# NATIONAL VOLUME FUNCTION FOR ESTIMATING TOTAL STEM VOLUME OF *PINUS RADIATA* STANDS IN NEW ZEALAND

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## ABSTRACT

Volume data from young trees were combined with predictions from existing individual-tree functions in older stands and used to develop new individual-tree and stand-level volume functions for *Pinus radiata* D. Don in New Zealand. As part of this process, several new individual-tree volume datasets were compiled to examine variation in stem form factor after accounting for tree height. Analysis of data from a national trial series which included nitrogen fertiliser treatments and controls without fertiliser found no treatment-related differences in form factor at stand age 5 years, except at a nitrogen-deficient sand site located in Woodhill Forest. Analysis of a seedlot comparison experiment, which included unimproved through to highly improved genetic stock, found no differences in form factor at stand age 3 years. Analysis of a stocking rates experiment found no significant differences in form factor when age was taken into account. However, across the combined datasets, form factor decreased strongly with increasing tree height. This relationship between form factor and height was incorporated into new volume functions enabling them to perform well for trees ranging from establishment to clearfelling age, across a range of stocking rates and site types, excluding coastal sand sites.

**Keywords**: volume function; carbon sequestration; form factor; *Pinus radiata*.

## INTRODUCTION

Carbon sequestration in *Pinus radiata* stands can be predicted using the C_Change model (Beets *et al*. 1999). C_Change requires a yield table for each stand of interest, consisting of annual stem total volumes under-bark. The 300 Index stand-level growth model (Kimberley *et al*. 2005), which can be used to predict volume yield tables for C_Change, predicts mean top height (*MTH*), basal area (*BA*), and stocking (*N*) annually from stand establishment to felling age. The model predicts these stand

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variables on the basis of plot measurement data taken from the stand, and from stand history information on stocking, thinning, and pruning. Plot measurements required to estimate stem volume include stem diameter at breast height (dbh) of all plot trees, and height obtained from a sample of trees.

Under-bark stand volume (*V*) is estimated from mean top height, basal area, stocking, and age using a stand-level volume function. When predicting carbon sequestration nationally, a stand-level function that performs well for any site, regime, and age is required. It is important that the function performs well for young trees as well as semi-mature and mature trees, because C_Change models carbon pools over a full rotation commencing from the stand establishment date. In contrast, existing volume functions have generally been developed to predict volume at clearfelling age, or at mid-rotation, and these do not necessarily perform well in young stands.

Stem volume is determined by sectionally measuring a sample of trees in each stand, with stands selected to take account of variation in stem form due to site and management factors (Gordon & Graham 1986), including site fertility, silviculture, and tree age. Of these factors, the effect of tree age has been least researched. However, when plots are measured starting from a young age, a national volume function suitable for estimating stem volume over a wide range in tree size is required. While considerable stem volume data have been obtained in semi-mature and mature stands, few studies include small trees.

A number of functional forms have been used to predict stem volume of sectionally measured trees. For example, Spurr (1952) assumed stem volume was proportional to a cylinder with the same diameter and height. However, this assumption is not appropriate when estimating stem volume across a wide range in tree sizes. A more flexible form that can be applied over a wide range of sizes has been used in New Zealand for plantation species (Gordon & van der Colff 2005). This form accommodates small trees by allowing for the relative position of the breast height diameter measurement in smaller stems.

In this paper the development of a volume function based on data acquired for young *P. radiata* is described. These data were combined with predictions from existing regional volume functions developed in New Zealand, to provide improved estimates of stem total volume under bark across a wide range of stand ages, sites, and stand management regimes.

## METHODS

### Notation

The following notation is used in this paper:

$h$ height of an individual tree (m)

*dbh* diameter over bark at breast height (1.4 m) of an individual tree (cm)

*ba* basal area of an individual tree ($m^2$), $ba = 7.85\times10^{-5}\times dbh^2$

*v* total under-bark stem volume of an individual tree ($m^3$)

*f* individual-tree breast height volume form factor, $f = v/(ba\times h)$

*BA* basal area per hectare of trees in a stand or plot ($m^2$/ha)

*N* stocking of trees in a stand or plot (stems/ha)

*MTH* mean top height (m) of a stand or plot, defined as the predicted mean height of the 100 largest diameter stems/ha

*V* stem volume under-bark per hectare of a stand or plot ($m^3$/ha)

