SITE INDEX EQUATIONS FOR RADIATA PINE IN NEW ZEALAND

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ABSTRACT

Site index (height of the dominant stand at some specified reference age) is a practical and commonly used method for quantifying site quality in pure even-aged stands. Permanent plot records from radiata pine (**Pinus radiata** D.Don) stands were used to compute site index equations. The data were divided into groups that might exhibit different height-growth patterns, and separate coefficient estimates were computed for each group. Coefficients for the eight groups finally selected are presented, and limitations in the application of the equations are discussed.

INTRODUCTION

A numerical measure of site quality is necessary for judicious forest management planning. Volume productivity is the variable of ultimate concern but height, a major determinant of volume, is generally more easily measured and the dominant height of stands is reasonably independent of variation in stand density and thinning treatments. Consequently, the height of the dominant stand at some specified reference age (site index) is a commonly used concept for quantifying site quality in pure even-aged stands.

Site index studies for several species have shown that height-curve shape varies with site quality (Beck, 1971; Bull, 1931; Carmean, 1972; Graney and Burkhart, 1973; King, 1966; Stage, 1963; and others). Families of site index curves which exhibit different shapes for varying site quality are commonly termed polymorphic. Although the variation in curve shape for different site qualities is widely recognised, it is important to recognise also that a variety of curve shapes may be possible for land of the same site index.

Radiata pine (*Pinus radiata* D.Don) is the most significant exotic timber species in New Zealand and it is extremely important that a reliable means of quantifying the productive potential of land for this species be available. One practical means of quantifying productive potential is the employment of the site index method. Because radiata pine is grown under a wide variety of conditions throughout New Zealand, it seems apparent that any useful site index curves must be polymorphic, reflecting differences in curve shapes for different site-quality classes and allowing for differences in shape within site index classes.

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Beekhuis (1966) recommended that local height-age curves be derived when applying his yield prediction method. Following this recommendation, many local sets of height-age curves were, and continue to be, compiled for radiata pine. The alignment chart published by Lewis (1954) for site index determination in radiata pine stands throughout New Zealand is also extensively applied. Another published site index investigation is that of Bailey and Clutter (1974) for radiata pine on pumice sites in the central North Island. The purpose of the present study was to derive a set of site index equations that could be applied to the important radiata-pine-growing areas in New Zealand.

The height measure that is now most commonly used to determine site index in New Zealand is mean top height, defined as the predicted mean height of the 100 largest-diameter stems per hectare*. To determine mean top height, a sample of heights of well-formed trees and their diameters is measured and a height-diameter relationship, such as the following, is fitted:

$$(1/(h - 1.4))^{0.4} = b_0 + b_1(1/D)$$

where h is total tree height and D is diameter at breast height (1.4 m). The quadratic mean diameter of the 100 largest-diameter trees per hectare is computed and the height-diameter function evaluated at that diameter to determine mean top height.

Base age 20 years, a generally accepted standard for radiata pine in New Zealand, was adopted for this study. Age was determined as years since planting or, in regenerated stands, as age from 1 year after clearfelling.

DATA

Data from plots in the Forest Service Permanent Sample Plot computerised system were used to derive site index equations. Suitable sample plots were available for all Forest Service Conservancies except Westland. In total, 2784 observations of mean top height and age from 523 plots were used for the analyses reported here. Table 1 contains additional information about the data used.

All sample plots were in pure even-aged stands of radiata pine. The majority of the plots had been pruned and thinned; however, there is a great deal of variation in the application of these treatments in the sample plots. Plot records from both regenerated and planted stands were used. No plot data from fertiliser trials were included.

ANALYSES AND RESULTS

Preliminary Computations and Data Division

The following function was fitted for each plot:

$$\ln(H) = b_0 + b_1(1/A)$$

where H is mean top height and A is stand age. The plot height-age function was evaluated at age 20 to determine site index. To avoid large errors in estimating site index, only plots with measurements near age 20 were used. If height measurements occurred at ages below and above 20, the plot was retained. For plots with all measurements below age 20, the last measurement was required to be at age 18 or older to

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* Although mean top height is defined on a per-hectare basis, in practice sample plots much smaller than 1 ha are generally used in its determination. However, to date, a standard plot size has not been specified.

