MATHEMATICAL SOLUTION FOR OPTIMISING THE SAWING PATTERN OF A LOG GIVEN ITS DIMENSIONS AND ITS DEFECT CORE

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ABSTRACT

The mathematical solution for a two-dimensional cutting problem of an irregular space presented is capable of being extended to a third dimension. The computer program based on this algorithm uses information on marketable timber assortments (dimensions, prices for the various grades, and dressing) to optimise the sawing pattern for a specified log with a certain defect core. In the two-dimensional examples three grades are recognised: clear boards, one face clear, and knotty grade boards.

The run time on a single processor is extensive and real-time on-line applications will be realistic only using a multi-processor, for which the dynamic programming algorithm described is highly suited.

INTRODUCTION

After felling, trees undergo a number of processing operations before reaching the marketplace as a salable commodity. The collective term for these operations is conversion, which may involve any of a number of processing options.

Initially, the felled trees are cut to length (bucking in the United States) and the logs are then allocated, usually on the basis of size and quality, to various conversion processes such as sawmilling, pulping, or peeling. The process of log allocation is an important step, as it fixes the wood supply to the various forest industries and thus its economic contribution and the potential added value. For example, logs may be segregated into round produce (poles), pulpwood, peelers, or sawlogs.

At each stage of the conversion process an optimal solution is available. Allocation of logs to provide maximum value (i.e., revenue from end-use products) would be an optimal solution to the allocation problem. Similarly, the processing of a log lot to provide the mix of sawn sizes most suited to the market, thus obtaining highest value under existing market conditions, would be an optimal solution.

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The optimal conversion of a sawlog into timber assortments to obtain the highest value is constrained by the dimensions and characteristics of the pruned sawlog and its knotty core, or defect core. Around this defect core is a sheath of clear wood. If, in the sawing process, boards are cut in this clear zone, they have a higher value than if cut partly or wholly in the defect core. Therefore, in optimising the sawing pattern of a log, where there are different values for various grades, there is a geometrical problem – the position of the board or timber assortment in the log with respect to the defect core. This aspect of the problem excludes mathematical optimisation procedures which are not based on the position of the elements in the space to be cut, and optimisation procedures suitable only for rectangular spaces. The program based on the method described in this paper recognises three different qualities in terms of the positions of the assortments in the log: clear grade, one face clear grade, and knotty grade (knotty on both sides of the assortment).

The term flitch used in the paper refers to a piece taken from the log, which has two sawn faces and two waney edges.

Previous Work

In the 1960s optimal wood allocation models were most often linear programming models (Smith & Harrell 1961; Jackson & Smith 1961). The activities in such models represent the possible cutting patterns which therefore had to be predefined.

In 1972 Pnevmaticos & Mann showed how dynamic programming can be used to determine the optimal one-dimensional cross-cutting pattern of a log into short logs, though in his thesis in 1974 Pnevmaticos referred to work done by Pugovkin & Stepakov (1968) and Bailey (1970). This optimal cross-cutting algorithm was an application of such importance that it was adopted in various optimal log conversion and allocation projects (Bare *et al.* 1979; Gluck & Koch 1973; Geerts 1979; Landmesser *et al.* 1977). The use of the algorithm in an inventory system (Deadman & Goulding 1979) provided a completely new tool for assessing merchantable volume. In forestry modelling this cross-cutting algorithm has been used in combination with linear programming (Eng & Whyte 1982; Gluck & Koch 1973).

Determining optimal sawing patterns is in mathematical terms a two or more dimensional cutting stock problem. Mathematical solutions in the literature include a knapsack algorithm (Gilmore & Gomory 1965) which has the advantage that – being of an integer linear programming type – additional side constraints can be taken into account. For the sawing pattern optimisation, however, this method cannot be used as the location of the small elements (rectangles) in the available space will not be known.

Christofides & Whitlock (1977) designed a tree search algorithm for rectangles. Further research is needed to decide whether such an algorithm, or combinations of algorithms, can be designed for optimisation of the sawing pattern of an irregular shape. Such algorithms may be much faster than the one proposed here.

The proposed algorithm is of a two-stage guillotine cutting type, which means that in the first stage all cuts go from one edge of the space to be cut to the opposite edge and for the resulting flitches in the second stage all cuts must go from one edge of the flitch to be cut to the opposite edge. It can be extended to three dimensions, with cross-section and length optimisation for various grades, provided defect information from a defect scanner (Benson-Cooper *et al.* 1982; Harpole & McDonald 1981; McDonald 1978, 1979) together with a fast image-processing system (Hodgson & McNeill 1983) are available.

