the approach of Chikumbo et al., (2012) can be scaled up to identify the smart policy directions at a regional/national level that will help farms to achieve the global objectives of protecting the environment without compromising the local farm objectives.

Conclusion

In land and water management, risk can be regarded as a resource that is largely untapped. Although

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most research is focused on risk mitigation, it is important that value creation from risk minimisation, in both economic and non-economic terms, should be examined following risk quantification based on a systematic approach that combines quantification of expert judgement and simulation modeling. Lessons learned about quantifying, packaging and selling risk, in the financial sector, needs to be applied to land and water management The current downward trend in commodity prices necessitates the identification of new opportunities that could add significant economic value and environmental protection/vitality in a socially-acceptable way, but the hurdles still remain in establishing a robust paradigm for framing trade-offs of environmental services.

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APPENDIX 1: Models used

Beta distribution model

The Beta distribution was first described in the Seventeenth Century. The Probability Density Function (PDF) of the Beta model, widely used in the field of operations research, describes a continuous distribution with both ends fixed at exact locations (Fente et al., 1999). Flexibility is derived from the two parameters that control its shape (θ_1 and θ_2), both of which are real and greater than minus one. Depending on their values, the Beta PDF generates a U-shaped, J-shaped, triangular, or bell-shaped (symmetric or skewed) curve of the uni-modal function. In the absence of sufficient amounts of data, subjective information can be used to estimate the parameters i.e. the minimum, maximum, and any two of the mode, mean, variance or selected percentiles. The Beta Distribution PDF (Johnson et al., 1995) is :

$$f_x(x) = \frac{\Gamma(\theta_1 + \theta_2)}{\Gamma(\theta_1)\Gamma(\theta_2)(b-a)^{\theta_1+\theta_2-1}} (x-a)^{\theta_1-1}(b-x)^{\theta_2-1} \quad \text{for } a \le x \le b$$

where:

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t}\, dt \quad \text{(for } z > 0\text{) denotes the gamma function,}$$

x = random variable;
a = minimum; and
b = maximum.

Triangular distribution model

The origins of the Triangular distribution can be traced back to 1755 (Gupta & Nadarajah, 2004). Johnson (1997) investigated it as a proxy for the Beta distribution. It is specified by minimum, maximum and most likely values and can be symmetric or skewed to the left or right. In some situations it may be strongly skewed i.e. if the most likely value is close to either the minimum or maximum value. It is represented by straight lines between the mode and the minimum and maximum values. These assign too much probability to values near the extremes, and there is a tendency for the tails of the distribution to be over-emphasized. Johnson (1997) has pointed out that a Triangular distribution does not adequately approximate uniform J-shaped or U-shaped distributions.

Recent popularity of the Triangular distribution can be attributed to its use in Monte Carlo simulation modelling (Garvey, 2000; Vose, 1996), discrete system simulation (Altiok & Melamed, 2001; Banks et al., 2005; Kelton et al., 2002) and in standard analysis software. It has the following PDF:

$$f_x(x) = \frac{2}{\theta_2 - \theta_1} \frac{x - \theta_1}{m - \theta_1} \quad \text{for } \theta_1 \le x \le m$$

$$f_x(x) = \frac{2}{\theta_2 - \theta_1} \frac{\theta_2 - x}{\theta_2 - m} \quad \text{for } m \le x \le \theta_2$$

$$f_x(x) = 0 \quad \text{elsewhere}$$

where: m = mode.

Two-sided Power (TSP) distribution model

The four-parameter TSP distribution is an extension of the three-parameter Triangular distribution. It retains the intuitive appeal and meaningful parameters of the Triangular distribution (Dorp & Kotz, 2001) and describes uniform

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J- or U-shaped curves. Estimation of maximum likelihood (MLE) is straightforward and robust, involving only elementary functions. This is in sharp contrast to MLE for Beta distribution parameters (Dorp & Kotz, 2001). The TSP PDF (Dorp & Kotz, 2003) is:

$$f_x(x) = \frac{n}{\theta_2 - \theta_1}\left(\frac{x - \theta_1}{m - \theta_1}\right)^{n-1} \quad \text{for } \theta_1 \le x \le m$$

$$f_x(x) = \frac{n}{\theta_2 - \theta_1}\left(\frac{\theta_2 - x}{\theta_2 - m}\right)^{n-1} \quad \text{for } m \le x \le \theta_2$$

where:

for $n > 1$ the mode of the density function is at m and the value of the PDF at the mode is always $n/(\theta_2 - \theta_1)$,
for and $0 \le n < 1$ and $\theta_1 < m < \theta_2$ the mode of the density function is at θ_1 or θ_2 and $f_X(x) \rightarrow \infty$ at its modes,
for $n = 1$, $f_X(x)$ simplifies to a uniform $[\theta_1, \theta_2]$ distribution, and
for $n = 2$, $f_X(x)$ reduces to a Triangular Distribution.

APPENDIX 2: Simulation of random variables

The simplest way to simulate a random variable, x, with the PDF, $F(x)$, is known as the Reverse Transformation method. This involves two steps:

(i) generation of a pseudo-random number, r, from a uniform distribution in which $r \in [0,1]$; which is used in part (ii) to determine x; and

(ii) calculation of $x = F^{-1}(x)$,

where $F^{-1}(x)$ is the inverse cumulative distribution function of the same distribution, $f_X(x)$, and x the resulting random number.

This method is only applicable to continuous distributions. It cannot be used with discrete distributions e.g. Poisson, Binomial or Geometric (Casella & Berger, 2001).

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