TABLE 1—Summary of characteristics of sample data used in site index analyses, along with climatological and soils information for each forest

Conservancy/Forests	No. plots	No. obs.	Range in Age (years)	Range in Site index (m)	Mean annual prec. (mm)*	Mean annual temp. (°C)*	Soils
Group 1 — Auckland Conservancy			6-39	16.9-34.0			Yellow-brown sands
Woodhill	59	319			1313	14.3	
Waiuku	141	628			1332	14.3	
Group 2 — Auckland Conservancy			10-38	22.4-34.1			Clay soils
Tairua	9	83			1823	14.2	
Maramarua	4	30			1271	13.2	
Waipoua	14	70			1696	14.0	
Athenree	6	46			2163**	13.6**	
Kauaeranga	8	30			1278**	14.8**	
Group 3 — Rotorua Conservancy			6-45	18.2-37.2			Volcanic soils
Kaingaroa	93	571			1549	10.7	
Whirinaki	4	14			1542**	10.9**	
Group 4 — Rotorua Conservancy			6-37	28.2-40.2			Volcanic soils
Whakarewarewa	17	142			1511	12.1	
Rotoehu	11	77			1680	12.7	
Group 5 — Wellington Conservancy			7-27	24.3-36.9			Highly variable soils
Esk	15	46			1635	11.4	
Gwavas	29	113			1214	11.1	
Ngaumu	3	27			1188	11.0	
Group 6 — Nelson Conservancy			5-41	21.9-29.8			Gravels
Golden Downs	32	187			1343	10.4	
Rai Valley	4	31			1970	11.3	
Tawhai	3	9			1972	11.2	
Group 7 — Canterbury Conservancy			8-32	17.4-22.7			Fluvio-glacial gravels
Eyrewell	14	61			871	10.4	
Balmoral	13	56			657	10.4	
Ashley	1	10			820	11.2	
Burnham	7	45			689**	10.8**	
Group 8 — Southland Conservancy			10-44	20.3-26.8			Highly variable soils
Herbert	10	30			641	10.3	
Berwick	13	71			761	10.4	
Hokonui	3	32			995	10.0	
West Tapanui	8	30			903	9.8	
Rankleburn	2	26			918	9.5	

* Based on data from a New Zealand Meteorological Service station within the forest, except where otherwise noted.
** Based on data from the nearest Meteorological Service Station.

retain the plot; in plots with all measurements above age 20, the first measurement must have occurred at age 22 or younger to use the plot records for subsequent analyses.

Anticipating wide variation in height growth patterns in New Zealand, we divided the data according to Conservancy for preliminary analyses. Auckland Conservancy data were further divided into plots on sand dune sites and those on clay soils, because past experience has indicated that there are growth-pattern differences between these two site categories.

Height Function

The basic height-age function employed was Richards' (1959) extension of von Bertalanffy's (1938; 1957) growth function. The equation, which has been used successfully to describe height development for a variety of species (Beck, 1971; Carmean, 1972; Graney and Burkhart, 1973; and others), may be written:

$$H = b_{11}(1 - \exp(b_{12}A))^{b_{13}} \qquad (1)$$

where H is height at a given age A, and b_{11}, b_{12}, and b_{13} are constants to be estimated from sample data. The non-linear estimation procedures used in this study have been described by Marquardt (1963; 1966).

Equation (1) has many desirable characteristics as a height growth function. It is sigmoidal in shape, has an inflection point, passes through the origin, and reaches a finite upper asymptote.

Because the shape of the height-age curves was expected to vary by site index class, analyses were conducted to see if the constants (b_{11}, b_{12}, b_{13}) in Equation (1) could be expressed as functions of site index. A variety of linear and non-linear functions was tested by alternately expressing each coefficient as a function of site index. The functional form which gave the most consistent results, in terms of describing the height-age data for the initial data groups, was:

$$H = b_{21}H^{b_{22}}(1 - \exp(b_{23}\,S.A))^{b_{24}} \qquad (2)$$

where S is site index (mean top height at age 20), and other symbols remain as previously defined.

When evaluating Equation (2) at the base age, the predicted height will not necessarily equal site index. Equation (2) was conditioned to ensure that predicted height equals site index when age equals index age, by writing b_{21} in terms of the other coefficients. Conditioning leads to the following expression for height as a function of age and site index:

$$H = (S/1 - \exp(b_{31}\,S.20))^{b_{32}})\,(1 - \exp(b_{31}\,S.A))^{b_{32}} \qquad (3)$$

Equation (3) was fitted to each of the initial regions. Using the equation for each group, residuals were computed and plotted against age and site index for each forest separately. These residual analyses indicated that each of the initial groups was reasonably homogeneous and that a single equation for each group should be adequate, except for the Rotorua Conservancy group. Residual plottings for Rotorua Conservancy indicated that at least two groups should be recognised. The volcanic plateau — that is, Kaingaroa and Whirinaki Forests — was indicated as a group, and the other forests from the Conservancy appeared to comprise a second group. The data were accordingly divided and Equation (3) was fitted separately for the two groups. Subsequent residual plots showed that the forests within the two groupings were similar in height development. Therefore the final data groups for purposes of describing height development in radiata pine stands were:

Group 1 — Auckland Conservancy sand sites
Group 2 — Auckland Conservancy clay soils
Group 3 — Rotorua Conservancy volcanic plateau
Group 4 — Rotorua Conservancy non-plateau sites
Group 5 — Wellington Conservancy
Group 6 — Nelson Conservancy
Group 7 — Canterbury Conservancy
Group 8 — Southland Conservancy

Hypothesis Test

The total data set was used to estimate parameters in Equation (3) and the necessity to calculate 16 parameters (two for each of the eight groups) was tested against the null hypothesis that all the data could be combined, thus reducing to two the number of parameters required. Testing at the 1% significance level with the likelihood ratio procedure described by Gallant (1975a; b) resulted in rejection of the null hypothesis. Therefore we concluded that separate parameter estimates were needed for the defined groups and that a single, overall set of site index curves would not be adequate for describing the data.

