IS CURVATURE ADJUSTMENT NECESSARY IN GROWTH STRAIN MEASUREMENT?

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ABSTRACT

A mathematical procedure was established to calculate a threshold tree/log radius above which the curvature adjustment required in Nicholson's "primary" procedure of measuring longitudinal growth strain may be omitted. The radius derived is a function of longitudinal growth strain on the surface of tree stems or logs, initial distance between measuring points, and the allowable error. The indication is that for all practical circumstances, it may not be necessary to account for curvature, based on the Kubler distribution of growth stresses. Formulas were also constructed to derive longitudinal surface strain under different circumstances.

Keywords: growth stress; longitudinal growth strain; curvature adjustment; eucalypts.

INTRODUCTION

Eucalypts are among the most widely planted tree species in the world. In many countries, the initial and primary purpose for planting them was to establish a pulp and paper resource. However, there has been increasing interest in utilising eucalypt "pulp logs" for higher-value uses as appearance grade timbers, sliced and rotary veneers, and laminated products. There are different log requirements and more-involved log processing in making these products. There are new problems as well. One example is growth stresses, which are of little concern in wood chipping but emerge as a significant factor in log sawing. Growth stresses are self-generated in the cambium during cell maturation, are present in all tree species, and are far more severe in some eucalypt species than in many other hardwoods (Jacobs 1938; Kubler 1959, 1987). The continuous formation of growth stresses during tree growth results in uneven distribution of residual stresses across tree stems. When logs are sawn longitudinally, these residual stresses are partially released and the gradient of longitudinal residual stresses causes sawing inaccuracy, spring in quarter-sawn boards, and bow in back-sawn boards.

A number of methods have been developed to measure longitudinal growth strain on the surface of tree stems and logs with bark removed (*see* review by Kubler 1987). The predominant method used in Australia is that developed by Nicholson (1971).

With Nicholson's "primary" procedure, two steel studs are glued to the surface of a tree stem or a log which has had the bark removed. The studs are about 50 mm apart and aligned parallel to the wood grain. The linear distance between the studs is measured before and after a wood segment, approximately 10 × 19 × 90 mm with the two studs in the centre (Fig. 1), is extracted from the tree or log. Because of the gradient of longitudinal growth strain along the tree radius (Kubler 1959), the wood segment may develop a curvature on extraction (Nicholson 1971). The removal of this "secondary" curvature is accomplished using a special apparatus to bend the wood segment in the opposite direction. After curvature adjustment, the distance between the studs becomes the true linear measurement which is measured after strain release. Strain is calculated from the before-extraction and after-extraction linear measurements.

The curvature adjustment takes approximately 20% of the whole measurement time with the Nicholson's apparatus (1971). It becomes very time-consuming when large numbers of trees are to be measured. It would be very useful, therefore, to know under what circumstances the adjustment might be disregarded. We know from Kubler (1959) that longitudinal strain differential is a function of the magnitude of longitudinal surface strain and the radius of trees or logs. We set out to establish the tree/log radius, for a given magnitude of longitudinal surface strain, at which the curvature adjustment may be neglected with a specified limit of error in the strain when the strain is measured on an extracted wood segment.

METHODS

The following assumptions were made: (1) the wood segment prior to extraction from a tree or log is as illustrated in Fig.1.; (2) the wood segment is flat prior to extraction, i.e., its initial curvature is zero; (3) the segment is curved after its extraction, as a result of the release of the longitudinal stresses; (4) the tree or log is concentric.

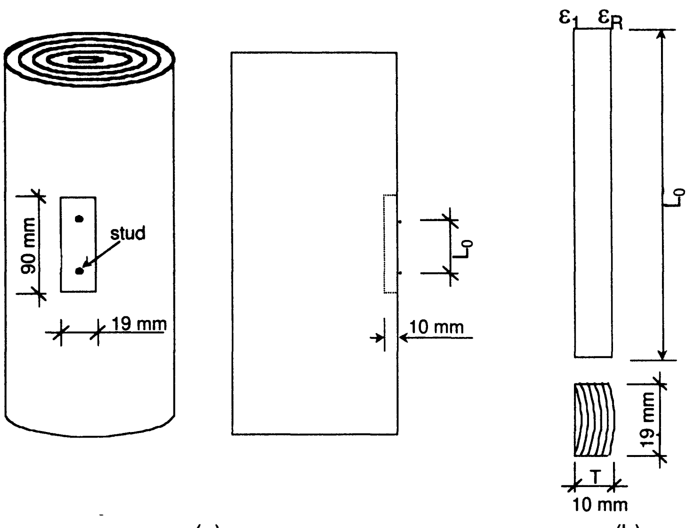

FIG. 1–(a) The wood segment to be extracted from the surface of a tree or log as specified in Nicholson's "primary" procedure (1971);
(b) The section within the wood segment involved in the strain measurement.

