INDICATIVE GROWTH AND YIELD MODELS FOR EVEN-AGED EUCALYPTUS FASTIGATA PLANTATIONS IN NEW ZEALAND

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ABSTRACT

Eucalyptus fastigata Deane & Maiden stand growth and yield data were collected from 66 permanent sample plots and 45 temporary plots, sampling even-aged plantations located in six geographic regions between Nelson/Marlborough and Northland, New Zealand. Height and volume growth, and volume yield functions were fitted to the *E. fastigata* data through non-linear least squares and multiple regression. Site index and volume growth curves that encompass the range of available data were created. Log grade recovery was predicted as a function of average tree size using MARVL (Method of Assessing Recoverable Volume by Log-types) log grade outturn data, and this indicated that recovery of sawlog grades increases markedly with increasing mean tree volume and diameter. Growth data and model predictions show that *E. fastigata* growth rates vary widely between sites, and that the species is capable of maintaining rapid growth rates to later ages on favourable sites.

Keywords: stand growth; grade recovery; yield models; yield tables; difference equation; non-linear mixed model; *Eucalyptus fastigata*.

INTRODUCTION

The growing of eucalypts in New Zealand has, over the years, been particularly challenging. Not only has the preferred species for planting changed, but also it has been necessary to adopt different approaches to address the research requirements for each of the species. Recently, the forestry industry has shown an interest in growing *Eucalyptus fastigata* in parts of the North Island for both fibre and sawlog production. *Eucalyptus fastigata* has attractive timber, is reasonably frost tolerant, and “has been successful on farm sites in many areas, especially in the central North Island” (Miller *et al.* 2000). *Eucalyptus fastigata* timber “is at least as stable as

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Pinus radiata (D. Don), both in the short and long term" (Haslett 1988). Also, volume and taper equations have been developed for *E. fastigata* (Smart 1992).

Demand for growth and yield information has prompted development of predictive models using a mixture of temporary and permanent sample plot data collected from *E. fastigata* stands around New Zealand. Temporary inventory plots were established in many of the known stands of *E. fastigata* around northern New Zealand, and MARVL (Method of Assessing Recoverable Volume by Log-types) assessment of the trees was undertaken (Deadman & Goulding 1979). These data complement successive measurement data collected from permanent sample plots (PSP) (Pilaar & Dunlop 1990) within *E. fastigata* stands, regime trials, and one Nelder spacing trial (Nelder 1962). Repeated measurement data from permanent sample plots are needed to develop basal area growth, mortality, and thinning functions that are combined with height and volume functions to generate predictions of stand growth. The single cross-sectional measurement data from temporary plots can be used only to fit yield models.

Rather than predicting stand growth over time, yield models predict values for the dependent variable such as height or volume at a given point in time, as a function of explanatory variables such as height, age, and stocking at the time of harvest. Tree or stand height is typically predicted as a function of age, and combined with age and target final-crop stocking to generate predictions of standing volume per hectare for a given clearfell age (e.g., West & Mattay 1993; McElwee 1998). An upper limit of tree size and density can be defined and used to constrain projections within realistic biological limits. Reineke (1933) proposed a stand density index (SDI), defined as the number trees per acre in an even-aged stand with an average diameter at breast height of 10 inches [25.4 cm], and reported maximum stand density index for various species growing in even-aged stands in California. Maximum stand density indices of approximately 500 and 1000 were reported for *E. globulus* Labill. and *Sequoia sempervirens* (D. Don) Endl., respectively. The stand density index has the advantage of being independent of site index and age (Reineke 1933).

Total standing volume predictions are useful for above-ground carbon or biomass estimation. However, total standing volume can include a significant volume of residues and cutting waste. Residues consist mainly of stumps, forks, and breakage, whereas cutting wastes are portions of stem lost during log-making for optimal solidwood grade recovery. Predictions of residue volume can be subtracted from TSV to yield estimates of whole-tree "fibre" volume. Cutting waste may be included within the fibre volume when whole trees or long logs are extracted for pulping, and should therefore be predicted separately. Predictions of volume recovery by log grade are useful when performing economic analyses of forestry investments and designing plantation management regimes. McElwee (1998) used

MARVL data collected from even-aged poplar (*Populus* spp.) plantations in New Zealand to develop log grade recovery models.

This paper outlines the development of indicative stand-level growth and yield models for even-aged *E. fastigata* plantations in northern New Zealand. Five sigmoid functions were evaluated as height and volume growth models. Models were developed to predict total standing volume yield as a function of stocking and age, and log grade recovery and waste as a function of average tree size. The individual tree volume function developed by Smart (1992) was rearranged to predict average tree size, and tested. These models can be used in combination to predict total standing volume per hectare, approximate tree size, and log grade recovery based on inputs of site index or height-age data and stand density (stocking). The data, model-fitting methodologies, resultant models and their limitations, and examples of *E. fastigata* growth and yield predictions are presented and discussed.