Coefficients for Equation (3) for each of the eight groups, along with the associated standard errors of the coefficients and the standard error of estimate, are shown in Table 2.

Testing with Independent Data

It is, of course, always desirable to test prediction equations with an independent data set. Since it was not feasible to obtain additional independent plot records, we divided the available plot data into two groups: derivation plots and evaluation plots. The evaluation plots consisted of 25% of the plot records selected randomly from each group.

TABLE 2—Coefficients for Equation (3) for projecting height development in radiata pine stands. The standard error is shown in parentheses after each coefficient

Group	b_{31}		b_{32}		Standard error of estimate (m)
1	—0.0021759	(0.0000868)	1.3264	(0.03883)	1.3665
2	—0.0027145	(0.0001436)	1.6636	(0.10332)	1.1820
3	—0.0018257	(0.0000773)	1.7725	(0.05208)	1.3931
4	—0.0020932	(0.0000789)	1.5226	(0.04722)	0.9270
5	—0.0009185	(0.0003235)	1.1965	(0.10926)	1.3957
6	—0.0017129	(0.0001130)	1.4888	(0.05957)	1.0310
7	—0.0024376	(0.0002222)	1.4965	(0.08461)	0.7553
8	—0.0011294	(0.0001529)	1.2963	(0.06748)	1.1642

Equation (3) was fitted to the derivation plots (75% of the original data set) and evaluated by comparing observed and predicted mean top height values for the evaluation plots. Plotting predicted mean top-height values against observed values showed strong linear relationships that were close to a 45° line. The values of the simple linear correlation coefficient (r) for each group were as follows:

Group:	1	2	3	4	5	6	7	8
Value of r:	0.978	0.975	0.990	0.993	0.912	0.993	0.990	0.986

These show that predictive ability is adequate for all of the groups.

Converting Predominant Mean Height to Mean Top Height

Although mean top height is now commonly used for site index computations, predominant mean height is the measure available in some instances. Beekhuis (1966) observed that mean top height and predominant mean height did not differ markedly. As an aid to the use of the site index equations reported here, an equation to convert predominant mean height to mean top height was computed. Data were available for 1247 observations from 493 plots from Forest Service Conservancies throughout New Zealand. The resultant equation, with r^2 value of 0.992 and standard error of estimate of 0.799 m, was

$$MTH = -0.3533 + 1.0179PMH$$

where MTH is the predicted mean height (m) of the 100 largest-diameter stems per hectare and PMH is the arithmetic mean height (m) of the tallest 100 stems/ha, selected at 1 stem per 0.01 ha. This conversion equation shows a slightly different relationship from the equation of Beekhuis (1966) in which mean top height is based on the 247 (rather than 100) largest-diameter stems per hectare. Although the slope coefficient of the conversion equation is significantly different from 1 and the intercept is significantly different from 0 (testing at the 1% significance level), one can readily see that the two measures of stand height are generally very similar in magnitude.

DISCUSSION AND CONCLUSION

The site index curves presented here are polymorphic, having different shapes by site index classes and different shapes between the eight groups for land of the same site index. As an example, Fig. 1 illustrates the variation in curve shape between Group 3 (Rotorua Conservancy volcanic plateau) and Group 7 (Canterbury Conservancy) for site index 24 m.

It is interesting to note that only two parameters were estimated to obtain a height-growth curve for each group. When considering the total data set, 16 parameters (two for each of eight groups) were deemed necessary. Investigations showed that inclusion of more parameters could improve the fit (in terms of reducing the residual sum of squares) but, as the number of parameters increased, illogical results (such as an incorrect sign, from a biological standpoint, for coefficients) often were obtained for one or more of the groups. For the sake of consistency, sacrifices were naturally made in terms of fitting individual groups of sample data. However, the purpose of these prediction equations — as with all prediction equations — is to describe population trends not sample data; hopefully that objective has, to some extent, been achieved.