*F* stand volume form factor, $F = V/(BA\times MTH)$

## Description of Study Sites with Small Tree Volume Data

Data were compiled on a total of 715 sectionally measured trees from biomass studies undertaken in juvenile through to mature-aged *P. radiata* stands located in the North Island and northern South Island of New Zealand (Tables 1 and 2). Soil fertility was moderate to high at most of these sites, with Puruki and Warkworth both fertile ex-pasture sites, while Kinleith, Golden Downs, and Tarawera were medium-fertility forest sites. Woodhill was a moderately low-fertility second-rotation sand site. Trees from Kinleith, Golden Downs, Tarawera, and Woodhill were from large experiments, and included nitrogen fertiliser treatments applied more-or-less annually in a split-plot randomised block design. The 35-ha Puruki pasture site was planted with *P. radiata* seedlings at a stocking of 2220 stems/ha, and included a range of thinning treatments in the first-rotation forest. Tree stocking in the unthinned (Inv) stands decreased through natural attrition to 1550 stems/ha by age 12. The Rua stand was thinned to 577 stems/ha at age 7; the Toru stand was thinned to 549, 292, and 162 stems/ha at ages 8, 11, and 15, respectively; and the Tahi stand was thinned to 495, 156, and 63 stems/ha at ages 6, 10, and 14 years, respectively (Beets & Brownlie 1987). The second-rotation forest at Puruki included plots with several seedlots (Museum, Puruki control, GF30, and High Density) in a randomised block design, each with approximately 900 stems/ha at age 3. The GF rating scale (Vincent 1987) ranges from GF0 (Museum) to GF30 (high growth rate and stem form), and the high density (HD) seedlot originated from improved parents with high wood density (Beets *et al*. 2007).

## Stem Measurements

These trees were measured as part of biomass studies; stem diameter under-bark was measured at the base (0.1 m above ground) of the tree and at fixed height intervals along the stem, with the length of the interval depending on the tree age.

TABLE 1–Number of trees in *P. radiata* biomass studies from Puruki.

| Rotation | Stand or treatment | Stocking (stems/ha) | Stand age (years) | | | | | | | | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | 3 | 10 | 11 | 12 | 13 | 17 | 22 | 23 | |
| First rotation | Inv | 1550 | | | | 5 | | 5 | | | 10 |
| | Rua | 577 | | 7 | 7 | 5 | 7 | 5 | | | 31 |
| | Toru | 549, 292, 162 | | 7 | 7 | 5 | 7 | 5 | | | 31 |
| | Tahi | 495, 156, 63 | | 7 | 7 | 5 | 7 | 5 | 10 | 10 | 51 |
| Second rotation | Museum | 900 | 16 | | | | | | | | 16 |
| | Puruki control | 900 | 48 | | | | | | | | 48 |
| | GF30 | 900 | 46 | | | | | | | | 46 |
| | High density | 900 | 46 | | | | | | | | 46 |
| Total | | | 156 | 21 | 21 | 20 | 21 | 20 | 10 | 10 | 279 |

TABLE 2–Number of trees in *P. radiata* biomass studies other than Puruki.

| Location | Treatment | Stocking (stems/ha) | Stand age (years) 5 | 8 | 45 | Total |
|---|---|---|---|---|---|---|
| **Warkworth** | Irrigation trial | 1500 | | 6 | | 6 |
| Warkworth total | | | | 6 | | 6 |
| **Woodhill** | Lupin | 270 | | | 7 | 7 |
| | Fertiliser | 2500 | 60 | | | 60 |
| | No fertiliser | 2500 | 60 | | | 60 |
| Woodhill total | | | 120 | | 7 | 127 |
| **Kinleith** | Fertiliser | 2500 | 36 | | | 36 |
| | No fertiliser | 2500 | 36 | | | 36 |
| Kinleith total | | | 72 | | | 72 |
| **Tarawera** | Fertiliser | 2500 | 80 | | | 80 |
| | No fertiliser | 2500 | 79 | | | 79 |
| Tarawera total | | | 159 | | | 159 |
| **Golden Downs** | Fertiliser | 2500 | 36 | | | 36 |
| | No fertiliser | 2500 | 36 | | | 36 |
| Golden Downs total | | | 72 | | | 72 |
| **Total by age and overall total** | | | 423 | 6 | 7 | 436 |

The height intervals used were 0.70 m in the 3-year-old stands at Puruki, 1 m in the 5-year-old stands at Golden Downs, Kinleith, Tarawera, and Woodhill, 2 m in the 10- to 23-year-old trees at Puruki, and 3 m in the 27- and 45-year-old trees at Kaingaroa and Woodhill.

Stem diameter was measured with a diameter tape placed around the circumference of the stem under bark and at least 10 cm away from branch whorls. The actual heights where stem diameter measurements were made were also recorded. The volume of each stem was calculated by summing sectional volumes, and section volumes were calculated using the formula for a frustrum of right circular cone, i.e., $V_{section} = \pi L(D_1^2 + D_1D_2 + D_2^2)/12$, where $L$ is the section length, and $D_1$ and $D_2$ are the diameters at each end of the section.

## Existing Stem Volume Functions

Although the biomass data contained a very complete set of measurements of small trees, there were far fewer trees from more mature stands represented. Therefore, to ensure that the volume function performed well for trees of all ages and sizes, data were extracted from the Ensis Permanent Sample Plot (PSP) system (Dunlop 1995). This extensive database includes height and diameter measurements of trees from large numbers of plots, and has available many individual-tree volume functions for predicting stem volumes. These functions have been developed for use in different regions, crop types, and stand ages (see Dunlop 1995, Appendix 6), and predict individual-tree under-bark stem volume from height and diameter at breast height.