METHODS

Data

The *E. fastigata* stand growth and yield data were collected from 66 permanent sample plots within managed stands, regime trials in Northland (n=12 plots) and the central North Island (n=14 plots), one central North Island Nelder spacing trial (n=15 plots), and 45 temporary MARVL plots, across six geographic regions of New Zealand (Table 1). The permanent sample plot and MARVL stand summary data were combined, giving stand-level data from a total of 111 plots within 14 forests (Table 2). The majority of data were collected from stands younger than 20 years old. Mean top height and diameter were defined as the average height and the diameter at breast height 1.4 m (dbh) of the 100 largest-diameter trees per

TABLE 1–*Eucalyptus fastigata* permanent sample plot (PSP) and MARVL sample plot count, oldest stand age recorded, and maximum mean annual volume increment by geographic region.

Region	PSP	MARVL	Total No. plots	Max. age (years)	Max. MAI (m^3/ha)
Northland	12		12	9.4	49.4
Bay of Plenty	18	1	19	18.2	41.2
Central North Island	34	27	61	66	38.6*
East Coast		13	13	19	37.5
Wanganui/Manawatu		2	2	61	24.9
Nelson/Marlborough	2	2	4	60	19.4
All regions	66	45	111	66	49.4*

* Excluding one PSP: age 9 data; 4000 stems/ha; max. MAI = 88.7 m^3/ha.

TABLE 2–Combined *E. fastigata* permanent sample plot and MARVL yield modelling dataset summary (n = 438).

	Age (years)	Stocking (stems/ha)	Mean top diameter (cm)	Mean top height (m)	Basal area (m^2/ha)	Volume (m^3/ha)	Volume MAI (m^3/ha)
Mean	12.4	1111	29.4	19.4	29.3	204	14.4
s.d.	9.8	1000	17.2	10.4	24.9	230	12.1
Min.	2	21	2.5	2.6	0.06	0.1	0.04
Max.	66	5000	100	58.4	138	1520	88.7

hectare, respectively. Total standing volume per hectare was calculated as the sum of individual tree volumes (inside bark) predicted by a tree volume equation.

Live MARVL plot stems were separated into quality zones defined by height above ground, size, and quality codes (*see* Table 3). Stem sections were then assigned to log grades in accordance with log size and quality criteria (listed in Table 4). Total standing volume and volume yield by log grade were calculated for each MARVL plot — three poorly-represented sawlog grades (B, S, and L) were combined. Log grade yields, cutting waste, and residue volumes from each plot were converted to percentages of total standing volume (Table 5).

TABLE 3–MARVL assessment quality codes for live stems. Branch diameter estimated at junction with stem. Sweep assessed with respect to log diameter (D).

Quality code	Quality criteria
A	Pruned or unbranched
B	Unpruned, small branches (less than 7 cm), sweep under D/4
C	Unpruned, large branches (more than 7 cm), sweep under D/4
S	Unpruned, sweep D/4 or greater
T	Unpruned, large branches (more than 7 cm), sweep D/4 or greater
P	Pulp
W	Waste

TABLE 4–MARVL assessment log size and quality codes. Log diameter estimated at small end (s.e.d.). Quality criteria defined in Table 3.

Log grade	Log length (m)	Minimum s.e.d. (cm)	Quality codes
Unbranched	3.1–6.5	40	A
A grade sawlog	3.1–6.5	35	AB
B grade sawlog	3.1–6.5	35	ABC
S grade sawlog (with sweep)	3.1–6.5	30	ABCS
L large branch sawlog (with sweep)	3.1–6.5	30	ABCST
Pulp	3.0–11.0	15	ABCSTP

TABLE 5–MARVL log grade data summary. Log grade outturn expressed as percentages of total standing volume.

	Unbranched sawlogs	A grade sawlogs	B, S, & L grade sawlogs	Pulp	Waste
Mean (%)	5.3	13.2	4.8	56.7	20.0
s.d.	11.4	17.7	5.8	15.0	10.9
Min.	0	0	0	12.1	4.4
Max.	47.0	82.7	22.8	78.5	47.3

Analysis

Height and volume growth models were developed using permanent sample plot data. Five sigmoidal functions commonly used to model biological growth were tested: the Chapman-Richards (Richards 1959; Pienaar & Turnbull 1973), Gompertz (Winsor 1932), Hossfeld II (Hossfeld 1822, cited by Peschel 1938), and Schumacher (1939) functions, and the cumulative form of the 3-parameter Weibull probability density function (Weibull 1939; Yang *et al.* 1978). These equations are shown in yield form with yield (Y) as a function of age (T):