The conclusion that two parameters should be adequate to describe height growth in each data group is supported by research of Karish and Borden (1976). They

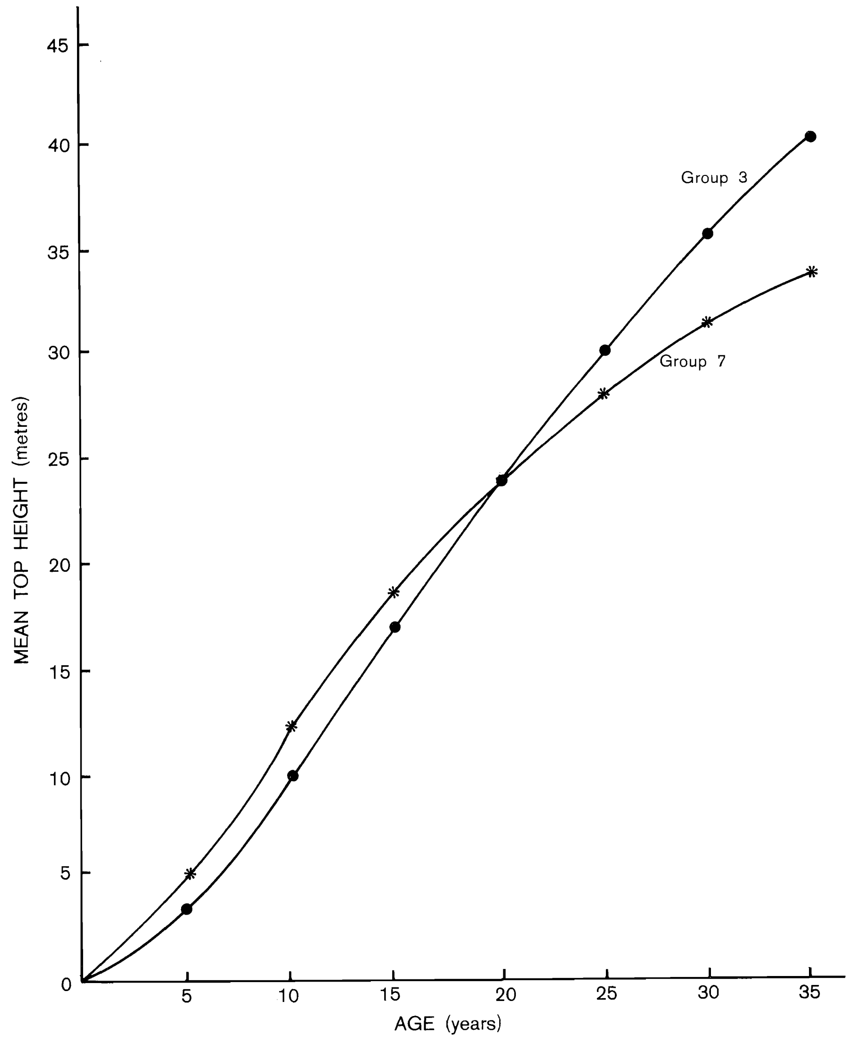

FIG. 1—Site index 24 m curve for Group 3 (Rotorua Conservancy volcanic plateau) and for Group 7 (Canterbury Conservancy).

examined parameter estimate correlations in several biological growth models, including the generalised Bertalanffy function, and concluded that it appears possible to reduce the dimensionality of the models to two.

Ideally, one would like to have measurements of continuous variables that could be entered into a height growth equation to describe growth pattern differences effectively for varying conditions. Unfortunately, such variables were not recorded for the sample plots. Consequently, it was necessary to divide the data somewhat arbitrarily into groups and to fit equations for each group. The group divisions were based on consultation with individuals familiar with radiata pine growth habits throughout New

Zealand and on analyses of residuals from the fitted equations. The differences between groups could be "true" differences in height growth patterns or they could, at least in part, be due to confounding with factors such as stand treatment differences between the groups.

It should be noted that the site index equations presented here do not purport to be valid for all growth conditions throughout the broad groupings used. However, the equations should give reasonably good predicted values for the range of conditions included in the sample data. Table 1 gives information regarding the number of plots from each forest within each group and a general indication of climatic and edaphic conditions for each group.

Climatological data in Table 1 were taken from New Zealand Meteorological Service (1973) where available. For forests without recordings in the Meteorological Service publication, the Conservancy offices kindly supplied data when possible. In four instances it was necessary to infer climatological values from the nearest Meteorological Service station.

To select an appropriate site index equation for a region from which data were not available for this analysis, one must rely on judgment and experience with growth characteristics for radiata pine in New Zealand. The data in Table 1 were compiled to aid in height-curve selection. However, for areas where plot records are not now available, height growth data should be collected as early as possible. These data can aid in selecting appropriate coefficients from Table 2 or, if necessary, they can be used to compute new coefficients.

In conclusion, the site index curves presented here should provide an adequate representation of height development for radiata pine over a substantial portion of the conditions under which it is grown in New Zealand. When possible, height growth data should be collected in local areas to verify the validity of these equations.

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