Stand volume estimates were derived using these functions for a set of nine final-crop stocking trials located across the country, chosen to represent a wide range of site types, in terms of both Site Index and 300 Index (Kimberley *et al.* 2005). Most trials contained stocking treatments of 50, 100, 200, 400, and 600 stems/ha, and there were included a total of 88 PSPs and 515 plot measurements. Basal area, mean top height, age, and stocking were obtained for each plot measurement. Stand volume was obtained using 53 different *P. radiata* volume functions by summing the individual-tree volume predictions. Across all measurements, mean top height averaged 29 m and ranged from 20 to 46 m, and basal area averaged 36 $m^2$/ha and ranged from 7 to 92 $m^2$/ha.

As most PSP volume functions were developed for use with trees extracted for processing, either as commercial thinnings or at clearfelling, the volume functions in the PSP database are not generally reliable for very small trees. The heights of the shortest and tallest trees used in deriving each function were recorded by Dunlop (1965, Appendix 6) and, when functions were applied outside their range, they generally produced extremely unstable volume predictions compared to those from more broadly based functions. Therefore, volumes were extracted from PSP measurements only when the mean top height was within this range for each function. In the final analysis, an overall minimum mean top height of 20 m was also applied. The purpose of the PSP data was to provide good performance for the volume function for larger trees by indirectly using the very large database of trees from which these PSP volume functions were derived. The biomass data provided excellent information on smaller trees, particularly for trees between ages 3 and 10 years, an age range over which existing functions are unlikely to perform well.

## Statistical Analysis

For each tree in the biomass dataset, breast height volume form factors $f = v/(ba \times h)$ were calculated, where $v$ is stem volume, $ba$ is tree basal area, and $h$ is tree height. Mean $h$, $ba$, $v$, and $f$ were calculated for each plot. Tests of differences in mean $f$ between treatments in each experiment were performed using analysis of covariance, with $h$ included as a covariate using the SAS (Version 9) procedure PROC MIXED. An individual tree volume function was then developed by plotting $f$ against $h$, and testing various functions for predicting $f$ from $h$. From these, a suitable functional form for predicting $v$ from $ba$ and $h$ and other plot variables was chosen and tested. This function was fitted using the SAS procedure PROC

NLIN. Because the variance of $v$ was strongly non-homogeneous, and in general proportional to the square of the mean, the nonlinear regression was fitted using iterative reweighting by the inverse of the predicted value squared. Note that parameters fitted this way give approximately unbiased estimates of $v$, whereas the alternative approach of using unweighted regression with log($v$) as the dependent variable produces slightly biased estimates. The performance of the individual-tree volume model was explored by calculating the percentage prediction error, $100\times(v_{Actual}-v_{Pred})/v_{Pred}$, for each tree. Differences between treatment and site in mean prediction error were tested using analysis of variance.

Mean top height was calculated for each plot using the method described by Dunlop (1995). To develop a stand-level volume function, stand volume form factors $F$ (defined as $F = V/(MTH\times BA)$, where $V$ is stand volume and $BA$ is stand basal area) were calculated for each biomass plot. Stand form factor was also calculated for each of the 53 existing volume functions applied to each PSP measurement. The form factors of the volume functions were then examined and functions that appeared unrepresentative or abnormal were excluded from further analysis. The remaining volume functions were not balanced across all PSPs because of differences in their valid height ranges. Therefore, the PSP volumes were processed by 2-way analysis of variance using the SAS procedure PROC MIXED, with model factors for PSP measurement and volume function. The least squares means for each PSP measurement obtained from this analysis had equal weighting for each volume function, and were used in the subsequent analysis.

Stand form factor of the combined biomass and PSP data was then plotted against *MTH* and various functional forms were tested for predicting $F$ as a function of *MTH* and other plot variables. From these, a function was developed for predicting $V$ as a function of *BA* and *MTH*. The function was fitted to the combined biomass and PSP data using the SAS NLIN procedure with iterative reweighting. Percentage errors were calculated and analysed for this volume function against the biomass data, against the PSP predicted volumes, and also against Kaingaroa volume function T237 which is often regarded as an “average” tree volume equation for *P. radiata* in New Zealand (A. Gordon, pers. comm.).

## RESULTS

### Individual-tree Volume Function

The following function was found to predict individual-tree form factor well for the biomass data:

$$f = a \times [(h-1.4)^{-b} + c] \qquad [1]$$

To test for differences between experiments, separate estimates of the parameter $a$ were fitted for each experiment using dummy variables. The 45-year-old trees at Woodhill were treated as a separate experiment in this analysis. This model produced no significant improvement in fit over the general model, indicating that form factor did not differ significantly between experiments once tree height was taken into account ($F_{7,135} = 1.30$; $p = 0.26$).