Symbols

H = Deflection of the segment (*refer* Fig. 2)

L_0 = Linear distance (mm) along the wood grain between the two steel studs, measured when the wood segment is in the tree/log (*see* Fig. 1)

L_R = Arc length (mm) between the two studs on the cambium-side surface of the segment corresponding to ε_R, after the extraction of the segment from the tree/log (*see* Fig. 2)

L_1 = Arc length (mm) between the two studs on the pith-side surface of the segment corresponding to ε_1, after the extraction of the segment (*see* Fig. 2)

L = Linear distance (mm) between the two studs, measured on the cambium-side surface after the extraction of the segment without curvature correction (*see* Fig. 2)

R = Radius of the tree/log, excluding the bark (mm)

T = Thickness of the wood segment in the radial direction (mm)

γ = Substitution parameter (27)

δ = Error in strain when there is no curvature adjustment

ε_e = Strain without curvature adjustment

ε_N = Longitudinal strain as determined using Nicholson's "primary" procedure (1971), on the surface of a tree or log with curvature adjustment

ε_R = Longitudinal strain on the surface of a tree or log

ε_1 = Longitudinal strain at a distance T inwards from the tree/log surface

θ = Angle corresponding to half of the segment curvature

ρ = Radius of the curvature of the segment

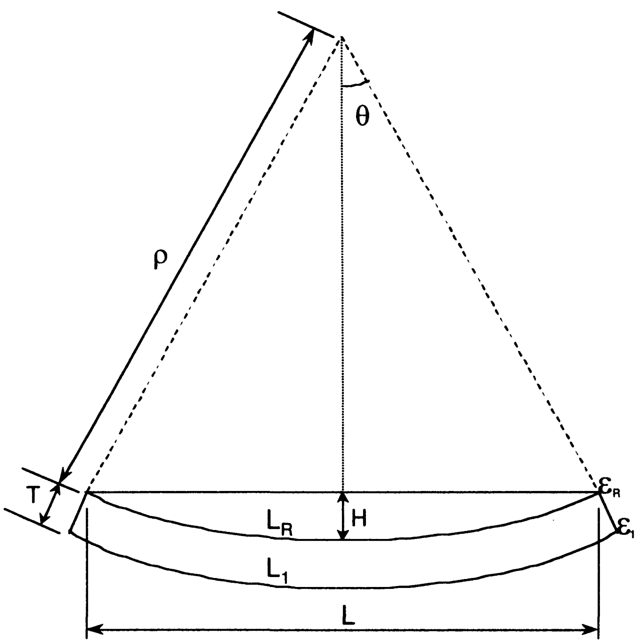

FIG. 2–The wood segment after removal, showing the variables used in the derivation.

Derivation of Threshold Radius

From Fig. 2 (L_0 is not shown)

$$L_l = L_0\,(1-\varepsilon_l) \tag{1}$$

$$L_R = L_0\,(1-\varepsilon_R) \tag{2}$$

$$2\rho\theta = L_R = L_0\,(1-\varepsilon_R) \tag{3}$$

$$2\,(\rho+T)\,\theta = L_l = L_0\,(1-\varepsilon_l) \tag{4}$$

From Equations (1), (2), (3), and (4)

$$\rho + T = \rho\left[\frac{1-\varepsilon_l}{1-\varepsilon_R}\right] \tag{5}$$

and so

$$\rho = \frac{T(1-\varepsilon_R)}{\varepsilon_R-\varepsilon_l} \tag{6}$$

From Equations (3) and (6)

$$\theta = \frac{L_0(1-\varepsilon_R)}{2\rho} = \frac{L_0(1-\varepsilon_R)(\varepsilon_R-\varepsilon_l)}{2T(1-\varepsilon_R)} = \frac{L_0(\varepsilon_R-\varepsilon_l)}{2T} \tag{7}$$

