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PRELIMINARY GROWTH AND YIELD MODELS FOR EVEN-AGED *ACACIA MELANOXYLON* PLANTATIONS IN NEW ZEALAND

JOHN-PASCAL BERRILL*, IAN D. NICHOLAS, and HAM H. GIFFORD

Ensis—the Joint Forces of CSIRO and Scion,
Private Bag 3020, Rotorua, New Zealand

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ABSTRACT

Preliminary stand growth and yield models were constructed for even-aged *Acacia melanoxylon* R. Br. plantations in New Zealand. Models that predict mean top height and basal area growth, initial basal area, post-thinning basal area, mortality, and total standing volume per hectare were fitted to permanent sample plot data biased towards younger ages. The data consisted of 1722 plot measurements from 229 sample plots. Site index estimates predicted using a polymorphic Chapman-Richards difference equation fitted as a non-linear mixed model ranged from 11.2 m to 35.5 m mean top height at age 30. The average site index was 24 m. All suitable data were used to fit models, preventing separation of independent validation data. The models were tested as a system of equations by comparing total standing volume predictions with data used to fit the models. Volume predictions were imprecise but unbiased on average. Predictions of standing volume and tree size development demonstrated the influence of site quality and stocking on *A. melanoxylon* growth and yield. Stands located on average sites, thinned to 200 stems/ha at age 10, were predicted to have 290 m^3/ha total standing volume and 49 cm average diameter at breast height at age 35 years, giving a mean annual volume increment of 8.3 m^3/ha.

Keywords: growth and yield model; stand growth; difference equation; non-linear mixed model; *Acacia melanoxylon*.

INTRODUCTION

Predictive models that account for different growth rates between stands allow forest managers to evaluate the influence of stand density (stocking) and timing of thinning on growth and yield for specific sites. Various factors influence the growth of *Acacia melanoxylon* (Australian blackwood) in New Zealand, including establishment and management practices, forest health, and site quality (Messina

* Corresponding author: pberrill@nature.berkeley.edu

& Barton 1985; Appleton *et al*. 1997; Nicholas & Brown 2002). Demand for growth and yield information prompted development of predictive models using all available data collected from permanent sample plots located in even-aged *A. melanoxylon* stands and silvicultural field trials around New Zealand.

The timber of New Zealand-grown *A. melanoxylon* is not known to present important processing problems (Haslett 1983). It resembles that grown in Australia in appearance and wood properties; its attractive appearance, fairly even texture, hardness, and good machining properties make it suitable for high-quality end uses (Haslett 1986). The interest in producing dark-coloured furniture timber to substitute for native New Zealand timbers such as rimu (*Dacrydium cupressinum* Lamb.) led to regular, typically small-scale planting of *A. melanoxylon* by New Zealanders over many decades. This plantation resource is extensive but fragmented, being planted mainly in small woodlots and along riparian zones on farms.

This paper outlines the development of preliminary stand-level growth and yield models for even-aged *A. melanoxylon* plantations in New Zealand. Models were developed to predict mean top height and stand basal area growth, initial basal area, post-thinning basal area, mortality, and total standing volume. These models can be used in combination to predict total standing volume per hectare and quadratic mean diameter, based on actual stand data or site index estimates and prescribed thinning regimes. The data, model-fitting methodologies, resultant models, and their limitations are discussed.

METHODS

Data

Stand-level data were extracted from the New Zealand Forest Research Institute Permanent Sample Plot System (Pilaar & Dunlop 1990). The per-hectare summary data consisted of 1722 plot measurements from 229 sample plots within *A. melanoxylon* stands and silvicultural field trials around New Zealand (Table 1). Mean top height and mean top diameter were calculated as the average height and

TABLE 1–*Acacia melanoxylon* measurement data summary (n = 1722). Mean annual volume increment (MAI) statistics calculated from maximum MAI for each sample plot (n = 229 plots).

	Age (years)	Stocking (stems/ha)	Mean top diameter (cm)	Mean top height (m)	Basal area (m^2/ha)	Volume (m^3/ha)	Volume MAI (m^3/ha)
Mean	9.2	788	15.6	9.1	8.6	42.9	5.23
s.d.	5.5	744	8.8	5.2	9.3	69.0	4.24
min.	3.0	20	1.3	2.1	0.0	0.1	0.14
max.	34.0	6000	50.7	31.8	55.4	462.2	21.09

diameter respectively of the 100 largest-diameter stems/ha. Average tree diameter was defined as quadratic mean diameter, the diameter of average tree basal area. Basal area was calculated as the hectare sum of cross-sectional stem area at breast height 1.4 m. Total standing volume per hectare (inside bark) was calculated as the sum of individual tree volumes predicted by a tree volume equation. A height-diameter regression predicted total tree height for sample plot stems without height data. Mean annual volume increment (MAI) was calculated for each measurement within each sample plot. Numbers of plots in each geographic region, the range of stand ages at the time of measurement, and MAI summary data are listed in Table 2.

TABLE 2–*Acacia melanoxylon* sample plot count, minimum and maximum stand age at the time of measurement, and mean annual volume increment (MAI) summary statistics by geographic region of New Zealand.