Differences between treatment means of form factor were tested using analysis of covariance with *h* included as a covariate. For the Woodhill, Golden Downs, Tarawera, and Kinleith trials, this analysis testing the effect of fertiliser on form factor of young trees was conducted using plot means. Only the Woodhill trial (AK1029) showed a significant fertiliser effect on form factor, the trees with fertiliser having a smaller factor than the trees without (Table 3).

Analysis of second-rotation trees at Puruki (FR443/5) using plot means indicated no significant differences between seedlots in form factor (Table 4).

TABLE 3–Least squares means (with standard errors in parentheses) of form factor adjusted to a common mean height for 5-year-old trees with and without fertiliser at four locations (experiment number in parentheses).

| | Woodhill (AK1029) | Kinleith (FR188) | Nelson (FR220) | Tarawera (FR41) |
|---|---|---|---|---|
| With fertiliser | 0.431 (0.008) | 0.492 (0.015) | 0.444 (0.011) | 0.440 (0.011) |
| No fertiliser | 0.459 (0.008) | 0.498 (0.015) | 0.476 (0.011) | 0.433 (0.011) |

TABLE 4–Least squares means (with standard errors in parentheses) of form factor adjusted to a common mean height for four seedlots of 3-year-old second-rotation trees at Puruki.

| Seedlot | Mean form factor |
|---|---|
| Museum | 0.637 (0.088) |
| Puruki control | 0.753 (0.047) |
| GF30 | 0.706 (0.049) |
| High density | 0.763 (0.048) |

Analysis of the first-rotation trees at Puruki was performed using individual trees, as the treatments in this trial consisted of different stockings in large unreplicated stands. Separate analyses were carried out for each age. Apart from a minor difference in the 10-year-old measurements (the form factor for Tahi was significantly lower than for Rua and Toru), no significant differences in form factor were found between treatments at the same age in this trial (Table 5).

The following general individual-tree volume function was fitted to the combined biomass data:

$$v = h \times ba \times (a \times (h - 1.4)^{-b} + c) \qquad [2]$$

Parameter estimates and standard errors are given in Table 6.

TABLE 5–Least squares means (with standard errors in parentheses) of form factor, adjusted to a common mean height for each age, for different stocking treatments in first-rotation trees at Puruki.

| Age (years) | Inv (1550 stems/ha) | Rua (577 stems/ha) | Tahi (thinned progressively to 63 stems/ha) | Toru (thinned progressively to 162 stems/ha) |
|---|---|---|---|---|
| 10 | | 0.393 (0.010) | 0.363 (0.010) | 0.401 (0.010) |
| 11 | | 0.377 (0.015) | 0.379 (0.017) | 0.392 (0.018) |
| 12 | 0.363 (0.013) | 0.391 (0.013) | 0.360 (0.014) | 0.357 (0.014) |
| 13 | | 0.360 (0.012) | 0.352 (0.012) | 0.358 (0.013) |
| 17 | 0.355 (0.015) | 0.380 (0.016) | 0.350 (0.014) | 0.335 (0.015) |
| 22 | | | 0.329 (0.010) | |
| 23 | | | 0.333 (0.007) | |

TABLE 6–Parameter estimates for Equation [2].

| Parameter | Estimate | Standard error |
|---|---|---|
| a | 0.860 | 0.019 |
| b | 0.972 | 0.028 |
| c | 0.304 | 0.008 |

Analyses of variance were used to test for treatment differences in the prediction error for each experiment and age, and showed no significant differences ($\alpha = 0.05$) except for a minor difference between the age-10 Puruki stands. This demonstrated that the individual-tree volume equation worked equally well across the wide range of stockings in first-rotation Puruki, across all four seedlots in second-rotation Puruki, and for treatments with and without fertiliser in each of the four fertiliser trials. Prediction errors are summarised by study and age in Table 7. Mean prediction errors showed no clear trends with either site fertility or age. Only three of the means shown in Table 7 differed significantly from zero. Prediction errors at Puruki tended to be positive, and grouped together were marginally significantly greater than zero ($t = 2.18$, $p = 0.031$).

## Stand-level Volume Function

All the 49 *P. radiata* volume tables listed by Dunlop (1995, Appendix 6), along with several more recent tables, were initially considered. Stand-level form factors plotted against mean top height for these functions are shown in Fig. 1, with each function plotted over its experimental height range. Six functions which

TABLE 7–Percentage prediction errors of individual tree volume function by experiment and age.

| Experiment | Age (years) | Mean prediction error (%) | Standard deviation (%) |
|---|---|---|---|
| Puruki | 10 | 1.5 | 7.4 |
| | 11 | 4.1 | 10.5 |
| | 12 | 2.1 | 8.3 |
| | 13 | 0.0 | 8.9 |
| | 17 | 3.7 | 9.5 |
| | 22 | –1.5 | 9.3 |
| | 23 | –0.5 | 6.6 |
| Woodhill | 5 | –2.8** | 10.0 |
| | 45 | 4.9* | 5.0 |
| Kinleith | 5 | 0.8 | 11.8 |
| Tarawera | 5 | –3.0** | 11.4 |
| Nelson | 5 | 1.4 | 11.9 |
| Warkworth | 8 | 7.4 | 8.1 |