Chapman-Richards: $Y = a(1 - e^{-bT})^c$

Hossfeld II: $Y = \dfrac{aT^c}{b + T^c}$

Gompertz: $Y = ae^{-(e^{(c-bT)})}$

Schumacher: $Y = ae^{(-bT^c)}$

Weibull: $Y = a(1 - e^{-bT^c})$

Mean top height and age data from permanent sample plots with two or more measurements were organised into pairs of consecutive measurements. Anamorphic and polymorphic difference forms of the five candidate sigmoid functions were fitted to pairs of consecutive mean top height and total standing volume measurements through Gauss-Newton non-linear least squares regression analysis executed by the SAS statistical analysis software PROC NLIN procedure (SAS Institute Inc. 1989). Functions were also fitted to the repeated sample plot measurements as mixed models using the SAS macro NLINMIX (Littell *et al.* 1996), where for each sample plot a random error term entered the asymptote (a) or slope (b) parameter of anamorphic and polymorphic forms respectively. Allowing the asymptote (a) or slope (b) parameter to change between sample plots or pairs of consecutive measurements produces families of curves that reflect growth differences between sites, ages, or management. This "local" parameter can be replaced by yield at age T_1 when the yield form is arranged in difference form to predict yield at age T_2. The anamorphic difference form with local asymptote produces curves with a common slope parameter and different asymptotes across the range of input values. The polymorphic form with local slope produces curves with different slopes that converge at one upper asymptote (Clutter *et al.* 1983). Starting values (yield at age T_1) are needed for difference equation projections. Site index, the height of dominants at a given base age, or height-age data reflect local site quality, and provide starting values for height growth projections. Starting values for volume growth models can be collected from local stands or predicted from regressions that consider stocking, age, and site quality.

Site index, defined as mean top height at base age 15 years, was predicted for each measurement using the best mean top height growth model. Average tree size and stand density data were plotted on logarithmic scales, and examined for density-dependent self-thinning patterns. Volume yield models that predict starting values for the volume growth model were fitted as multiple linear and non-linear regression models to all available data, and to data collected in young (age 5–20 years) stands. The data collected from *E. fastigata* stands less than age 20 covered a wide range of stockings and site index estimates. Stocking, mean top height, site index, and age were tested as explanatory variables.

The relationships between MARVL log grade outturn data and explanatory variables average tree diameter, average tree volume, mean top height, stocking, and stand age were examined. Log grade recovery data were expressed as a percentage of total standing volume, and as a cumulative percentage that included recovery of more valuable grades — e.g., cumulative percentage recovery of pulp included recovery in all sawlog grades. Linear, quadratic, exponential, logarithmic, power, and logistic regressions of cumulative percentage recovery in each log grade as a function of each explanatory variable were developed and compared in terms of goodness of fit. The generalised logistic function of the form:

$$Y = \frac{a}{1 + e^{b-cX}}$$

was tested. The individual tree volume function developed by Smart (1992) was rearranged to predict approximate mean tree diameter from total standing volume, stocking, and mean top height:

$$d = \sqrt{\frac{V/N}{aH + b}}$$

where d = mean tree diameter (cm dbh),
V = total standing volume (m^3/ha),
N = stocking (stems/ha),
H = mean top height (m),
and parameter estimates a = 0.0000199; b = 0.0000777 (Smart 1992).

Candidate growth models were tested for goodness of fit across all pairs of measurements, across the range of predicted values, ages, and stockings, and by comparing predictions for the last measurement using the first measurement for each Permanent Sample Plot as starting values. Yield models were tested for goodness of fit across the range of predicted values, ages, stockings, average tree sizes, and site index estimates. Prediction errors were calculated in real terms, as the difference between predicted and actual values. Errors were summarised as the average and standard deviation of all prediction errors, and as the square root of the mean error sum of squares (*RMSE*) calculated by taking the square root of the sum of squared errors divided by the number of degrees of freedom:

$$RMSE = \sqrt{\frac{SSE}{df_{Error}}}$$

The coefficient of determination (R^2) was calculated for each model, adjusted for degrees of freedom, as:

$$R^2_{adj.} = 1 - \frac{SSE / df_{Error}}{SST / df_{Total}}$$

where SSE = error sum of squares,

SST = total sum of squares,

df_{Total} = n-1 observations or consecutive pairs of time series data,

df_{Error} = n-k-1 where k = number of explanatory variables in multiple linear regression models, or the number of model parameters in non-linear regression models including the number of fixed effects in non-linear mixed models.

Overall model significance tests (F-tests) consistently returned probabilities for the model F-value failing to exceed the critical F-statistic (Pr >F) of <0.0001, and were therefore not reported. The statistical significance of individual parameter estimates was evaluated at the 95% level of confidence, when the t-value for linear model parameter estimates exceeded the critical t-statistic (Pr >t) of 0.05, or when the approximate 95% confidence interval for non-linear model parameter estimates did not include zero.

The models were applied in combination to demonstrate the influence of stocking and site quality on total standing volume (m^3/ha) and approximate mean diameter (cm) development in managed stands. Projections were based on age-10 starting values for a range of final-crop stockings (100–1100 stems/ha) on high quality and average sites, defined as the upper limit and average of site index estimates for all data, respectively.

RESULTS

Height Growth

Difference equations were fitted to 311 pairs of consecutive mean top height and age data. Mixed models were fitted to 376 mean top height data from 66 permanent sample plots. Of all models tested, the polymorphic Schumacher mean top height model, fitted in difference form with the slope parameter b isolated (local) in the algebraic difference formulation, exhibited the lowest mean prediction error and *RMSE*. The model also made the most accurate and precise predictions of mean top height at the last measurement based on the first measurement in each sample plot. The polymorphic difference form predicts mean top height H_2 at age T_2 dependent on starting values of mean top height H_1 and age T_1 (Equation 1).

$$H_2 = a^{1-\left[\frac{T_2}{T_1}\right]^c} H_1^{\left[\frac{T_2}{T_1}\right]^c} \qquad (1)$$

Parameter estimates and their standard errors for the polymorphic Schumacher mean top height model ($R^2 = 0.99$) were $a = 118.1$ (s.e.(a) =15.3) and $c = -0.4941$ (s.e.(c) = 0.035). Model fit statistics were calculated for measurement pairs and plots used to fit the model (Table 6).