Region	No. plots	Minimum age (years)	Maximum age (years)	Mean MAI (m^3/ha)	Maximum MAI (m^3/ha)
Northland	12	5.0	18.0	7.14	15.86
Auckland	51	3.3	30.3	5.57	15.80
Waikato	37	3.0	34.0	5.79	18.09
Bay of Plenty	101	3.0	24.2	4.91	21.09
Gisborne	12	4.1	13.0	4.14	8.92
Taranaki	5	18.0	23.1	4.59	5.55
Westland, S.I.	11	3.0	20.0	3.87	7.13

Analysis

Mean top height and basal area growth models were developed using Permanent Sample Plot data. Five sigmoidal functions commonly used to model biological growth were tested: the Chapman-Richards (Richards 1959; Pienaar & Turnbull 1973), Gompertz (Winsor 1932), Hossfeld II (Hossfeld 1822 cited by Peschel 1938), and Schumacher (Schumacher 1939) functions, and the cumulative form of the 3-parameter Weibull probability density function (Weibull 1939; Yang *et al.* 1978). These equations are shown in yield form with yield (Y) as a function of age (T):

Chapman-Richards: $Y = a = (1 - e^{-bT})^c$

Hossfeld II: $Y = \dfrac{aT^c}{b+T^c}$

Gompertz: $Y = ae^{-(e^{(c-bT)})}$

Scumacher: $Y = ae^{(-bT^c)}$

Weibull: $Y = a(1-e^{-bT^c})$

Mean top height and basal area data from Permanent Sample Plots with two or more measurements were organised into pairs of consecutive measurements. Anamorphic and polymorphic difference forms of the five candidate sigmoid functions were fitted to pairs of consecutive mean top height and basal area measurements through Gauss-Newton non-linear least squares regression analysis executed by the SAS statistical analysis software PROC NLIN procedure (SAS Institute Inc. 1989). Functions were also fitted to the repeated sample plot measurements as mixed models using the SAS macro NLINMIX (Littell *et al.* 1996), where for each sample plot a random error term entered the asymptote (a) or slope (b) parameter of anamorphic and polymorphic forms respectively. Allowing the asymptote (a) or slope (b) parameter to change between sample plots or pairs of consecutive measurements produces families of curves that reflect growth differences between sites, ages, or management. This “local” parameter can be replaced by yield at age T_1 when the yield form is arranged in difference form to predict yield at age T_2. The anamorphic difference form with local asymptote produces curves with a common slope parameter and different asymptotes across the range of input values. The polymorphic form with local slope produces curves with different slopes that converge at one upper asymptote (Clutter *et al.* 1983). The y-intercept (H_0) of the mean top height model was set to 0.3 m to reflect average seedling height at planting. An x-intercept term (T_0) was incorporated into basal area equations. This accounted for the average time taken for stems to reach breast height 1.4 m, below which stand basal area is zero.

Starting values (yield at age T_1) are needed for difference equation projections. Site index, the height of dominants at a given base age, or height-age data reflect local site quality, and provide starting values for height growth projections. Site index, defined as mean top height at base age 30 years, was predicted for each measurement using the best mean top height growth model. Average site index for each plot was summarised by geographic region. Starting values for basal area models can be collected from local stands or predicted from regressions that consider stocking, age, and site quality. Initial basal area models that predict starting basal area values for an average site as a function of stocking and age, and as a function of stocking and mean top height, were fitted as multiple linear and non-linear regression models to data from young unthinned stands.

Average tree size and stocking data were plotted on logarithmic scales, and examined for density-dependent self-thinning patterns. Stocking and age data were organised into pairs of consecutive measurements. Linear and non-linear regression models that predict stocking reduction from natural mortality and non-catastrophic windthrow were fitted and tested. Linear and non-linear thinning models were fitted to the ratio of basal area after and before thinning as a function of the ratio of stocking after and before thinning, and rearranged to predict post-thinning basal area from stocking before and after thinning and pre-thinning basal area. Linear and non-linear models of total standing volume as a function of mean top height and stand basal area were fitted and tested.

Individual models were tested for goodness of fit across all pairs of measurements, across the range of predicted values, ages, and stockings, and by comparing predictions of the last measurement based on the first measurement for each sample plot. Prediction errors were calculated in real terms, as the difference between predicted and actual values, and in percentage terms, dividing the real error by the predicted value. Errors were summarised as the average and standard deviation of all prediction errors, and as the mean error sum of squares (*RMSE*) calculated as the square root of the sum of squared errors divided by the number of degrees of freedom. The coefficient of determination (R^2) was calculated for each model, adjusted for degrees of freedom, as

$$R^2_{adj.} = 1 - \frac{SSE \,/\, df_{Error}}{SST \,/\, df_{Total}}$$

where SSE = error sum of squares;

SST = total sum of squares;

df_{Total} = n–1 observations or consecutive pairs of time series data;

df_{Error} = n–k–1 where k = number of explanatory variables in multiple linear regression models, or the number of model parameters in non-linear regression models including the number of fixed effects in non-linear mixed models.

Overall model significance tests (F-tests) consistently returned probabilities of the model F-value failing to exceed the critical F-statistic (Pr.>F) of <0.0001, and were therefore not reported. Individual models were considered for further testing only when individual parameter estimates were statistically significant at the 95% level of confidence. This criterion was met when the t-value for linear model parameter estimates exceeded the critical t-statistic (Pr.>t) of 0.05, or when the approximate 95% confidence interval for non-linear model parameter estimates did not include zero.

The most suitable individual models were applied in combination as a system of equations to predict total standing volume per hectare and quadratic mean diameter at the last measurement for all sample plots with stocking, mean top height, and basal area data. Predicted quadratic mean diameter was calculated as the diameter of a stem with average basal area for any predicted stocking and basal area per hectare. Data from the first measurement in each sample plot were used as model starting values, permitting comparison of the latest measurement data with model predictions of mean top height, basal area, thinning, mortality, volume, and diameter for that age. Volume model predictions were based on predicted mean top height and basal area. Actual and predicted final measurements from a range of starting ages were compared to examine the influence of starting age and projection period length on model predictions. Minimum starting ages of 3 (all data), 5, 10, 15, 20, and 25 were tested; data from the first measurement above the minimum starting age were used as starting values for each sample plot. Actual and predicted volume growth was depicted graphically for eight sample plots with a long history of re-measurement.

The models were applied in combination to demonstrate total standing volume and quadratic mean diameter development in managed stands. Projections were based on age-5 starting values for 1100 stems/ha located on relatively good and average sites, defined as the ninetieth and fiftieth percentiles of available height and basal area data, respectively. Thinning to six final-crop stockings at age 10 was simulated.