* Significantly different from zero ($\alpha = 0.05$)
** Significantly different from zero ($\alpha = 0.01$)

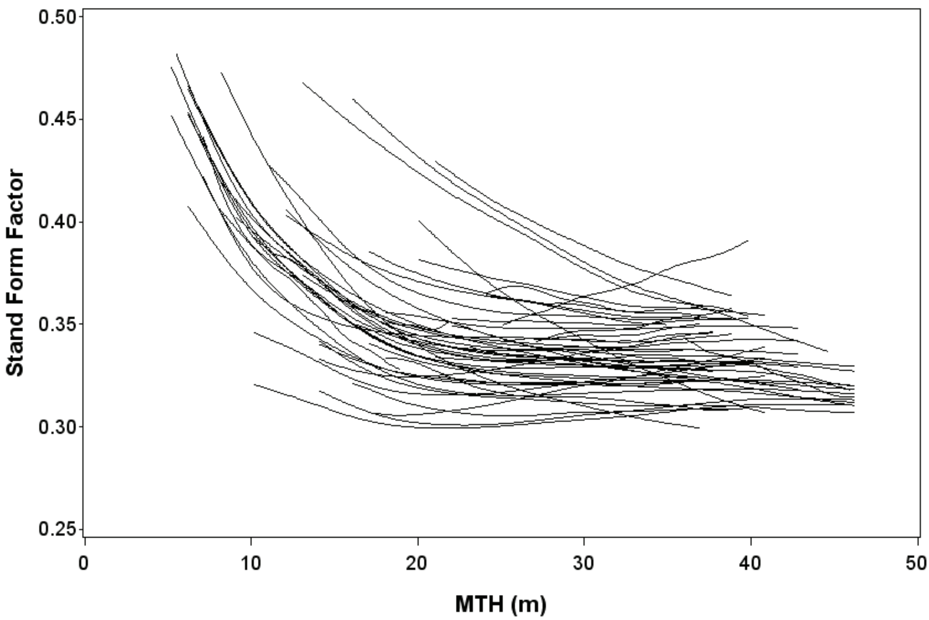


FIG. 1–Stand volume form factors plotted against *MTH* for 53 *P. radiata* volume functions. The lines shown are smoothing spline functions fitted to volume predictions for 515 measurements from 88 plots. Each function is plotted against a *MTH* range equivalent to the height range of trees used to derive the function.

showed behaviour markedly different to the others (three with form factors much greater than normal, one with a very strong positive slope, and two with excessive negative slopes) were excluded from further consideration. Mean form factors for the remaining 47 functions were calculated for a mean top height of 35 m. Examination of these form factors indicated that the seven functions developed for coastal sand forests had form factors averaging 0.349, well above the overall average. Fourteen functions developed in the 1950s and 1960s and two "Old Crop" Kaingaroa functions derived in 1975 had form factors averaging 0.320, consistently lower than the overall average. Therefore, these earlier functions and the coastal sand functions were excluded from subsequent analysis. The form factors at 35 m mean top height of the remaining 24 functions averaged 0.336 with standard deviation 0.011, and ranged from 0.312 to 0.355. No consistent pattern of difference was detected between North and South Island form factors. The average form factor of these functions confirms a rule-of-thumb attributed to H. Beekhuis (L. Knowles, pers. comm.) that stand volume in mature *P. radiata* can be calculated using $BA \times MTH/3$. The form factors for these selected functions are shown in Fig. 2.

Form factors for the PSP data (restricted to mean top height >20 m) and the biomass data are shown in Fig. 3. The two datasets are compatible with each other, with the form factor declining steeply with increasing mean top height in young trees,

