NOTE

SIMPLIFIED METHOD-OF-MOMENTS ESTIMATION FOR THE WEIBULL DISTRIBUTION

OSCAR GARCIA

Forest Research Institute, New Zealand Forest Service,
Private Bag, Rotorua, New Zealand

(Received for publication 10 December 1981)

It is often necessary to find a Weibull distribution with specified mean, variance, and location parameter (e.g., Ek *et al.* 1975; Goulding & Shirley 1979). The usual approach involves the use of some iterative numerical procedure for computing the parameters. An approximation which simplifies the computation and makes it less costly is presented here. This method can be used with a pocket calculator. Greenwood & Durand (1960) used a similar method for maximum likelihood estimation of Gamma parameters.

The (three-parameters) Weibull distribution is

$$F(x) = 1 - \exp\left[-a(x-c)^{b}\right], \quad c \leqq x < \infty ,$$

with density

$$f(x) = ab(x-c)^{b-1}\left[-a(x-c)^{b}\right].$$

The mean and variance are

$$\mu = c + a^{-1/b}\,\Gamma\,(1 + 1/b) \tag{1a}$$

$$\sigma^2 = a^{-2/b}\left[\Gamma\,(1+2/b) - \Gamma^2(1+1/b)\right] \tag{1b}$$

The moment estimators of a and b (c assumed known) are obtained by solving Equations (1) for a and b, using the sample mean and variance.

One way of solving (1) is:

(i) Solve for b

$$\Gamma\,(1 + 2/b)/\Gamma^2\,(1 + 1/b) - 1 = \left[\sigma/(\mu - c)\right]^2 \tag{2}$$

(ii) Compute a from

$$a = \left[\Gamma\,(1 + 1/b)/(\mu - c)\right]^{b} \tag{3}$$

Equation (2) may be solved approximately using any of a number of standard iterative root-finding procedures. The complexities and computational cost of root-finding procedures can be avoided, however, if an explicit approximation for b as a function of $\sigma/(\mu-c)$ is used.

Approximations of the form

$$1/b = z\left\{1+(1-z)^2 \sum_{i=0}^{n} k_i z^i\right\} \tag{4}$$

were obtained, where $z = \sigma/(\mu-c)$ (the coefficient of variation of $x-c$), and the k_i are n coefficients given below. The approximations are good for $z \leqq 1.2$ (i.e., $b \geqq 0.83$). This range includes all the unimodal forms of the density function; the Weibull density has an inverted **J**-shape for $b \leqq 1$ (*see*, for example, Johnson & Kotz 1970).

Four sets of coefficients are shown below, giving increasing levels of accuracy at the cost of increased computational effort. The maximum absolute differences for the approximation of $1/b$ within the range $0 \leqq z \leqq 1.2$ are also shown.

(i) $n=3$, maximum difference $= 3.67 \cdot 10^{-5}$
$k_0 = -0.221016417$
$k_1 = 0.010060668$
$k_2 = 0.117358987$
$k_3 = -0.050999126$

(ii) $n=4$, maximum difference $= 7.08 \cdot 10^{-6}$
$k_0 = -0.219854571$
$k_1 = -0.004622506$
$k_2 = 0.166368610$
$k_3 = -0.110324204$
$k_4 = 0.023514463$

(iii) $n=5$, maximum difference $= 3.64 \cdot 10^{-6}$
$k_0 = -0.220009910$
$k_1 = -0.001946641$
$k_2 = 0.153109251$
$k_3 = -0.083543480$
$k_4 = 0$
$k_5 = 0.007454537$

(iv) $n=5$, maximum difference $= 3.15 \cdot 10^{-6}$
$k_0 = -0.220040320$
$k_1 = -0.001433169$
$k_2 = 0.150611381$
$k_3 = -0.078575996$
$k_4 = -0.004305716$
$k_5 = 0.008804944$

Summarising, the procedure for computing the Weibull parameters consists of computing b using (4), and then finding a from (3). The polynomial in (4) is best evaluated using Horner's method:

$$\Sigma k_i z^i = k_0 + z(k_1 + z(k_2 + z(\ . \ . \ . \ ))).$$

If the Gamma function is not available, the following approximation may be used (Abramowitz & Stegun 1972):

$$\Gamma(1+x) = 1 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5$$

$$\begin{aligned} a_1 &= -0.5748646 \\ a_2 &= 0.9512363 \\ a_3 &= -0.6998588 \\ a_4 &= 0.4245549 \\ a_5 &= -0.1010678 \end{aligned}$$

$$\text{error} \leqq 5 \cdot 10^{-5} \text{ for } 0 \leqq x \leqq 1$$

$$\left[\text{for } x > 1 \text{ use the fact that } \Gamma(z+1) = z\,\Gamma(z)\right].$$

REFERENCES

ABRAMOWITZ, M.; STEGUN, I. A. (Eds) 1972: "Handbook of Mathematical Functions". Dover, New York.

EK, A. R.; ISSOS, J. N.; BAILEY, R. L. 1975: Solving for Weibull diameter distribution parameters to obtain specified mean diameters. **Forest Science 21:** 290–2.

GOULDING, C. J.; SHIRLEY, J. W. 1979: A method to predict the yield of log assortments for long term planning. **In** Elliott, D. A. (compiler) "Mensuration for Management Planning of Exotic Forest Plantations". **New Zealand Forest Service, FRI Symposium No. 20.**

GREENWOOD, A. J.; DURAND, D. 1960: Aids for fitting the gamma distribution by maximum likelihood. **Technometrics 2**: 55–65.

JOHNSON, N. L.; KOTZ, S. 1970: "Continuous Univariate Distributions - 1". Houghton Miffling Co., Boston.