TABLE 6–Mean top height growth model error statistics for measurement pairs (all pairs) and entire plot history (by plot). Errors expressed in real terms: predicted-actual (m). *RMSE* = root mean square error.

	All pairs	By plot
n	311	66
Mean error (m)	–0.056	–0.100
s.d. of errors	0.833	2.107
RMSE	0.699	0.299

Site index was predicted for each permanent sample plot and MARVL plot, and summarised by region (Table 7). Site index curves that approximately encompass the range of mean top height and age data are given in Fig. 1. Prediction errors were plotted against predicted mean top height for all data pairs (Fig. 1).

TABLE 7–Site index (mean top height at age 15) summary statistics by geographic region.

Region	Total No. plots	Average site index (m)	s.d. (m)	Min. (m)	Max. (m)
Northland	12	34.2	2.2	32.0	38.5
Bay of Plenty	19	30.2	2.8	26.3	36.5
Central North Island	61	25.1	2.4	19.7	30.7
East Coast	13	23.7	3.3	17.8	26.9
Wanganui/Manawatu	2	18.5	1.0	17.8	19.2
Nelson/Marlborough	4	22.1	3.0	19.2	25.7
All data	111	26.6	4.4	17.8	38.5

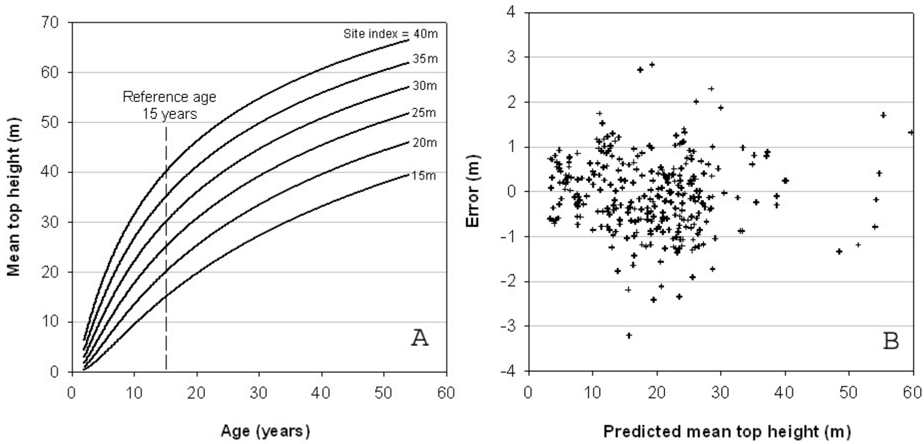

FIG. 1–Mean top height growth model predictions — site index curves that approximately encompass the range of *E. fastigata* height-age data: 15, 20, 25, 30, 35, 40 m site index; base age 15 years (A). Mean top height prediction errors (n=311) (B).

Volume Growth

The polymorphic Chapman-Richards model, fitted in difference form, exhibited the lowest mean prediction error and *RMSE*. The model also made the most accurate and precise predictions of total standing volume at the last measurement based on the first measurement in each sample plot. The polymorphic difference form predicts total standing volume V_2 at age T_2 dependent on starting values of volume V_1 and age T_1 (Equation 2).

$$V_2 = a \left[1 - \left[1 - \left(\frac{V_1}{a} \right)^{\frac{1}{c}} \right]^{\left(\frac{T_2}{T_1} \right)} \right]^{c} \qquad (2)$$

Parameter estimates and their standard errors for the polymorphic Chapman-Richards model ($R^2 = 0.99$) were $a = 1898.5$ (s.e.(a) =184.7) and $c = 2.8787$ (s.e.(c) = 0.122). Model fit statistics were calculated for measurement pairs and plots used to fit the model (Table 8). Volume growth curves that approximately encompass the range of total standing volume and age data are given in Fig. 2. Prediction errors were plotted against predicted total standing volume for all data pairs (Fig. 2).

Volume Yield

The most satisfactory volume yield model predicts total standing volume for stand ages 5–20 years and stockings below 2500 stems/ha as a multiplicative function of the natural logarithm of mean top height H and stocking N (Equation 3).

$$V = e^{a} \, ln(H)^{b} \, ln(N)^{c} \qquad (3)$$

TABLE 8–Total standing volume growth model error statistics for measurement pairs and entire plot history. Errors expressed in real terms: predicted-actual (m^3/ha).