If no adjustment for curvature is made, the estimated value for ε_R is according to Fig. 2 (L_0 is not shown)

$$\varepsilon_e = \frac{L_0-L}{L_0} \tag{8}$$

whereas from (2) above, the true value of the strain is

$$\varepsilon_R = \frac{L_0-L_R}{L_0} \tag{9}$$

The fractional error in strain when the curvature adjustment is not made is

$$\delta = \frac{\varepsilon_e-\varepsilon_R}{\varepsilon_R} \tag{10}$$

With the substitutions from (8) and (9), we have from (10)

$$\delta = \frac{\dfrac{(L_0-L)}{L_0}-\dfrac{(L_0-L_R)}{L_0}}{\dfrac{(L_0-L_R)}{L_0}} = \frac{L_R-L}{L_0-L_R} = \frac{L_0(1-\varepsilon_R)-L}{L_0\,\varepsilon_R} \tag{11}$$

Again, according to Fig. 2

$$L = 2\rho\sin\theta \tag{12}$$

With substitutions from (6), (7), and (12), we have from (11)

$$\delta = \frac{L_0(1-\varepsilon_R)-2\rho\sin\theta}{L_0\,\varepsilon_R} = \frac{L_0(1-\varepsilon_R)-2\dfrac{T(1-\varepsilon_R)}{\varepsilon_R-\varepsilon_l}\sin\theta}{L_0\,\varepsilon_R}$$

$$= \frac{(1-\varepsilon_R)}{\varepsilon_R}\left[1-\frac{2T}{L_0(\varepsilon_R-\varepsilon_l)}\sin\theta\right] = \frac{(1-\varepsilon_R)}{\varepsilon_R}\left[1-\frac{\sin\theta}{\theta}\right] \tag{13}$$

Since ε_R and ε_l are both very much less than unity with reference to (7), so also is θ. It follows that

$$\sin\theta \approx \theta - \frac{\theta^3}{6} \cdots \tag{14}$$

With the expansion (14) applied to (13), and neglecting higher-order terms

$$\delta = \frac{(1-\varepsilon_R)}{\varepsilon_R}\left[1 - \frac{1}{\theta}\left(\theta - \frac{\theta^3}{6}\right)\right] = \frac{(1-\varepsilon_R)}{\varepsilon_R}\frac{\theta^2}{6} \tag{15}$$

and so

$$\theta = \sqrt{\frac{6\varepsilon_R\delta}{1-\varepsilon_R}} \tag{16}$$

Eliminating θ between (7) and (16)

$$\frac{L_0(\varepsilon_R - \varepsilon_l)}{2T} = \sqrt{\frac{6\varepsilon_R\delta}{1-\varepsilon_R}} \tag{17}$$

Kubler's distribution of longitudinal growth stress is derived on the assumption that new cambium layers are added having a constant value of longitudinal growth stress (Kubler 1959). The distribution of longitudinal growth strain through the tree follows as a consequence of equilibrium (Fig. 3). From this, ε_l is seen to be related to ε_R by

$$\varepsilon_l = \varepsilon_R\left(1 + 2\ln\frac{R-T}{R}\right) \tag{18}$$

so that

$$(\varepsilon_R - \varepsilon_l) = -2\varepsilon_R\ln\frac{R-T}{R} \tag{19}$$

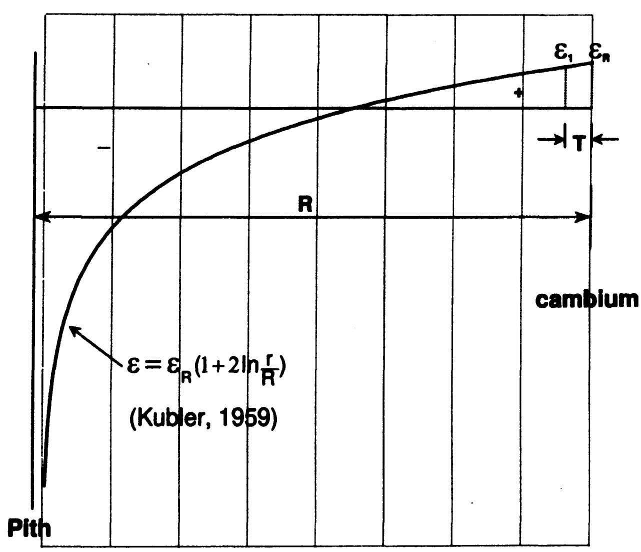

FIG. 3–Distribution of longitudinal growth strain along the radius of a tree (Kubler 1959).