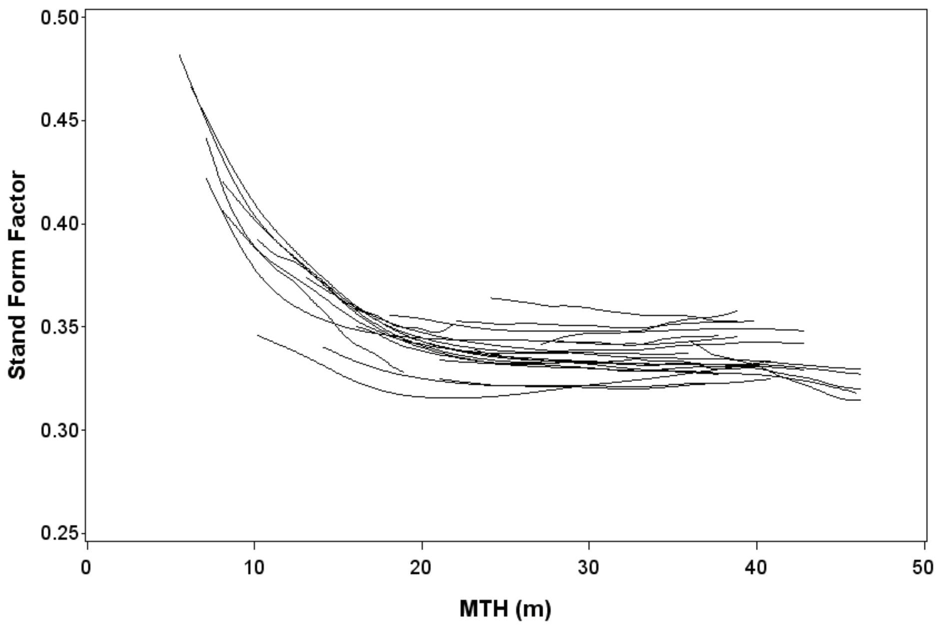


FIG. 2–Stand volume form factors plotted using smoothing splines against *MTH* for 24 selected *P. radiata* volume functions.

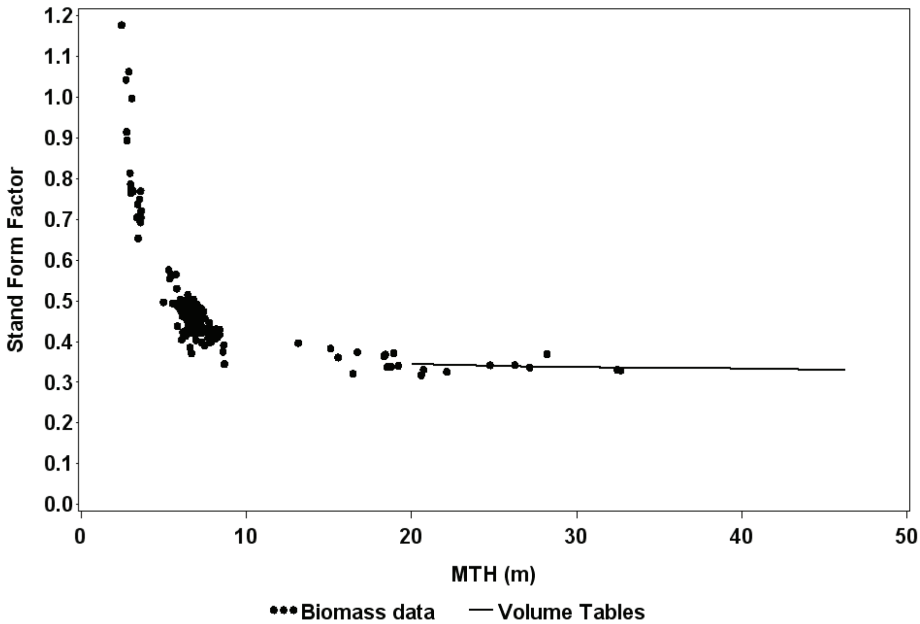


FIG. 3–Stand volume form factor *versus MTH*. Asterisks show biomass plot means, and the solid line is a smoothing spline function fitted through the PSP volume table data.

and flattening towards an asymptote in older stands. The following function was fitted to the combined dataset:

$$V = MTH \times BA \times (a \times (MTH - 1.4)^{-b} + c) \qquad [3]$$

The parameter estimates are shown in Table 8. Other variables were tested for inclusion in this model including stocking and age, but they did not contribute anything to the fit. Power coefficients for stand basal area and mean top height were also tested but found not to improve the fit.

Given the generally close agreement between the volume functions shown in Fig. 2 from which Equation [3] is derived, along with additional data from mainly smaller trees in the biomass data set, it is believed that this function will provide a reasonably reliable estimate of under-bark stem volume for *P. radiata* anywhere

TABLE 8–Parameter estimates for Equation [3].

| Parameter | Estimate | Standard error |
|---|---|---|
| a | 0.942 | 0.020 |
| b | 1.161 | 0.021 |
| c | 0.317 | 0.001 |

in New Zealand. Estimates using this new stand-level volume function were in close agreement with the results found across the entire range of stand volumes in the combined dataset, which ranged from 1.8 $m^3$/ha in the lowest volume 3-year-old Puruki plot, to over 1200 $m^3$/ha for some of the older PSP measurements (Fig. 4).

Prediction errors were calculated for each biomass plot, and showed no trends of bias with tree size (Fig. 5), although errors in individual plots were sometimes significant (mean error = 0.1%, s.d. = 6.8%). Errors for biomass study sites closely followed those of the individual tree model discussed above, with no consistent patterns of bias evident. Prediction "errors" were also calculated against the volume estimates from the PSPs used for developing the model. These also showed little trend except for a slight tendency to under-predict (positive "error") at extreme stand mean top height. Predictions were also compared with the Kaingaroa volume function T237 which is often regarded as an "average" tree volume equation for *P. radiata* in New Zealand, and this revealed little bias over the range of heights valid for this function.