PROBLEM, AND METHOD FOR SOLUTION

The problem of finding the optimal sawing pattern can be defined as "to saw a log with certain parameters (log profile and defect core) into timber assortments so as to give a maximum return, given a table of timber assortments, their dimensions, and the prices for the various grades (or any other value for which the log has to be optimised, e.g. volume)". In the sawmill the dimensions and values for various grades are usually defined in an order book, and the amount of dressing and the sizes of the saw kerf can vary. The optimum conversion of the log into timber assortments can be unconstrained, as the conversion of a single log might be regarded as a very small element in the whole conversion process of a log lot. Market constraints can be taken into account by excluding certain assortments at optimisation time or by changing the relative values of the assortments. This decision can be made for each individual log with information from an optimal production/stock/market-demand model, giving on-line information about an optimum production schedule (Buffa & Miller 1979).

A one-dimensional dynamic programming algorithm is used to optimise the breakdown of the log into flitches, with the value of the flitch as the variable to be maximised. This is the first level of the algorithm. In this algorithm there is a one-to-one correspondence between the stage and the state variable. The breakdown of the flitches into timber assortments with the value of the timber assortments, e.g., price for a certain grade as the variable to be maximised, forms a second level of optimisation. It is solved by a similar algorithm. The output of this level is used as the variable to be optimised in the first level. At the second level the variable to be maximised is the variable to be optimised (e.g., value) of the current timber assortment. Because the over-all algorithm optimises a multi-dimensional space (two dimensions are described in this article) it is called multi-dimensional, and because of its construction for the two-level problem it is of a two-stage guillotine type.

It should be noted that the proposed solution assumes open market conditions, i.e., prices are not affected by the total amount produced.

Mathematical Definition

The two-level approach consists of a similar dynamic programming algorithm for each level. The first level maximises the breakdown of the log into flitches and the second level maximises the breakdown of each flitch into timber assortments, with the maximum value for the variable to be optimised as the result for each flitch. This value is used in the first level as the variable to be optimised.

Level one

An integer deterministic dynamic programming formulation for level one follows:

Let F_N be the maximum value of the objective function at the end of the optimisation procedure for a log, being the N-stage maximum value, $0 \leq n \leq N$, where N is the total number of stages.

Then F_n is the maximum value of the objective function for a subsection of the log between stage 0 and stage n. The value of the state variable at stage n = n.k1, where k1 is the distance between the stages at level one. There is a one-to-one correspondence between the stage and the state variable (e.g., n and n.kl or n – w(s)/kl and n.kl – w(s)).

The deterministic recursive relationship for level one is as follows:

$$F_n(n.k1) = \underset{sES}{MAX} \left\{ F_{n-w(s)/k1}(n.k1 - w(s)) + G_M(M.k2,\ n.k1,\ n.k1 - w(s)) \right\},$$
$$0 \leq n \leq N \text{ with } F_0(.) = 0$$

where:

n = stage variable, the stages are defined along the x-axis (n = 0, ..., N) and represent the value of the x co-ordinate/k1. The distance between the stages is k1 mm;

n.k1 = state variable, the state variable at stage n equals the actual distance between stage 0 and stage n in millimetres, which is equal to n.k1;

w(s) = decision variable for this level. The decision variable is equal to the width of the flitch to be cut between two stages;

S = total set of timber assortments;

s = subset of S. All timber assortments of which the width or depth are equal or smaller than the decision variable w(s) belong to s;

k1 = the increment value of the first-level stage variable n in millimetres;

F_n(n.k1) = the n-stage maximum value of the log section between stage 0 and stage n;

G_M(M.k2, n.k1, n.k1 – w(s)) = the M-stage maximum value of the objective function for the flitch defined between first-level stage n and the first-level stage n – w(s)/k1, as a function of the state variable M.k2.

Level two

An integer deterministic dynamic programming formulation for level two follows: Let G_M be the maximum value of the objective function at the end of the optimisation procedure for the flitch, defined between the first-level stages n and n – w(s)/k1, being the M-stage maximum value $0 \leq m \leq M$, where M is the total number of stages.