Bringing together (17) and (19)

$$\ln\frac{R-T}{R} = -\frac{T}{L_0}\sqrt{\frac{6\delta}{\varepsilon_R(1-\varepsilon_R)}} \tag{20}$$

and therefore

$$R = \frac{T}{1-e^{-\frac{T}{L_0}\sqrt{\frac{6\delta}{\varepsilon_R(1-\varepsilon_R)}}}} \tag{21}$$

R is the threshold tree or log radius above which the curvature adjustment may be ignored at a given allowable error and an assumed surface longitudinal strain ε_R.

A series of "R" values can be calculated using Equation (21) once the four terms "δ", "ε_R", T, and "L_0" are given. The initial distance between the two studs (L_0) varies each time when the two studs are glued to the tree or log surface, usually between 51.0 mm and 52.0 mm. It can be seen from Equation (21) that L_0 is positively related to "R". Using a greater L_0, say 52.0 mm, in Equation (21) would yield a greater "R", therefore assuring a conservative threshold radius.

An example of these "R" values, obtained when L_0 = 52.0 mm, T = 10.0 mm, ε_R ranges from 10×10^{-6} to 5000×10^{-6}, and δ is 5% and 1% respectively, is given in Fig. 4. Since some very small terms are involved in the calculation, at least six decimal points need to be kept to avoid loss of accuracy.

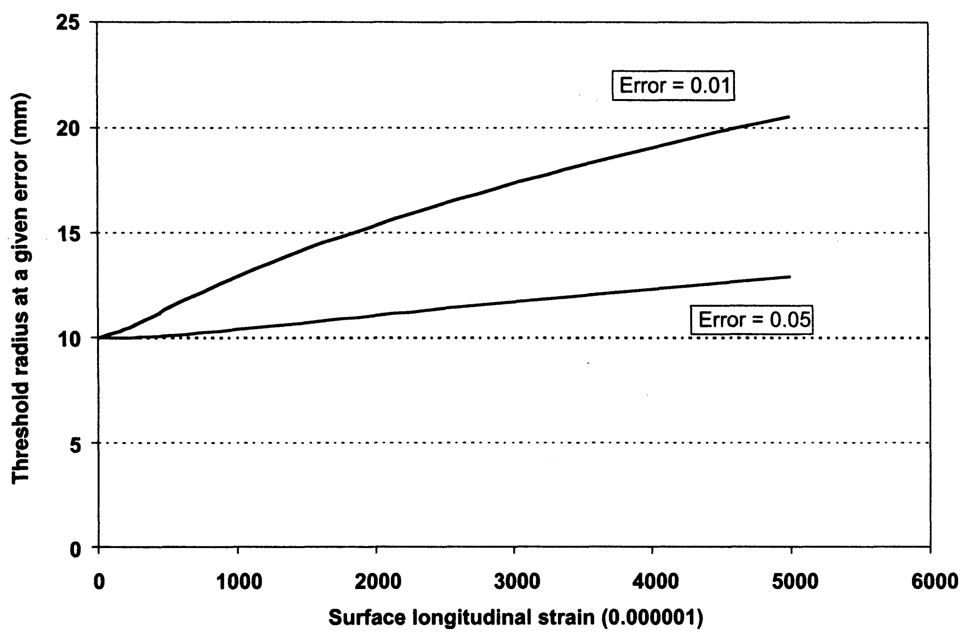

FIG. 4—The threshold radii calculated from Equation (18) as a function of longitudinal growth strain. Parameter values are: T = 10 mm, L_0 = 52.0 mm, and the allowable errors in the strain measurements are 5% and 1% respectively.

We have not intended to gather specific data to validate Equation (21). Such a validation requires trees that are very small and also have high stresses (*see* Fig. 4). In order to find suitable trees, a large number of trees would have to be measured. Further, removing a 10 × 19 × 90-mm segment (Fig. 1) from a very small tree (e.g., radius about 15 mm) for the strain measurement would be quite traumatic for the tree.

Thus, we handled validation by demonstrating that the H values corresponding to the threshold radii are far greater than the H values we have encountered.