RESULTS

Mean Top Height Model

The polymorphic Chapman-Richards mixed mean top height model with local slope (b) exhibited the least prediction error and greatest precision across all measurement pairs, stockings, and ages, and when predicting mean top height of the last measurement for each sample plot. The polymorphic difference form with local slope predicts mean top height H_2 at age T_2 dependent on starting values of mean top height H_1 and age T_1 (Equation 1).

$$H_2 = a\left[1 - \left[1 - \left[\frac{H_1 - 0.3}{a}\right]^{\frac{1}{c}}\right]^{\frac{T_2}{T_1}}\right]^{c} + 0.3 \qquad (1)$$

Parameter estimates (and their standard errors) for the polymorphic Chapman-Richards model (R^2=0.99) were

a = 68.3895 (s.e.(a) = 2.90)
b = 0.01145 (s.e.(b) = 0.0014)
c = 0.9064 (s.e.(c) = 0.011)

Mean top height at age 30 (site index) was predicted for all height-age data. Average site index estimates for each sample plot were summarised by geographic region (Table 3). Mean top height growth curves that approximately encompass the range of site index estimates for all forests, and prediction errors plotted against predicted mean top height values are shown in Fig. 1. Mean top height model fit statistics across all pairs of measurements and by plot, from the first to the last plot measurement, are listed in Table 4.

Basal Area Model

All difference and mixed models of *A. melanoxylon* basal area growth fitted the data poorly, especially over longer projection periods and when compared against

TABLE 3–Site index summary statistics by region for *A. melanoxylon* plots with mean top height and age data. Site index is defined as mean top height at base age 30 years.

Region	No. plots	Average site index (m)	Minimum site index (m)	Maximum site index (m)	Standard deviation of site index (m)
Northland	12	29.1	25.1	32.0	2.39
Auckland	51	21.3	11.2	31.4	5.71
Waikato	36	23.1	16.3	31.7	4.03
Bay of Plenty	101	24.8	15.8	35.5	3.30
Gisborne	12	29.4	24.7	33.3	2.80
Taranaki	2	28.3	28.0	28.7	0.49
Westland, S.I.	11	20.7	17.2	22.7	1.68
All data	225	24.0	11.2	35.5	4.60

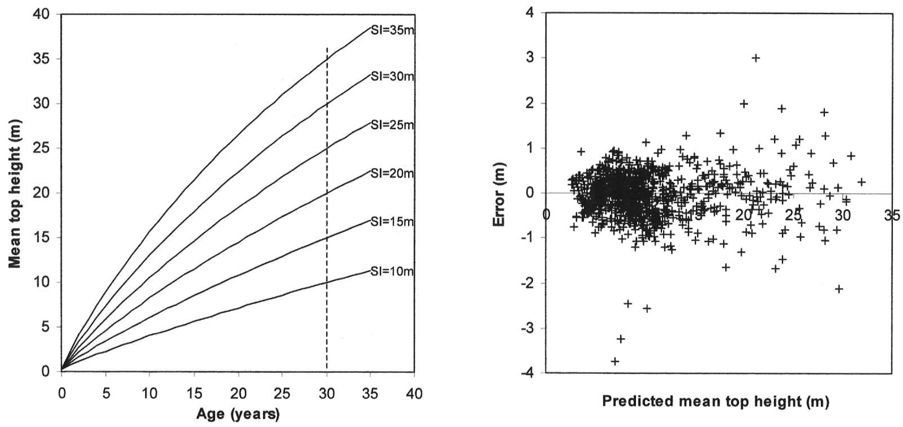

FIG. 1–Mean top height-age curves for *A. melanoxylon*. Site index (SI) defined as mean top height (m) at base age 30 years. Chapman-Richards polymorphic mean top height model error chart (n = 970).

TABLE 4–Mean top height model error statistics for *A. melanoxylon* measurement pairs and entire plot history. Errors expressed in real terms: predicted–actual (m).

	All pairs	By plot
n	970	208
Mean error (m)	–0.036	–0.087
s.d. of errors	0.502	1.486
RMSE	0.504	1.492

sample plots exhibiting relatively rapid basal area growth. Model fit was improved by discarding measurement data from plots with stockings above 4000 stems/ha and replicated field trial data from stands under age 7 years exhibiting very slow growth. The remaining data comprised 533 measurements organised into 409 pairs of consecutive measurements from 124 plots; these data had a greater average age and lower mean stocking than the entire dataset. Of all models tested, the polymorphic Schumacher basal area model, fitted in difference form with the slope parameter b isolated (local) in the algebraic difference formulation, exhibited the lowest mean prediction error and RMSE. The model also made the most accurate and precise predictions of basal area at the last measurement based on the first measurement in each sample plot. The model predicts basal area BA_2 at age T_2 dependent on starting values of basal area BA_1 and age T_1 (Equation 2).

$$BA_2 = a^{1-\left[\frac{T_2 - T_0}{T_1 - T_0}\right]^c} BA_1^{\left[\frac{T_2 - T_0}{T_1 - T_0}\right]^c} \quad (2)$$

The x-intercept (T_0) parameter represents the average age when stems reach breast height (1.4 m), below which basal area is zero. Parameter estimates (and their standard errors) for the polymorphic Schumacher basal area model ($R^2 = 0.99$) were

a = 145.20 (s.e.(a) = 24.6)

c = –0.5917 (s.e.(c) = 0.064)

T_0 = 1.0302 (s.e.(T_0) = 0.25)

Basal area model fit statistics across all pairs of measurements used to fit the model and by plot, from the first to last plot measurement, are listed in Table 5. Prediction errors were plotted against predicted basal area for all data pairs (Fig. 2).

TABLE 5–Basal area model error statistics for all *A. melanoxylon* measurement pairs and entire plot history. Errors expressed in real terms: predicted–actual (m^2/ha).