Then G_m is the maximum value of the objective function for a subsection of this flitch between the second-level stage 0 and stage m, the latter with state variable m.k2, where k2 is the distance between the stages at level two. There is a one-to-one correspondence between the stage and the state variable (e.g., m and m.k2 or m – d (s,I)/k2 and m.k2 – d (s,I).

The deterministic recursive relationship for level two is as follows:

$$G_m(m.k2,\ n.k1,\ n.k1 - w(s)) = \underset{\substack{sES \\ IES}}{MAX} \left\{ G_{m-d(s,I)/k2}(m.k2 - d(s,I)) + V(I).p(I,\ n.k1,\ m.k2,\ w(s),\ d(s,I)) \right\}$$
$$0 \leq m \leq M \text{ with } G_0(.) = 0$$

where

m = stage variable, the stages are defined along the y-axis (m = 0, ..., M) and represent the value of the y co-ordinate/k2. The distance between the stage is k2 mm;

m.k2 = state variable. The state variable of stage m equals the actual distance between stage 0 and stage m in millimetres, which is equal to m.k2 mm;

I = decision variable for this level. The decision variable is a timber assortment belonging to the subset s;

d (s,I) = depth or width of the current timber assortment I belonging to the subset s;

k2 = the increment value of the second-level stage variable m;

V(I) = the value of the variable to be optimised for assortment I, located between the first-level stages n and n − w(s)/k1 and the second-level stages m and m − d(s,I)/k2;

G_m (m.k2, n.kl, n.kl − w(s)) = the m stage maximum value of a log section between the first-level stages n and n − w(s)/k1 and the second-level stages 0 and m;

p(I, n.k1, m.k2, w(s), d(s,I)) = the correction factor for V(I) as a function of assortment I and the current place in the log as defined by the co-ordinates n.k1 and m.k2 and assortment size w(s) and d(s,I).

Optimisation is done by nesting the two levels which means that the output of level two is used as input to level one and by backtracking at each level until the optimal solution is found, meaning that when the N stage maximum value F_N is computed, the optimum decisions for each stage are defined by subtracting the optimum decisions one by one from the total state variable until the 0 stage is reached.

The variables k1, k2 must be common denominators of the depth and width of the assortments and saw kerfs in both directions. As the saw kerfs can be different in both directions, k1 and k2 can be equal but do not have to be.

When the profile of the log is known, the third dimension can be taken into account by replacing V(I) .p(I, n.k1, m.k2, w(s), d(s,I)) with the maximum result of an extra step along the length axis of the stem, and the variable to be optimised at this level would then be V(I) .p(I, n.kl, m.k2, L.k3, w(s), d(s,I), L(s,I)) where L is the stage variable of the third level, k3 is the increment value of L, and L(s,I), is the length of assortment I belonging to s.

COMPUTATIONAL RESULTS

A program has been developed as a research tool (referred to as Dynamic Optimisation System for Sawmills, DOSS). To identify the effect of changes in saw kerf size, defect core size, and amount of dressing, as well as to approximate the solution of the continuous problem as closely as possible to the described discrete solution, a small increment of 1 mm for the interval between the stages is used.

Log and defect core profiles are represented in the program by circles, although the defect core can be displaced from the centre, as shown in the accompanying diagrams. The algorithm allows for dealing with an irregular shape for the two profiles as can be defined by polygons. The algorithm is two-dimensional and the values in the output examples (Fig. 1 and 2) represent values per metre of log length. No third-dimension and therefore no taper was taken into account. Because of the length between stages of 1 mm and the multi-dimensional nature of the algorithm, the run time is high on a single processor (PDP 11/34) and increases with log size and defect core size. Although the software is not yet optimised, processing (CPU) times are given for each run. As the positions of the boards are known at run time, the algorithm can deal with various values for the different assortments; however, this is at present restricted to three different values for positions in relation to the defect core. The program selected its assortments from a table of 45 assortments, detailing their width, thickness, values for clears, one face clear, and knotty grade, and also the amount of dressing for each assortment (Table 1). Because of a dressing of 5 mm for more assortments, certain assortments which encroach into the defect core by no more than 5 mm are calculated as clear boards. The theoretical recovery percentage, given for each optimisation, represents the ratio between total board area and log cross-sectional area. The cost of a saw cut, possibly different for the different saw cuts, can be taken into account by subtracting this value from the objective functions as a penalty. This feature was not used in the sample problems solved.