	All pairs	By plot
n	308	66
Mean error (m^3/ha)	–1.4	–37.9
s.d. of errors	17.3	78.8
RMSE	17.4	92.2

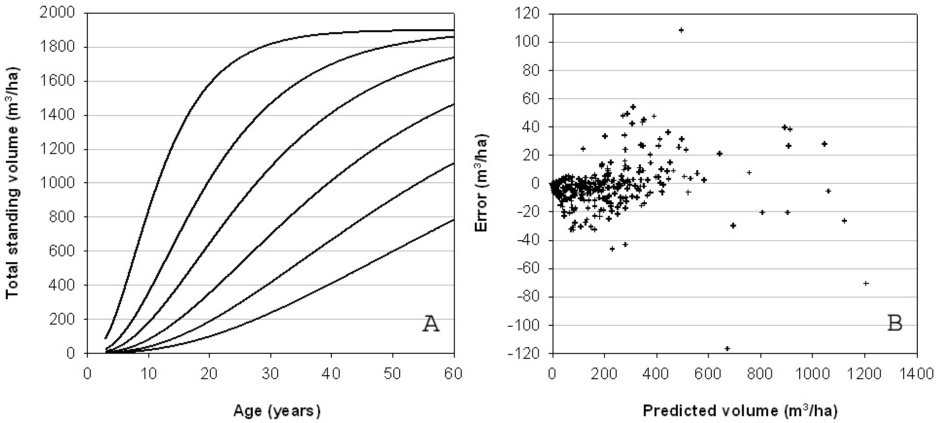

FIG. 2–*Eucalyptus fastigata* volume growth model predictions for a range of starting values that approximately encompass the range of volume-age data (A). Volume growth model prediction errors for all pairs of volume-age data (n=308) (B).

Parameter estimates and their standard errors for the volume yield model ($R^2 = 0.87$) were $a = 0.5840$ (s.e.(a) = 0.04), $b = 1.2667$ (s.e.(b) = 0.05), and $c = 0.4159$ (s.e.(c) = 0.03). Model fit statistics were calculated for all data and for three groups of stockings (Table 9).

Volume yield model predictions for a range of stockings (100–1100 stems/ha) across the range of mean top height data used to develop the model (ages 5–20 years) are given in Fig. 3. Prediction errors were plotted against predicted total standing volume for all data (Fig. 3).

TABLE 9–Total standing volume yield model error statistics for all data (ages 5–20; <2500 stems/ha), and for stockings below 500 stems/ha, 500–1000 stems/ha, and 1000–2000 stems/ha. Errors expressed in real terms: predicted-actual (m^3/ha).

	All data <2500 stems/ha	<500 stems/ha	500–1000 stems/ha	1000–2000 stems/ha
n	302	92	86	86
Mean error (m^3/ha)	–1.8	14.3	–9.9	–11.3
s.d. of errors	64.8	42.5	62.5	82.6
RMSE	61.0	45.3	64.0	84.3

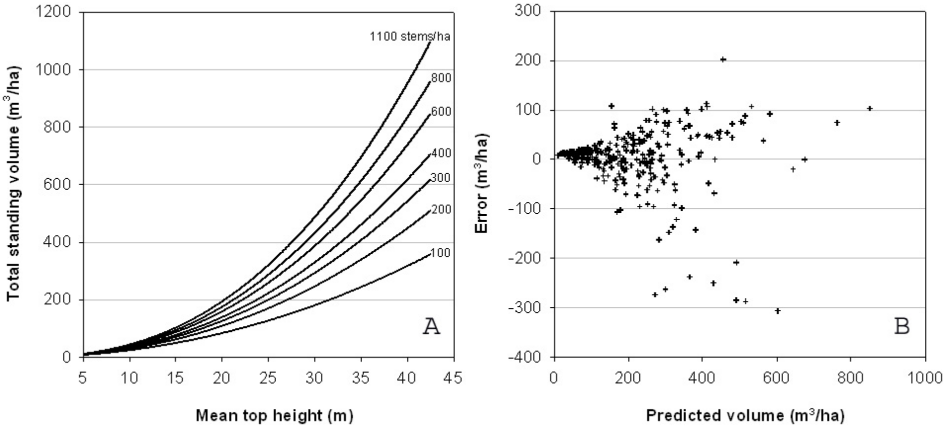

FIG. 3–*Eucalyptus fastigata* volume yield model predictions for final-crop stockings 100–1100 stems/ha across the range of mean top height data used to develop the model (A). Total standing volume yield model prediction errors (n=302) (B).

Volume Recovery by Log Grade

Mean tree volume and mean tree diameter were the most useful explanatory variables predicting log grade recovery. Mean tree volume was the best predictor of percentage recovery for A-grade sawlogs. Mean tree diameter was the best predictor of grade recovery for unbranched sawlogs, pulp logs, and waste grades. Linear regressions were selected to predict the percentage recovery of unbranched logs. The linear models were fitted using weighted least squares with a weight W_i of mean tree volume and diameter squared, respectively, to obtain more efficient estimates from data with unequal error variance (heteroscedasticity), after removing one influential record (zero percentage unbranched log volume; mean dbh 75.8 cm; Cook's D_{volume} = 4.8 and Cook's $D_{diameter}$ = 5; exceeded critical $D_{2,45df}$ = 3.2). Logarithmic models of cumulative percentage of A-grade sawlogs (the sum of A-grade and unbranched sawlog percentage recovery), and total sawlog recovery (the sum of B+S+L grade, A-grade, and unbranched sawlog percentage recovery) as a function of the natural logarithm of mean tree volume and diameter were fitted as ordinary least squares regressions. Logistic functions made the best predictions of cumulative percentage recovery of pulp logs (sum of pulp log and total sawlog percentage recovery), and cumulative percentage cutting waste as a function of mean tree volume and diameter, respectively. Parameter estimates and fit statistics for models predicting cumulative log grade recovery as a function of mean tree volume and mean tree diameter are listed in Table 10. Cumulative percentage recovery by log grade predicted for each grade across the range of mean tree diameters, and prediction errors for unbranched log grade recovery, are given in Fig. 4.