	All pairs	By plot
n	409	124
Mean error (m^2/ha)	–0.022	–0.145
s.d. of errors	1.349	3.765
RMSE	1.351	3.783

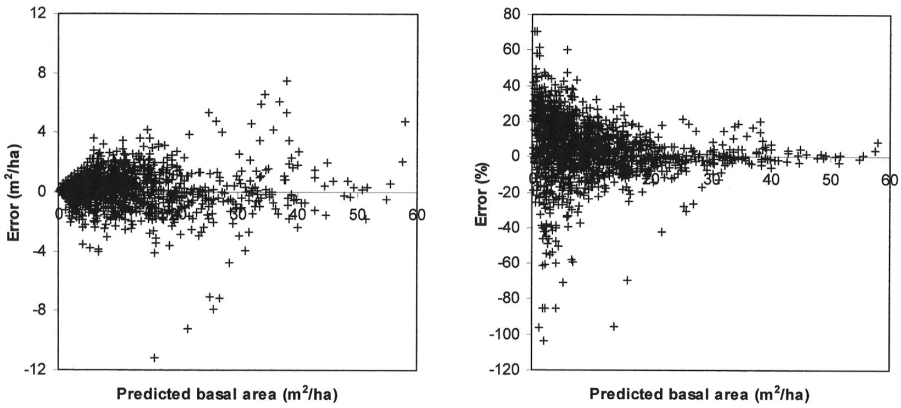

FIG. 2–Schumacher polymorphic *A. melanoxylon* basal area model error chart (n = 1481).

Initial Basal Area Model

Basal area and stocking data were square root transformed. A multiple linear regression of the square root of basal area as a function of age and the square root of stocking made imprecise predictions of basal area for young unthinned stands ($R^2 = 0.47$). Predictions of initial basal area improved when mean top height replaced age as an explanatory variable in the multiple linear regression ($R^2 = 0.88$). The most satisfactory model predicts basal area BA as a multiplicative function of stocking N and mean top height H for unthinned stands between 400 and 2000 stems/ha between ages 5 and 8 years (Equation 3).

$$BA = \left(a\sqrt{N^b}\,(H - H_0)^c \right)^2 \tag{3}$$

Parameter estimates (and their standard errors) for the regression of square root-transformed basal area as a function of mean top height and the square root of stocking ($R^2 = 0.88$) were

a = 0.12470 (s.e.(a) = 0.017)
b = 0.68834 (s.e.(b) = 0.029)
c = 0.47680 (s.e.(c) = 0.039)
H_0 = 2.6595 (s.e.(H_0) = 0.25)

Model fit statistics are listed in Table 6. Predictions were plotted for a range of stockings, and prediction errors were plotted against predicted initial basal area (Fig. 3).

TABLE 6–*Acacia melanoxylon* initial basal area model error statistics. Errors expressed in real terms: predicted–actual (m^2/ha).

n	179
Mean error (m^2/ha)	–0.055
s.d. of errors	1.278
RMSE	1.290

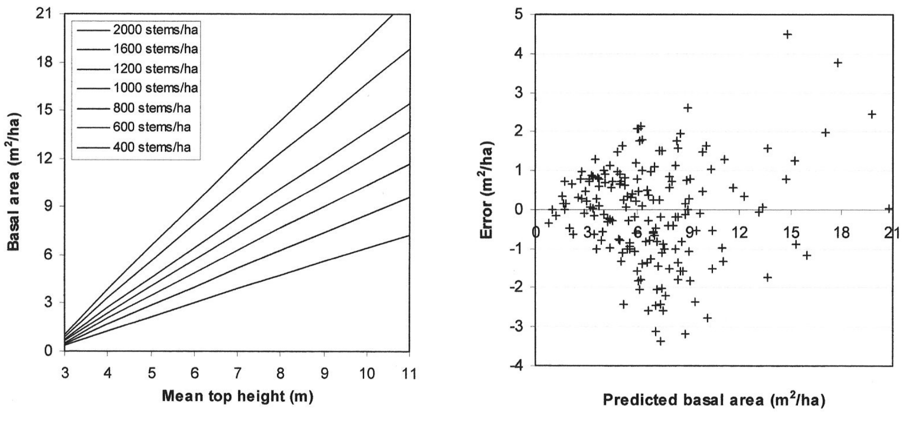

FIG. 3–*Acacia melanoxylon* initial basal area model predictions for a range of stockings, and prediction error charts (n = 179).

Thinning Model

The thinning model predicts post-thinning basal area (BA_2) as a function of pre-thinning basal area (BA_1) and stocking before (N_1) and after (N_2) thinning in stands 4–21 years old (Equation 4).

$$BA_2 = BA_1 \left[a\left[\frac{N_2}{N_1} \right] + b\left[\frac{N_2}{N_1} \right]^2 \right] \tag{4}$$

Parameter estimates (and their standard errors) for the quadratic thinning model ($R^2 = 0.97$) were

$a = 1.29309$ (s.e.(a) = 0.020)

$b = –0.28592$ (s.e.(b) = 0.026)

Model fit statistics were calculated for predictions across all thinned basal area data pairs (Table 7). Ratios of post- to pre-thinning stocking and basal area data and model predictions, and prediction errors plotted against predicted post-thinning basal area are shown in Fig. 4. One outstanding prediction error (–6.5 m^2/ha; 21% under-prediction) was made for a stand with 55 m^2/ha basal area, thinned from 1700 to 800 stems/ha at age 21 years (Fig. 4).