In Fig. 1A–C all the variables except the defect core are kept constant. For Fig. 1B and C, a similar run is made with the same table of assortments (Table 1), except for the last three square assortments (total number of assortments 42). These examples give an idea of the effect of the defect core on the value outturn. Figure 2A gives the effect on value outturn of changing the saw kerf size in the primary breakdown saw to 3 mm. Changing this vertical saw kerf to 6 mm resulted in 15 sawn assortments with a total actual value of NZ$19.73. Figure 2B is an optimisation by volume.

Very small changes in a log parameter (i.e., log or defect core diameter) or saw parameter (i.e., width of saw cut) affect the value outturn of the log, irrespective of market demands. This should be regarded as the power of the eventual system – optimising the utilisation of the resource. Market demand should be regulated by stock policy or supply regulation.

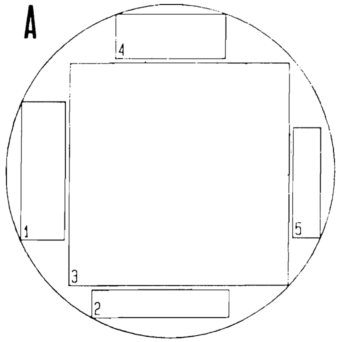
DISCUSSION, CONCLUSION, AND IMPLEMENTATION

Although developed for a research environment, the present algorithm has great potential for sensitivity analyses of variables such as defect core size, saw kerf size for both primary and secondary breakdown saws, and amount of dressing. In its present form the run time is too high to consider its application in a sawmill. However, the rapid increases in computer processing power through speed improvements and multi-processor configurations indicate possibilities for the future. The algorithm is highly suitable for implementing on a multi-processor, as the computations at each stage of the first level to compute the value of different flitch sizes in the second level are independent. Similarly, the various stages in the second level are independent. The commercial availability of these multi-processors seems only a matter of time (Galil & Wolfgang 1983; Manuel 1983; Taylor & Wilson 1982). When developed for three dimensions, the described algorithm can be used for real-time optimisation in a sawmill,

TABLE 1—Table of timber assortments

Depth (mm)	Width (mm)	Clear value (NZ$)	One-face-clear value (NZ$)	Knotty value (NZ$)	Dressing (mm)
25	75	0.221	0.147	0.073	5
25	100	0.295	0.196	0.098	5
25	125	0.369	0.240	0.123	5
40	75	0.354	0.236	0.118	5
40	100	0.472	0.314	0.157	5
40	125	0.590	0.393	0.196	5
50	75	0.443	0.295	0.147	5
50	100	0.590	0.393	0.196	5
50	125	0.738	0.492	0.246	5
75	75	0.664	0.442	0.221	5
75	100	0.885	0.590	0.295	5
75	125	1.106	0.737	0.367	5
40	140	0.745	0.496	0.248	5
40	150	0.798	0.532	0.266	5
50	140	0.931	0.620	0.310	5
50	150	0.998	0.665	0.332	5
75	140	1.397	0.931	0.465	5
75	150	1.496	0.997	0.498	5
40	160	0.947	0.631	0.315	5
40	170	1.006	0.670	0.335	5
40	180	1.066	0.710	0.355	5
40	190	1.125	0.750	0.375	5
40	200	1.184	0.789	0.394	5
50	160	1.184	0.789	0.394	5
50	170	1.258	0.838	0.419	5
50	180	1.332	0.888	0.444	5
50	190	1.406	0.937	0.468	5
50	200	1.480	0.986	0.493	5
75	160	1.776	1.184	0.592	5
75	170	1.887	1.258	0.629	5
75	180	1.998	1.332	0.666	5
75	190	2.109	1.406	0.703	5
75	200	2.220	1.480	0.740	5
40	225	1.386	0.924	0.462	5
50	225	1.733	1.155	0.577	5
75	225	2.599	1.732	0.866	5
40	250	1.600	1.066	0.533	5
50	250	2.000	1.333	0.667	5
75	250	3.000	2.000	1.000	5
40	300	1.956	1.304	0.652	5
50	300	2.445	1.630	0.815	5
75	300	3.668	2.445	1.222	5
200	200	4.000	4.000	4.000	0
250	250	6.250	6.250	6.250	0
300	300	9.000	9.000	9.000	0

A



TOTAL NUMBER OF ASSORTMENTS IN THE SET 45

SELECTED :