TABLE 10–Regression coefficients and fit statistics for cumulative percentage total standing volume recovery by log grade, as a function of mean tree volume and diameter. Standard errors for parameter estimates in parentheses. Percentage unbranched sawlog models n = 44; other models n = 45.

Log grade	Model	a	b	c	R^2	*RMSE*
Explanatory variable: mean tree volume						
Unbranched	Linear	-5.254^{ns} (6.4)	9.548 (1.9)	–	0.36	20.3
A-sawlogs	Logarithmic	25.86 (1.6)	22.21 (1.9)	–	0.76	10.1
Sawlogs	Logarithmic	31.23 (1.6)	24.03 (1.9)	–	0.79	10.1
Pulp	Logistic	92.08 (2.0)	0.183^{ns} (0.2)	3.636 (0.7)	0.70	6.0
Cut waste	Logistic	98.77 (1.0)	–0.578 (0.3)	4.454 (0.9)	0.68	3.1
Explanatory variable: mean tree diameter						
Unbranched	Linear	–19.59 (5.1)	0.724 (0.1)	–	0.49	405.5
A-sawlogs	Logarithmic	–179.9 (18.5)	56.497 (5.3)	–	0.72	10.9
Sawlogs	Logarithmic	–197.0 (17.1)	62.717 (4.9)	–	0.79	10.1
Pulp	Logistic	92.48 (2.2)	2.4313 (0.6)	0.1404 (0.03)	0.72	5.8
Cut waste	Logistic	98.41 (0.8)	2.9605 (0.7)	0.2057 (0.03)	0.78	2.6

ns = coefficient not significant at 95% level (p >t >0.05).

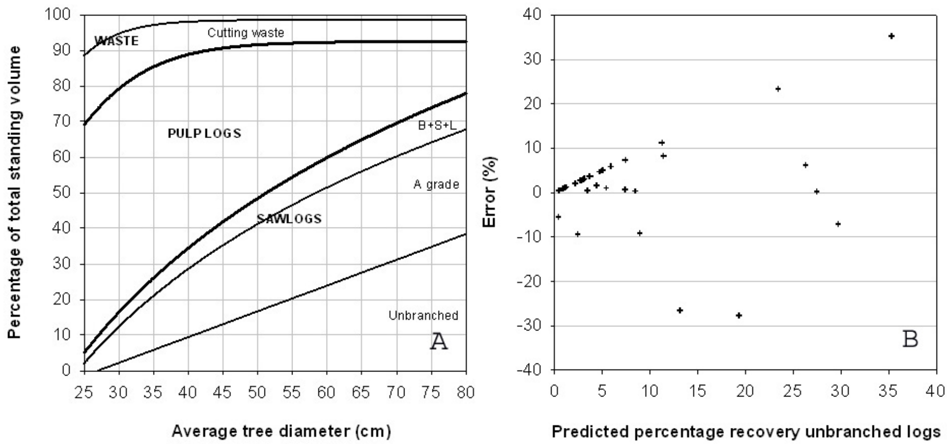

FIG. 4–Influence of mean tree diameter on predicted log grade recovery as a cumulative percentage of *E. fastigata* total standing volume. Example: Stand with mean dbh = 60 cm has approx. 24% of total standing volume in unbranched sawlogs, 28% branched A-grade sawlogs, 8% B+S+L sawlog grades, 32% pulp logs, 6% cutting waste, and <2% residue waste. Predicted unbranched log volume expressed as a percentage of total standing volume; A-grade sawlog volume, sawlog volume, pulp log volume, and cutting waste all expressed as cumulative percentages with preceding (more valuable) grades, as a percentage of total standing volume, e.g., cumulative percentage sawlogs = percentage unbranched + percentage A-grade sawlogs + percentage sawlogs (A). Prediction errors for percentage unbranched log grade recovery as a function of mean tree diameter (n=44) (B).

Prediction of Average Tree Diameter

Prediction error statistics for approximate mean tree diameter are listed in Table 11, predicted from mean top height, total standing volume, and stocking using the individual tree volume function developed by Smart (1992).

TABLE 11–Mean tree diameter (dbh) prediction error statistics for all data, permanent sample plot data, and MARVL data. Errors expressed in real terms, i.e., predicted-actual (cm).