TABLE 7–*Acacia melanoxylon* thinning model error statistics. Errors expressed in real terms: predicted-actual (m^2/ha).

n	279
Mean error (m^2/ha)	–0.022
s.d. of errors	0.611
RMSE	0.613

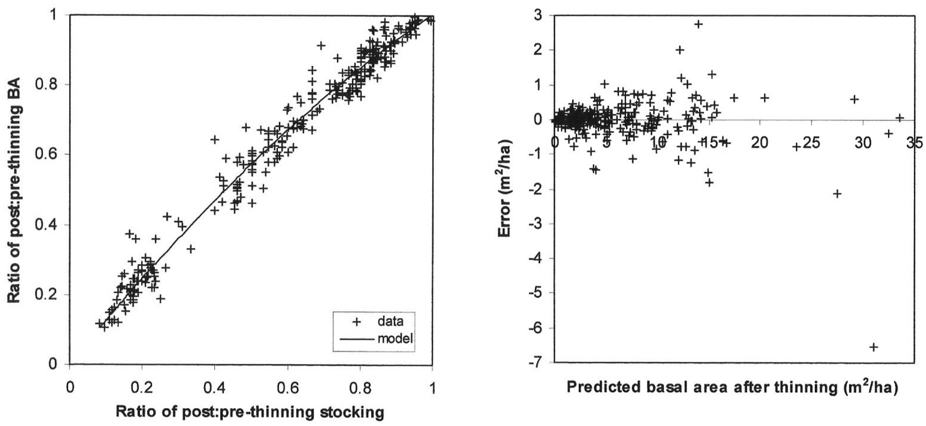

FIG. 4–*Acacia melanoxylon* thinning model predictions and data for ratio of post- to pre-thinning stocking and basal area, and thinning model error chart (n = 279).

Mortality Model

Stocking remained constant over 81% of all pairs of consecutive stocking measurements (n = 1477), in 68% of all periods before, between, or after thinning (n = 548), and in 35% of all plots (n = 214 plots). After data from plots with less than six consecutive measurements were discarded, models were fitted to 195 pairs of stocking data from 24 plots, including 15 plots where little or no mortality was observed. The most satisfactory model, proposed by Woollons (1998) for *P. radiata* D.Don, predicts stocking N_2 at age T_2 from stocking N_1 and age T_1 for stands with stockings below 2200 stems/ha (Equation 5).

$$\frac{1}{N_2} = \left[\frac{1}{\sqrt{N_1}} + a \left[\left[\frac{T_2}{100} \right]^2 - \left[\frac{T_1}{100} \right]^2 \right] \right]^2 \tag{5}$$

The parameter estimate (and its standard error) for the mortality model ($R^2 = 0.99$) were $a = 0.0846$ (s.e.(a) = 0.0061). Mortality model fit statistics were calculated in real and percentage terms across all pairs of stocking data used to fit the model, and for predictions of stocking at the last measurement based on the first measurement for each period before, between, or after thinning in all plots (Table 8). Predicted mortality over time for a range of stockings, and prediction errors, are shown in Fig. 5.

TABLE 8–Mortality model error statistics for consecutive pairs of *A. melanoxylon* stocking-age data from 24 plots with six or more measurements used for model fitting, and for predictions from the first to last stocking measurement in periods before, between, or after thinning. Errors expressed in real and percentage terms: real error = predicted–actual (stems/ha); percentage error = 100*(predicted–actual)/predicted.

	All pairs from plots with six or more measurements		All periods before, between, or after thinning	
	Real error	Percentage error	Real error	Percentage error
n	195	195	548	548
Mean error (stems/ha)	–1.09	–0.35	8.69	–4.01
s.d. of errors	30.32	3.22	84.60	6.94
RMSE	30.34	3.24	85.00	8.02

Volume Model

The most satisfactory volume model, a multiplicative model fitted to square root-transformed data by non-linear least squares, predicts total standing volume V as a function of mean top height H and basal area BA for stands older than age 10 years with stockings below 2300 stems/ha (Equation 6).

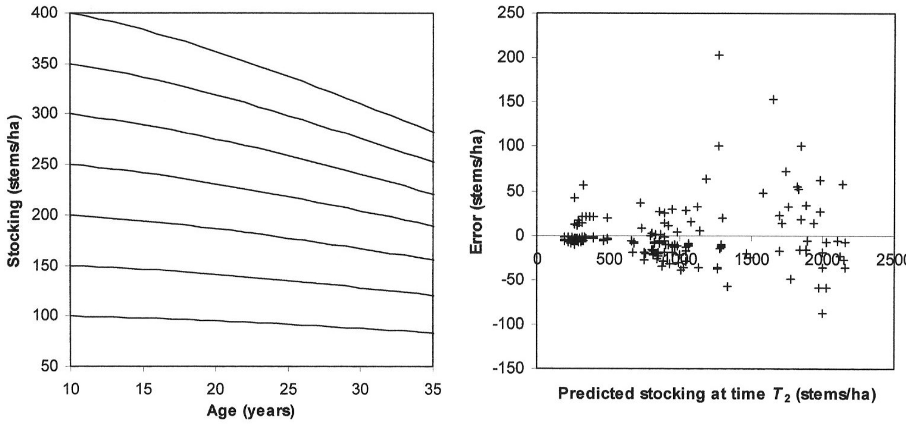

FIG. 5–Mortality model predictions for *A. melanoxylon* stands with stockings of 100–400 stems/ha at age 10, and mortality model prediction error chart (n = 195).

$$V = (a\sqrt{H}^{b}\sqrt{BA}^{c})^{2} \tag{6}$$

Parameter estimates (and their standard errors) for the volume model ($R^2 = 0.99$) were

$a = 0.62607$ (s.e.(a) = 0.0035)
$b = 1.03740$ (s.e.(b) = 0.0049)
$c = 0.94652$ (s.e.(c) = 0.0027)

Volume model fit statistics are listed in Table 9. Prediction errors were plotted against predicted standing volume (Fig. 6).

TABLE 9–Volume model error statistics for all *A. melanoxylon* data, and data from stands older than age 10 years. Errors expressed in real terms: predicted–actual (m^3/ha).