Although Equation [3] can be used for reliably estimating volume for *P. radiata* stands with mean top height greater than 1.4 m, at and below this height the form

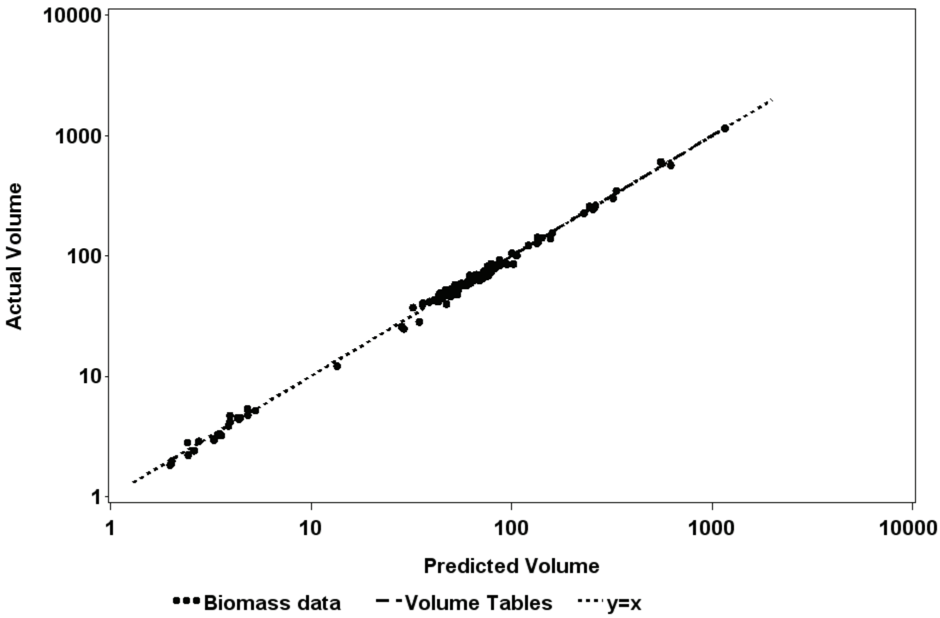


FIG. 4–Actual versus predicted stand volume obtained using Equation [3]. Asterisks show biomass plot means and the solid line is a smoothing spline function fitted through the PSP data. The y = x identity is given by the dashed line.

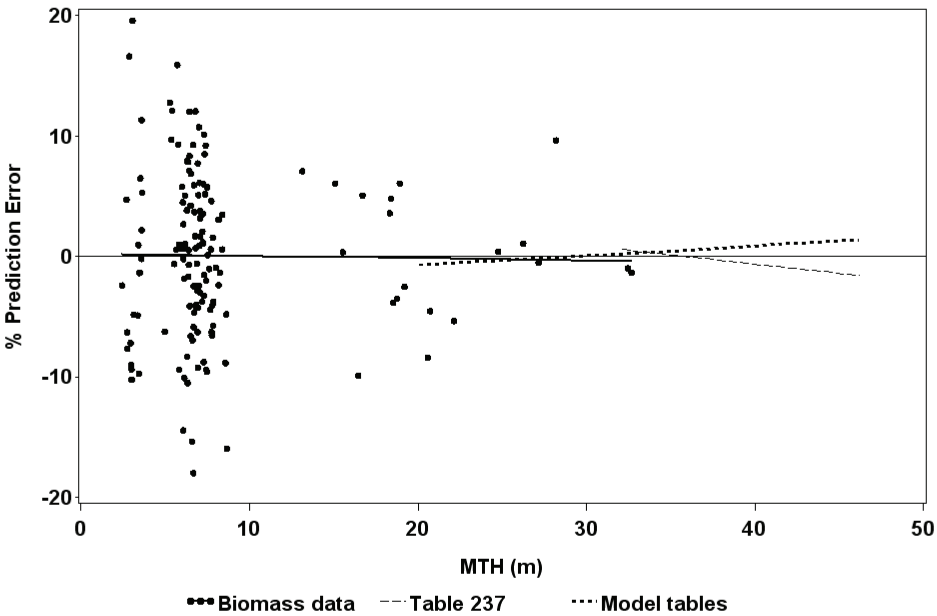


FIG. 5–Stem volume prediction error versus *MTH* for each biomass plot. Also shown are linear regressions for prediction error in the biomass plots, prediction error for the PSP volumes that were estimated from the tables used to develop the model, and prediction error for the PSP volumes that were calculated using volume function T237.

factor is effectively infinite because the basal area is zero or undefined. Therefore, in the first year or two of growth when the mean top height is less than 1.4 m, and the basal area is undefined, an alternative stem volume function is required. This alternative function is based on the initial volume and growth rate of seedlings in trials. The mean volume of 120 nursery seedlings at time of lifting from the nursery beds at Rotorua (0.064 $cm^3$) was estimated from seedling mean stem weight (reported by Beets *et al.* 2007) and density. Seedling annual root collar diameter and height growth measurements from the Kinleith fertiliser trial (data not shown) showed that the mean volume of seedlings typically increases exponentially during the first few years, with the increase being proportional to age raised to the power of 2.7. Volumes of small trees from planting until diameter at breast height reaches about 2 cm were calculated as follows. Firstly, the age $T_1$ when trees reach about 2 cm average dbh is calculated from the 300 Index growth model. The mean individual tree volume $v_1$ at age $T_1$, is found by predicting stand volume using Equation [3] and dividing by stocking, $N$. Stand volume $V$ at age $T$ between age zero and $T_1$ is then predicted using the following equation:

$$V = N \times (0.064 + (v_1 - 0.064)T^{2.7} / T_1^{2.7}) \quad [4]$$

Equation [3], which should be applied when the mean diameter at breast height is greater than 2 cm, and Equation [4], which should be applied when the mean diameter at breast height is less than 2 cm, together provide reasonable estimates of stem volume of *P. radiata* stands across the entire age range from planting to clearfelling age.