	All data	PSP	MARVL
n	423	378	45
Mean error (cm dbh)	–0.61	–0.54	–1.21
s.d. of errors	0.71	0.51	1.49
Min. error	–7.81	–2.15	–7.81
Max. error	2.11	2.11	1.55
RMSE	0.94	0.75	1.97

Influence of Age, Stocking, and Site Index

Total standing volume growth and approximate mean tree diameter, predicted from age-10 starting values for stockings of 100–1100 stems/ha on high quality and average sites, are given in Fig. 5.

DISCUSSION

The data used for model development were collected from a mixture of permanent and temporary sample plots located within even-aged *E. fastigata* stands and field trials located mainly in the central North Island (Table 1). These data were characterised by a low average age, high stocking, and low average tree size (Table 2). Site index estimates varied widely between sample plots (Table 7). Differences in mean annual increment data and site index estimates between regions should not be interpreted as differences in overall productivity because some regions had only a small number of plots. Additionally, mean annual increment may not have reached its maximum in younger stands, or in thinned stands. However, differences within and between regions imply that *E. fastigata* growth can vary widely between stands. The inclusion of mean top height as an explanatory variable in the volume yield model should account for some of the variation in volume yield attributed to differences in site quality and genetics. Shelbourne *et al*. (2000) recorded important differences in growth between *E. fastigata* seedlots planted on the same site. The polymorphic mean top height growth model can be used to obtain mean top height estimates for any age based on local height-age data, or on site index data presented in Table 7 when starting values are limited to stocking and age. Polymorphic forms are renowned for superior

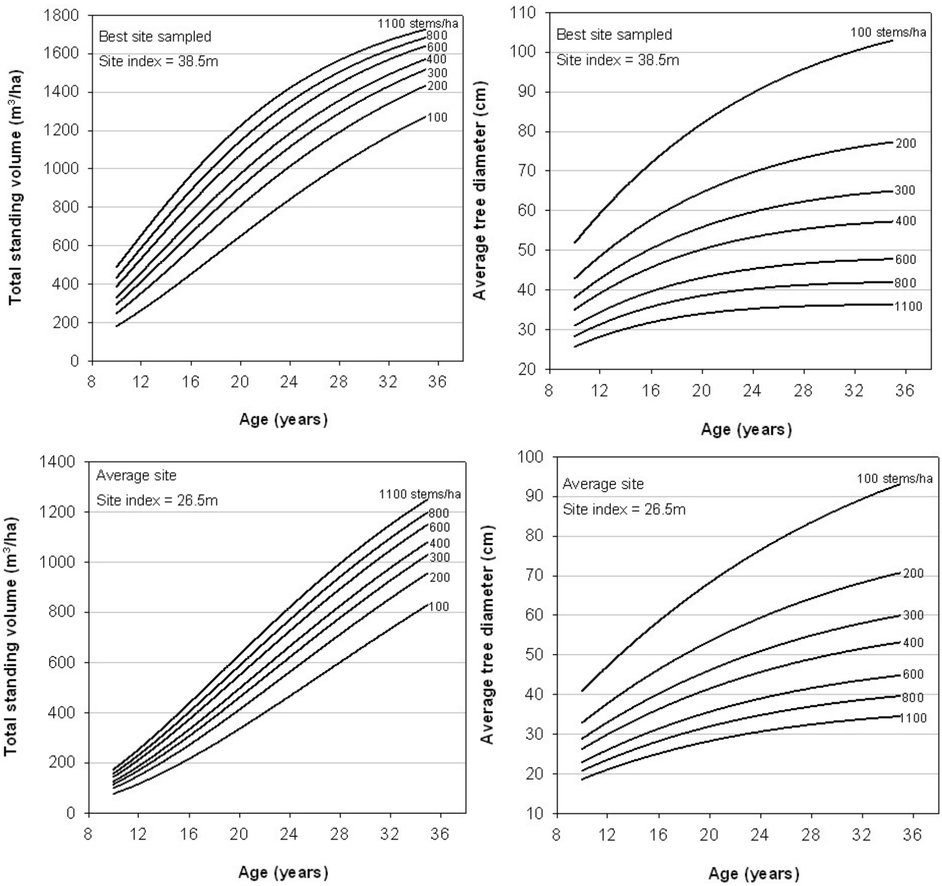

FIG. 5–Predicted *E. fastigata* volume growth and mean tree diameter for a range of final-crop stockings on high quality (site index = 38.5 m) and average (site index = 26.5 m) sites. Volume yield model predicted age-10 starting values for volume growth projections. Mean tree diameter predicted from total standing volume, age-10 stocking, and mean top height predicted using height growth model.

representation of variability in site and other factors influencing biological growth (Mason & Whyte 1997; Ngugi *et al*. 2000; Berrill 2004). The volume yield model can be used to evaluate the influence of stocking, site index, and age on volume production for regimes with rotation ages less than 20 years. The volume yield model also provides starting values for the volume growth model when projections beyond age 20 are required.