According to Fig. 2,

$$H = \rho - \rho\cos\theta = \rho(1-\cos\theta) \approx \rho(1-(1-\frac{\theta^2}{2})) = \frac{\rho\theta^2}{2} \tag{22}$$

With substitutions from (6) and (7), we have from (22)

$$H = \frac{1}{2}\frac{T(1-\varepsilon_R)}{\varepsilon_R-\varepsilon_l}\left[\frac{L_0(\varepsilon_R-\varepsilon_l)}{2T}\right]^2 = \frac{L_0^2(1-\varepsilon_R)(\varepsilon_R-\varepsilon_l)}{8T} \tag{23}$$

Eliminating ε_l and ε_R between (19), (20), and (23), we find for H

$$H = \frac{3\delta T}{2\ln\frac{R}{R-T}} \tag{24}$$

A typical value for the threshold value of H might be calculated assuming R = 200 mm, T = 10 mm (segment thickness), and an allowable error of 1%. Substituting these figures into (24) we find H = 2.92 mm.

By examining our data collected in the past from larger logs and standing eucalypt trees (radius greater than 130 mm), we found that H values never exceeded 0.1 mm. In other words, the curvature in those segments was well below the "threshold" curvature as calculated above. Despite those segments having low to moderate strains (between 0.0003 to 0.001), their very low values of H provided an indirect support of the mathematical derivation of a threshold radius we have presented in Equation (21). Nevertheless, it is clear that Equation (21) has not been subjected to rigorous experimental validation, and when trying to use Equation 21, one needs to observe the assumptions that have been made when deriving the equations.

Derivation of Surface Strain ε_R When the Segment is NOT Straightened

If the sample is not straightened, the true strain ε_R can be obtained from the measurement of L using (7), (8), and (15). Neglecting the multiplicative term $(1-\varepsilon_R)$ during the derivation, the result is

$$\varepsilon_R = \frac{\sqrt{1+4\gamma\varepsilon_e}-1}{2\gamma} \tag{25}$$

where

$$\gamma = \frac{L_0^2}{6T^2}\ln^2\frac{R}{R-T} \tag{26}$$

In terms of the deflection of the segment H, we find a good approximation:

$$\varepsilon_R = \frac{\varepsilon_e - \dfrac{8H^2}{3L_0^2}}{1 - \dfrac{8H^2}{3L_0^2}} \qquad (27)$$

Derivation of Surface Strain ε_R When the Segment is Straightened

Straightening the segment in fact leads to a lower value for the strain than the true value ε_R because the strain in the surface fibres which was released by the removal of the segment is partially reinstated by the straightening process. What is measured in Nicholson's "primary" procedure (1971) is ε_N, not ε_R. To estimate ε_R from ε_N, one needs to go one more step, as illustrated below using Kubler's model (1959).

If the sample is straightened, the length will be very close to the curved length at the radius, $\rho + 0.5T$. With reference to Fig. 2 (L_0 is not shown), we have for ε_N,

$$\varepsilon_N = 1 - \frac{L_R + L_I}{2L_0} \qquad (28)$$

Hence with (1) and (2)

$$\varepsilon_R - \varepsilon_N = \frac{L_I - L_R}{2\,L_0} = \frac{1}{2}(\varepsilon_R - \varepsilon_I) \qquad (29)$$

Using (19)

$$\varepsilon_R - \varepsilon_N = -\,\varepsilon_R \ln\frac{R-T}{R} = \varepsilon_R \ln\frac{R}{R-T} \qquad (30)$$

from which

$$\varepsilon_R = \frac{\varepsilon_N}{1 - \ln\dfrac{R}{R-T}} \qquad (31)$$

To obtain the true strain ε_R at the surface from the strain ε_N calculated after straightening the sample, the correction is given by (31). If T is equal to 10 mm, the correction amounts to 5.4% for a 400-mm-diameter tree (R = 200 mm).

In terms of the deflection of the segment H, we find a good approximation

$$\varepsilon_R = \frac{\varepsilon_N + \dfrac{4HT}{L_0^2}}{1 + \dfrac{4HT}{L_0^2}} \qquad (32)$$

CONCLUSIONS

A procedure has been established, with specific reference to Nicholson's (1971) "primary" procedure for growth stress determination, to calculate the threshold radius. The radius derived is a function of longitudinal growth strain on the surface of tree stems or logs, initial distance between measuring points, and the allowable error. Above a threshold radius, the curvature adjustment required by Nicholson's "primary" procedure may be omitted. The derivation suggests that, for practical purposes, straightening of the sample may not be necessary unless the trees are very small. Nevertheless, caution must be exercised in actual experiments because trees are biological material, most of them do not perfectly fit the assumptions made for our derivation, and our results have not been rigorously validated with experimental data. In addition, discrepancy invariably exists between even the best mathematical model of growth strain distribution and actual measurements. These consequently could affect the "correctness" of our results under various circumstances.

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