	All data	Data from stands over 10 years old
n	1549	527
Mean error (m^3/ha)	–0.723	–0.021
s.d. of errors	2.517	3.622
RMSE	2.620	3.629

Testing Model Predictions

Volume prediction error statistics were calculated for each minimum starting age in real and percentage terms, as the mean and standard deviation of all prediction errors, minimum and maximum error, and RMSE, for the last measurement from *n* sample plots (Table 10). Volume prediction errors were plotted against projection

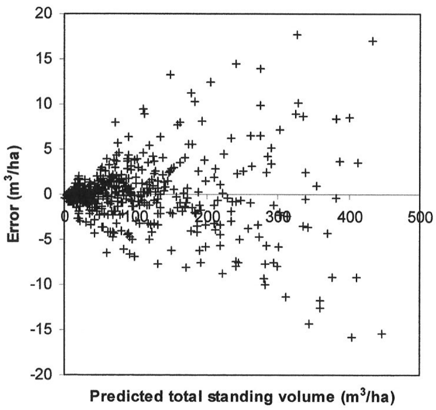

FIG. 6–*Acacia melanoxylon* total standing volume model error chart (n = 527).

period length for all plots in real and percentage terms (Fig. 7). Model predictions of total standing volume development were compared with data from eight sample plots with a long history of re-measurement (Fig. 8). The predicted influence of site quality and final-crop stocking on standing volume and average tree size development is demonstrated in Fig. 9. The ninetieth percentile (7.2 m; 7.5 m^2/ha) and average (5.6 m; 5.0 m^2/ha) of age-5 mean top height data and age-5 basal area for approximately 1100 stems/ha were used as starting values for high quality sites and sites of average quality, respectively (Fig. 9).

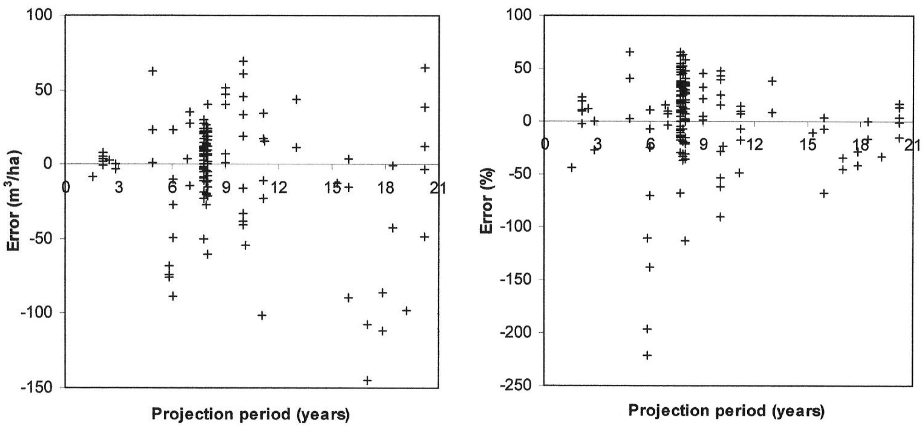

FIG. 7–Influence of projection period length on real and percentage errors for *A. melanoxylon* total standing volume predictions at the last measurement for all plots with pairs of volume-age data (n = 151).

DISCUSSION

The range of site index estimates for sample plots across New Zealand (approx. 11–35 m at age 30), obtained using the polymorphic Chapman-Richards mixed mean

TABLE 10–*Acacia melanoxylon* total standing volume (m^3/ha) and quadratic mean diameter (cm) prediction error statistics for the last measurement in each sample plot, predicted from minimum starting ages of 3, 5, 10, 15, 20, and 25 years. Errors expressed in real (m^3/ha or cm) and percentage terms.

Minimum start age	n	Mean error		Standard deviation of errors		Minimum error		Maximum error		RMSE	
		real	%	real	%	real	%	real	%	real	%
Volume (m^3/ha)											
3+	151	–0.8	2.2	35	43	–145	–222	70	66	36	43
5+	140	0.4	8.4	33	30	–145	–90	70	65	34	32
10+	126	–7.0	–7.4	27	16	–145	–50	44	34	29	18
15+	30	3.7	1.0	54	21	–118	–59	86	31	59	22
20+	20	6.9	2.8	43	13	–81	–21	71	23	49	15
25+	18	9.5	3.0	35	10	–83	–22	53	21	41	12
Diameter (cm)											
3+	157	1.2	4.7	3.4	16.0	–8.3	–55.7	12.9	42.5	3.6	16.8
5+	145	1.4	6.4	3.2	14.3	–8.3	–47.3	10.3	37.0	3.5	15.7
10+	135	–0.5	–2.3	1.5	6.1	–7.2	–22.8	2.8	13.5	1.6	6.6
15+	35	0.0	–0.3	2.2	7.1	–5.6	–16.8	7.1	18.4	2.3	7.2
20+	25	0.7	2.1	1.1	3.1	–1.3	–3.6	3.8	10.7	1.3	3.9
25+	18	0.6	2.0	0.4	1.5	–0.1	–0.3	1.5	5.4	0.8	2.6