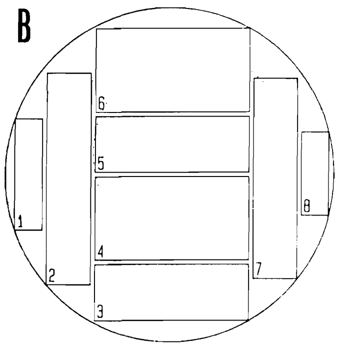
ASSORTMENT	CLEAR VALUE	ACTUAL VALUE
1) 40 * 125	0.590	0.590
2) 125 * 25	0.369	0.369
3) 200 * 200	4.000	4.000
4) 100 * 40	0.472	0.314
5) 25 * 100	0.295	0.295
TOTAL :	5.73	5.57

LOG DIAMETER ———————— : 300 mm
DEFECT CORE DIAMETER : 180 mm
RECOVERY PERCENTAGE : 77.3 %

		vert.	horz.
SAW KERFS	:	4 mm	4 mm
DEFECT CORE DISPL.	:	20 mm	20 mm
SAWCUT COSTS	:	0.00 $	0.00 $

CPU TIME (PDP 11/34) . 51.1 min.

B



TOTAL NUMBER OF ASSORTMENTS IN THE SET : 42

SELECTED :

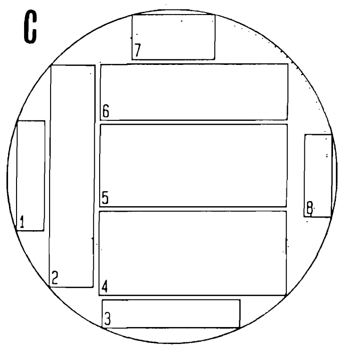
ASSORTMENT	CLEAR VALUE	ACTUAL VALUE
1) 25 * 100	0.295	0.295
2) 40 * 190	1.125	1.125
3) 140 * 50	0.931	0.931
4) 140 * 75	1.397	0.931
5) 140 * 50	0.931	0.310
6) 140 * 75	1.397	0.931
7) 40 * 180	1.066	0.710
8) 25 * 75	0.221	0.221
TOTAL :	7.36	5.45

LOG DIAMETER ———————— : 300 mm
DEFECT CORE DIAMETER : 180 mm
RECOVERY PERCENTAGE . 76.7 %

		vert.	horz.
SAW KERFS	:	4 mm	4 mm
DEFECT CORE DISPL.	:	20 mm	20 mm
SAWCUT COSTS	:	0.00 $	0.00 $

CPU TIME (PDP 11/34) : 51.8 min.

C



TOTAL NUMBER OF ASSORTMENTS IN THE SET : 42

SELECTED :

ASSORTMENT	CLEAR VALUE	ACTUAL VALUE
1) 25 * 100	0.295	0.295
2) 40 * 200	1.184	0.789
3) 125 * 25	0.369	0.369
4) 170 * 75	1.887	1.258
5) 170 * 75	1.887	0.629
6) 170 * 50	1.258	0.419
7) 75 * 40	0.354	0.236
8) 25 * 75	0.221	0.147
TOTAL :	7.46	4.14

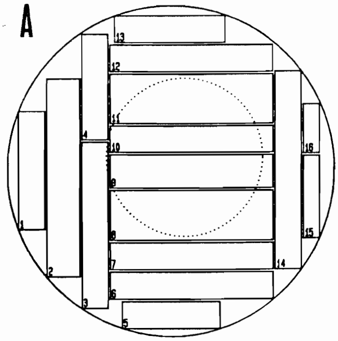
LOG DIAMETER ———————— : 300 mm
DEFECT CORE DIAMETER : 240 mm
RECOVERY PERCENTAGE : 74.3 %

		vert.	horz.
SAW KERFS	:	4 mm	4 mm
DEFECT CORE DISPL.	:	20 mm	20 mm
SAWCUT COSTS	:	0.00 $	0.00 $

CPU TIME (PDP 11/34) : 52.3 min.

FIG. 1—Optimisation by value with 45 and 42 assortments in the set, and defect core diameters of 180 mm and 240 mm.