## DISCUSSION

Plot summary statistics generated by the Permanent Sample Plot system include estimates of stem total gross and net volume under bark. Stem volume is estimated from tree diameter and height using either a tree or a stand-level volume function associated with each plot (Dunlop 1995). To ensure accuracy, volume functions need to be developed from sectionally measured trees sampled from the stands of interest. This presents problems in studies where permanent plots are installed in stands from an early age and monitored for many years, because tree volume functions generally do not apply across a wide range in tree sizes.

The volume functions within the PSP system examined in this paper illustrate this point, although a subset of these functions performed well across a moderately wide range in tree size. One conclusion from this study is that it is extremely dangerous to use volume functions outside their model range. In Fig. 2, form factors are plotted for only the range of the tree heights used in developing each function and, with a few exceptions, all show reasonable behaviour across their model range. However, many become extremely unstable when used outside their range. For example, at 15 m *MTH*, several functions produced form factors greater than 0.45 compared with the biomass study mean of about 0.35. Even volume function T237, which is considered an "average tree" volume function for New Zealand, shows poor behaviour in young trees, with a form factor of 0.39 at 15 m *MTH* indicating volume over-prediction of about 11%.

The approach described in this paper of combining data from young trees with predictions from existing individual-tree functions developed for older-aged stands growing on traditional forest and fertile sites provided a logical and sound method of developing a national volume function applicable to the majority of New Zealand's *P. radiata* forest estate. It combines existing knowledge based on a large sample of semi-mature and mature trees with new data obtained for young trees, and thereby provides a stand-level volume function that performs well across a wide range in tree sizes and ages.

Form factors based on diameter at breast height increase as tree height decreases, and this is accurately reflected by the new volume functions for trees exceeding 1.4 m in height. An alternative modelling approach was necessary to predict stem volume of trees less than 1.4 m in height, and these two approaches combined can be expected to provide robust volume estimates from time of planting to the clearfelling age for traditional forest and fertile sites, including Kyoto forest, throughout New Zealand.

Analysis of the experimental treatments shown in Tables 1 and 2 indicated that seedlot, stocking, and nitrogen fertiliser treatment-effects on stem form factor were rarely statistically significant, after taking into account tree height. It is recommended that 50 trees be sectionally measured when testing existing functions for suitability (Gordon & Penman 1987). Only trees growing on nitrogen-deficient sand differed significantly from trees with fertiliser, with greater form factors evident in non-fertiliser treatments, consistent with results from previous research (Gordon & Graham 1986). The seven volume functions derived for coastal sand sites showed consistently higher form factors in mature trees than did other functions. It is unclear whether this behaviour is due to the low fertility or is related to the high levels of exposure on these coastal sites, as suggested by A. Gordon (pers. comm.). However, because of these differences, sand sites were excluded from the analysis. The resulting volume functions are therefore applicable to all other sites including traditional forest sites and fertile sites.

Volume functions are usually developed to provide precise volume estimates for a particular site, and often for a particular crop type. They are also generally intended to be used only at clearfelling, unless specifically developed for a mid-rotation thinning. In contrast, the main objective in this study was to develop individual-tree and stand-level functions giving reasonable volume predictions nationally for any age from planting through to maturity. It was also hoped that the effects of stocking, genotype, and site would be small enough to allow national-level functions to perform adequately. The analysis indicated that this expectation was largely justified, other than on coastal sand sites where the models are expected to under-predict volume in mature stands.

This does not imply that there is not considerable tree-to-tree variation in form factor; the tree volume function estimates an individual-tree volume only to within ±20% (± 2 s.d., Table 7). However, given sufficient trees, prediction errors averaged for a site or crop type are expected to be within a few percentage points either side of zero, as found in the various site types and experimental treatments analysed in this study. Analysis of historical Permanent Sample Plot volume functions indicated that their form factors vary by ± 7% (± 2 s.d.). Note that this covers 95% of the functions; half the functions were within ± 3.5% of the mean. For mature trees, the form factor of the stand-level function developed in this study is essentially an average of these historic functions, and errors in volumes predicted for any particular site or crop type in New Zealand will therefore likely fall within this error range. In fact, this range is probably exaggerated as a component of the variation between historic functions is likely to have been contributed by methodological differences between studies and by individual tree variation within each study.

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