Too few data were available to model stocking reduction from natural mortality and non-catastrophic windthrow, or to model post-thinning basal area from stocking before and after thinning and pre-thinning basal area. Such models are needed for use in combination with basal area growth models to obtain accurate predictions of quadratic mean diameter. The simpler yield modelling approach adopted here circumvents the need for basal area, mortality, and thinning models by using volume growth and yield models to predict total standing volume, and a tree volume equation and stocking data to obtain average tree diameter estimates from total standing volume predictions. The individual tree volume function developed by Smart (1992) was used to obtain approximate estimates of mean tree diameter, assuming that the individual tree volume equation adequately described the relationship between average tree diameter, mean top height, and the mean individual tree volume (total standing volume/stocking). This approach gave relatively accurate and precise average diameter predictions across the range of data tested (Table 11).

Volume yield model predictions of *E. fastigata* total standing volume showed that stands on sites of average quality (site index 26.5 m) and high quality (site index 38.5 m) would produce 175 m^3/ha and 485 m^3/ha total standing volumes, respectively, for 1100 stems/ha at age 10. At age 25, total standing volumes of 660 m^3/ha and 1160 m^3/ha were predicted by the volume growth model for sites of average and high site index respectively, based on age-10 starting values for 300 stems/ha obtained from the volume yield model (Fig. 5). Since mortality was not predicted, stocking at harvest could be lower than the starting value used for the volume growth model. The inability to predict intra-specific competition and competition-induced mortality to later ages and at higher stockings could be mitigated by defining an upper limit to tree size and density data (Reineke 1933; Yoda *et al.* 1963). Tree size and stocking data showed that some stands had exceeded the upper limit of stand density index = 500 for *E. globulus* reported by Reineke (1933), and were approaching or appeared to have reached stand density index = 1000, the upper limit of growing space occupancy reported for second-growth *Sequoia sempervirens* in California: $\log(N) = -1.605 \log(D) + 13$, where N = stocking (stems/ha); D = average diameter (cm). This approximate upper limit could be used to constrain volume growth and yield projections for a given stocking within realistic biological limits (Fig. 6).

The MARVL sampling was opportunistic in that it included most known and accessible stands suitable for MARVL assessment. Estimates of sawlog yield as a percentage of total standing volume were low on average (Table 5). The range of sample stand ages was limited (mean 21.6 years; s.d. 12.6) and the sample stands generally had relatively high stockings and low average tree sizes (mean 35.3 cm dbh; s.d. 13.0 cm). As a consequence, models using tree size as a predictor of log grade recovery may overpredict unbranched sawlog yields from stands managed at wider spacings. Furthermore, clearwood recovery from unbranched logs cannot be assumed when the stem diameter at which pruning or self-pruning occurred was unknown. Log grade recovery data and models indicate that recovery of unbranched

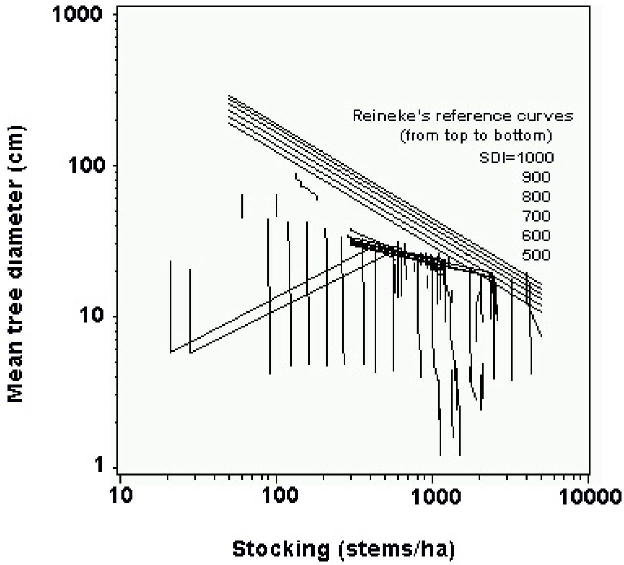

FIG. 6–*Eucalyptus fastigata* Permanent Sample Plot stocking and mean tree diameter data plotted on logarithmic scales (base 10), with Reineke's reference curves for SDI = 500, 600, 700…1000 superimposed.
SDI = stand density index: number of trees per acre in a stand with average dbh of 10 inches.

and knotty sawlog grades increases with average tree size (Fig. 4). For example, the approximate percentage recovery of sawlogs increases from 35% to 60% of total standing volume in stands with mean diameters of 40 cm and 60 cm respectively. Results also indicate that waste can account for a significant portion of total standing volume in stands with low average tree size (Fig. 4). Log grade recovery model fit statistics show that predictions were relatively imprecise for all grades, especially for unbranched sawlogs (Table 10).

The *E. fastigata* growth and yield models were developed with a combination of small datasets biased toward younger ages. While model coefficients were estimated correctly given the data at hand, model fit statistics show that predictions were relatively imprecise. Therefore model predictions should be regarded only as indicative until validated with independent data, especially data from older managed stands on good sites. Data and yield model predictions indicate that *E. fastigata* stand growth can vary widely between sites, and that *E. fastigata* is a productive plantation species capable of maintaining rapid growth rates at relatively high stockings to later ages on favourable sites*.

* Since this report was written, the *E. fastigata* growth model has been updated by the Eucalypt Co-operative using an extended dataset and now provides a more precise estimate of growth.

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