A



LOG DIAMETER ————	: 500 mm
DEFECT CORE DIAMETER	: 240 mm
RECOVERY PERCENTAGE	: 80.7 %

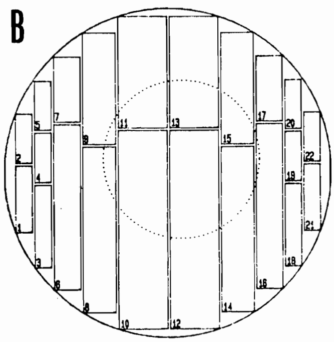
	vert.	horz.
SAW KERFS	: 3 mm	4 mm
DEFECT CORE DISPL.	: 20 mm	20 mm
SAWCUT COSTS	: 0.00 $	0.00 $

CPU TIME (PDP 11/34) : 245.3 min.

TOTAL NUMBER OF ASSORTMENTS IN THE SET : 42

ASSORTMENT	SELECTED : CLEAR VALUE	ACTUAL VALUE
1) 40 * 180	1.066	1.066
2) 50 * 300	2.445	2.445
3) 40 * 250	1.600	1.600
4) 40 * 160	0.947	0.947
5) 150 * 40	0.798	0.798
6) 250 * 40	1.600	1.600
7) 250 * 40	1.600	1.600
8) 250 * 75	3.000	2.000
9) 250 * 50	2.000	0.667
10) 250 * 40	1.600	0.533
11) 250 * 75	3.000	2.000
12) 250 * 40	1.600	1.600
13) 170 * 40	1.006	1.006
14) 40 * 300	1.956	1.956
15) 25 * 125	0.369	0.369
16) 25 * 75	0.221	0.221
TOTAL :	24.81	20.41

B



LOG DIAMETER ————	: 500 mm
DEFECT CORE DIAMETER	: 240 mm
RECOVERY PERCENTAGE	: 83.1 %

	vert.	horz.
SAW KERFS	: 4 mm	4 mm
DEFECT CORE DISPL.	: 20 mm	20 mm

CPU TIME (PDP 11/34) : 42.8 min.

TOTAL NUMBER OF ASSORTMENTS IN THE SET : 45

ASSORTMENT	SELECTED : CLEAR VALUE	ACTUAL VALUE
1) 25 * 100	0.295	0.295
2) 25 * 75	0.221	0.221
3) 25 * 125	0.369	0.369
4) 25 * 75	0.221	0.221
5) 25 * 75	0.221	0.221
6) 40 * 250	1.600	1.600
7) 40 * 100	0.472	0.472
8) 50 * 250	2.000	1.333
9) 50 * 170	1.258	0.838
10) 75 * 300	3.668	1.222
11) 75 * 170	1.887	0.629
12) 75 * 300	3.668	1.222
13) 75 * 170	1.887	0.629
14) 50 * 250	2.000	0.667
15) 50 * 170	1.258	0.419
16) 40 * 250	1.600	1.600
17) 40 * 100	0.472	0.472
18) 25 * 125	0.369	0.369
19) 25 * 75	0.221	0.221
20) 25 * 75	0.221	0.221
21) 25 * 100	0.295	0.295
22) 25 * 75	0.221	0.221
TOTAL :	24.42	13.76

FIG. 2A—Optimisation by value with vertical saw kerf size at 3 mm.
B—Optimisation by volume.

with the aid of a profile scanner and a defect scanner to provide the log profile and its defects.

With scanners, image capture systems, and multi-processors available, this algorithm and improvements open a field of research in system development. To make real-time optimisation possible, software, as well as hardware, must be available to provide a feasible run time.

Eventually this will allow accurate optimal sawing of a log into timber assortments, the sawmiller setting all parameters such as saw kerfs, dressing, and board size to the desired millimetre. Within these constraints he is assured of the highest return for each single log. Optimal conversion will then be possible as for each log the optimal sawing pattern would be computed and applied online, thus limiting the need for log sorting.

Future work will include research into faster multi-dimensional algorithms, and into the possible application of defect scanners and computer requirements (Benson-Cooper *et al.* 1982; Harpole & McDonald 1981; Hodgson & McNeill 1983; Szymani & McDonald 1981; McDonald 1978, 1979; Bates *et al.* 1983). Once the three-dimensional optimisation system is developed a comparison with real mill outturn is envisaged.

Future research will also be directed towards applications of micro-processors in the sawmill and the wood and furniture industries as computerised decision-making by optimisation routines can be applied in these areas as well. Again, the goal is to maximise the value and minimize waste in the conversion of a defective log or board with certain dimensions (McDonald 1979; Stern & McDonald 1978). To optimise material flow in the sawmills, implementation of various operations research techniques and models will be studied (Carino *et al.* 1979, 1981, 1982).

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