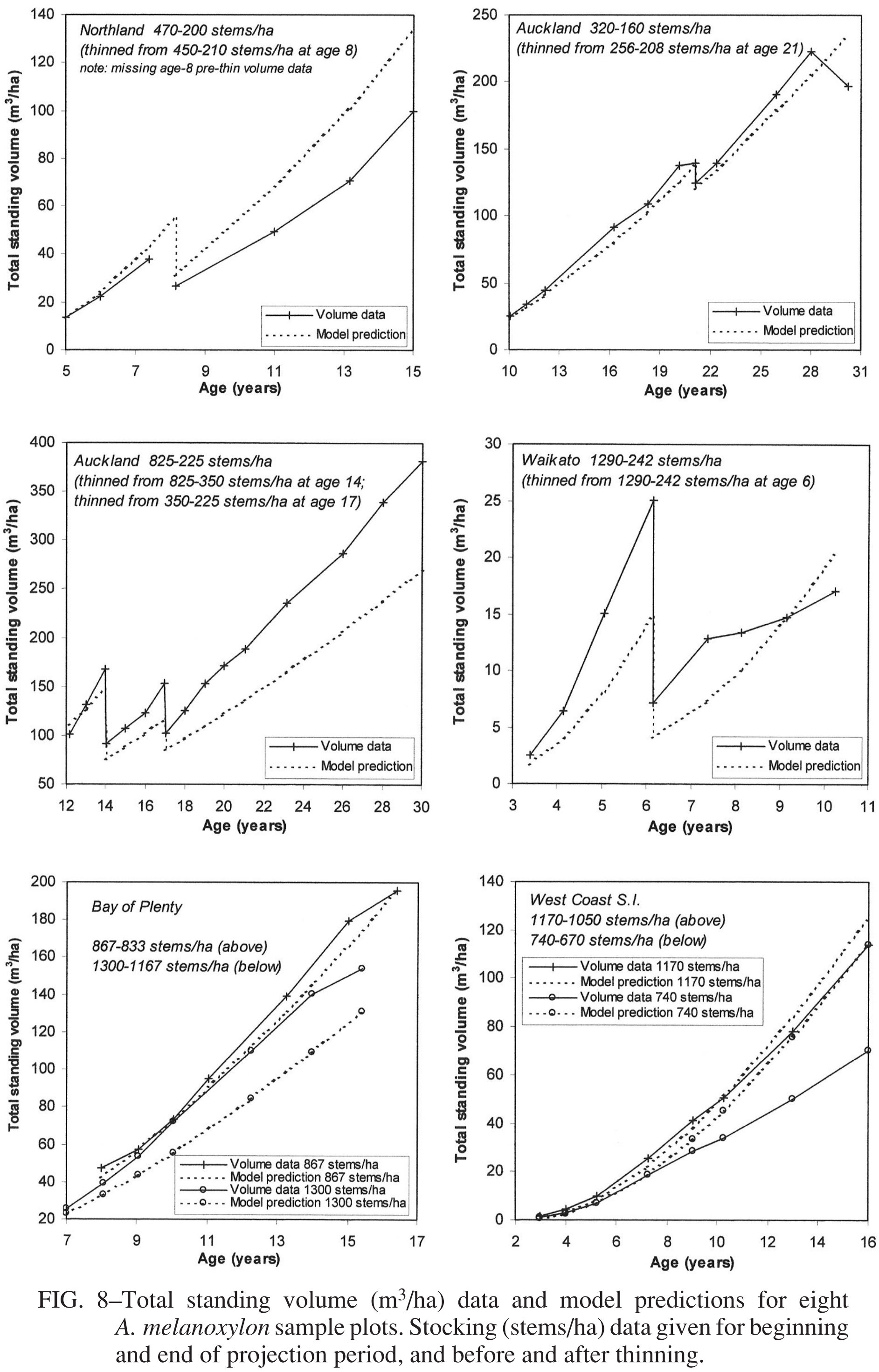

FIG. 8–Total standing volume (m³/ha) data and model predictions for eight *A. melanoxylon* sample plots. Stocking (stems/ha) data given for beginning and end of projection period, and before and after thinning.

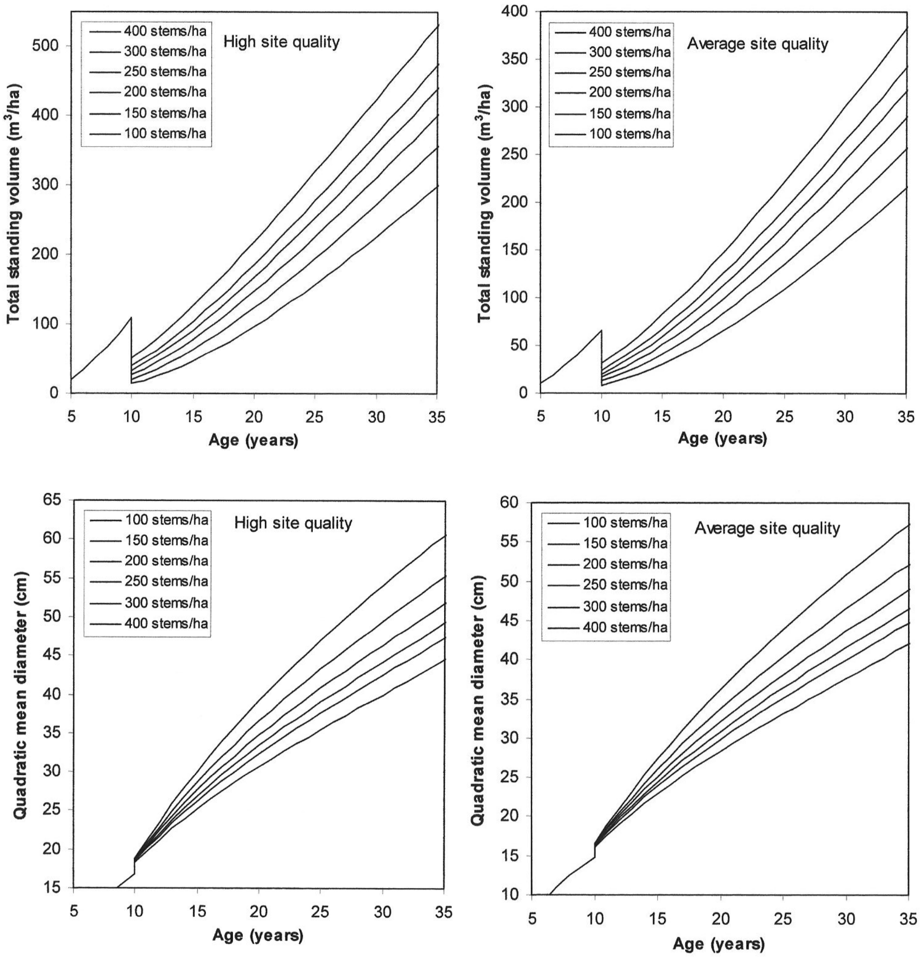

FIG. 9–Total standing volume (m^3/ha) and quadratic mean diameter (cm) predictions for *A. melanoxylon* stands starting at 1100 stems/ha at age 5, thinned to six final-crop stockings (100–400 stems/ha) at age 10. Site quality "High" and "Average" represent the ninetieth percentile and the mean, respectively, of age-5 mean top height and basal area data.

top height model, showed that *A. melanoxylon* height growth varied widely between stands (Fig. 1). Height-age data for *Cupressus lusitanica* Mill. and *C. macrocarpa* Hartw. plantations in New Zealand also varied widely between sample plots (15–35 m at age 30), and were best represented by a polymorphic Chapman-Richards equation fitted as a non-linear mixed model (Berrill 2004). All models fitted to the *A. melanoxylon* basal area data, including the best-fitting polymorphic Schumacher model, had large standard errors for fitted asymptote parameter estimates, indicating that more (older) data were needed to develop a robust basal area model. Too few post-thinning data were available to model thinning response. The basal area model assumes that post-thinning basal area growth is equivalent to the growth of stands with the same age and basal area as the residual thinned stand. The *A. melanoxylon* basal area growth model applies to pruned stands because the data originated predominantly from stands and silvicultural field trials that received some level of clear-bole pruning.

Large percentage errors for basal area growth model predictions in young stands (Fig. 2) and the high error variance and low R^2 (0.47) for initial basal area predicted from stocking and age indicated that basal area growth varied widely between young stands. Differences in site quality likely accounted for much of this variation. Predictions improved markedly (R^2=0.88) when mean top height and stocking replaced age and stocking as predictors of initial basal area. Local basal area and age data should provide less biased starting values than initial basal area model predictions. However, when local starting values are not available, the ninetieth percentile (7.2 m), average (5.6 m), and tenth percentile (3.9 m) of all age-5 mean top height data could be used as starting values for relatively good, average, and poor sites respectively. Mean top height estimates for young stands can also be obtained from Equation 1 or Fig. 1 when site index is known or estimated using site index summary data presented in Table 3.

Unlike data presented by Reineke (1933) for even-aged stands of native and exotic tree species growing in California, no clear upper limit of size-density relations was detected. Either too few data from fully stocked stands were available, or external factors such as wind were pre-empting mortality from intra-specific competition within *A. melanoxylon* plantations. Mortality model errors were not normally distributed because stocking remained unchanged between most measurements. Woollons (1998) overcame this problem by modelling the probability and level of *P. radiata* mortality separately in a two-step process. This approach was not applied because the level of mortality remained approximately constant over time in plots with six or more measurements, while the probability of mortality occurring between any two consecutive measurements was strongly related to the total number of measurements taken in plots with fewer than six measurements. Sample plots with fewer measurements had fewer mortality events. By fitting models to data from plots with six or more measurements, an appropriate functional form was discerned. Prediction errors for mortality in the 24 plots used to develop the model were approximately normally distributed (Fig. 5). The mortality model influences predictions of quadratic mean diameter calculated from stocking and stand basal area estimates.

Projections of standing volume and tree diameter development in *A. melanoxylon* stands indicated that site quality and final-crop stocking have an important influence on stand growth and yield (Fig. 9). Predictions indicated that an average tree diameter of 50 cm could be achieved in 29, 33, and 36 years in stands thinned to 100, 150, and 200 stems/ha respectively at age 10, on an average site. Total standing volume yields of 150, 230, and 300 m^3/ha were predicted for these three scenarios (Fig. 9). Predictions indicated that an average diameter of 60 cm could be achieved in 35 years only on high quality sites, defined as the top 10% of available data, by thinning to 100 stems/ha final-crop stocking, giving 300 m^3/ha total standing volume. The influence of silviculture and harvest age on recovery of merchantable volume was not addressed here, but should be considered when designing management regimes and performing economic analyses of forestry investments (Nicholas & Brown 2002). The sensitivity of projected rates of return or other objectives of management regime design to prediction errors should also be considered when using preliminary growth and yield models.

Comparing actual and predicted final measurements for each sample plot tested model predictions; the earliest measurement data above a range of minimum starting ages from each plot were used as starting values (Table 10). Overall, volume predictions were within 10% of actual values on average for each starting age tested, but prediction errors for individual plots were highly variable and extremely large in some plots (Fig. 7, 8). The variability in prediction errors decreased with increasing starting age. This result may be an artifact of the paucity of data from older stands, but implies that the oldest available data should be used for starting values. Large minimum and maximum errors across the range of starting ages were traced back to basal area model prediction errors. Even the most satisfactory basal area model could not completely account for the wide range of growth rates. The basal area model should be assigned highest priority for revision once more data are obtained.

Models were fitted to a dataset characterised by a low average measurement age and high stockings (Table 1). Some data were collected from older stands, but most data came from young silvicultural field trials and plantations with few plot measurements. While the model parameters were correctly estimated given the data at hand, they may not be efficient because of sampling bias towards younger ages. Inefficient parameter estimates could lead to serious prediction errors over long projection periods to later ages. Stand-level variables such as mean top height could not be calculated in some instances due to missing data. Thinning was recorded at only some measurements. Thus, individual models were fitted to different numbers of data, giving different degrees of freedom and operating ranges. Minimum and maximum stand-level data listed in Table 1 define model operating ranges unless otherwise stated — e.g., volume model predictions apply to stands older than 10 years with stockings less than 2300 stems/ha.

The preliminary models do not apply to all regions of New Zealand because data were not available for some regions (Table 2). All suitable data were required for model fitting, preventing separation of independent validation data. Comparing predictions with data used to fit the models tested but did not rigorously validate the preliminary models. Independent data should be collected from new sample plots within older *A. melanoxylon* stands, and by taking later measurements within existing permanent sample plots, for model validation and revision.

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