NOTE

AREA CONSERVATION MECHANISMS ASSOCIATED WITH FOREST MANAGEMENT

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ABSTRACT

Two area conservation mechanisms that are examined are commonly used within mathematical programming formulations of forest management problems. The mechanisms associated acyclic networks. That each mechanism allows for differing management alternatives is readily apparent from the network structures. Each mechanism is consistent with networks familiar to operations research, that is multi-item replacement networks. The network structures can be used with network programming and decomposition techniques to facilitate solutions to forest management problems.

Keywords: forest management; linear programming; network programming; decomposition.

INTRODUCTION

Ease of representation and tractability of solution techniques have made linear programming favoured amongst the class of mathematical programming techniques used to represent a problem, specific to forest management planning, known as a Forest Management Problem (FMP). This problem involves the scheduling of establishment, tending, and harvesting operations on a set of forest estates over some specified planning horizon. Many alternative representations using linear programming techniques have been developed, the central feature of each being the constraints that govern area conservation. Alternative representations for FMPs arise largely because of the variety of regulatory constraints that can be imposed (e.g., limits on yield at each period; forestwide yield requirements; limits on areas cut, standing, reforested), whereas there are surprisingly few sets of area conservation (structural) constraints that are used in practice. Three basic types are the Johnson & Scheurman (1977) Model I and II formulations and a formulation by Garcia (1984) which can be envisaged as an extension of a Model II.

This note is presented in two parts and examines the underlying structure of the Johnson & Scheurman (1977) Model II and the Garcia (1984) formulation. Initially,

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the area conservation network for each constraint set is presented. The replacement-problem nature of area regulation is identified. Subsequently, use of the area networks as subproblem within Dantzig-Wolfe decomposition and as an embedded network in a network problem with side constraints is discussed.

NETWORK AREA CONSERVATION MECHANISMS

Johnson & Scheurman (1977) provided the following formulations (1) and (2) of what are termed Model II structural constraints. This form of the constraints admits harvest possibilities before the minimum number of periods between regeneration harvests, z, has passed.

$$\sum_{j=1}^{T} x_{ij} \quad + w_{iT} \quad = A_i \tag{1}$$

$$\sum_{k=j+z}^{T} x_{jk} \quad + w_{jT} \quad = \sum_{i=-M}^{j-z} x_{ij} \tag{2}$$

$$i = -M, \ldots, 0; \; j = 1, \ldots, T$$

where

- x_{ij} are the units of area reforested in Period i followed by harvest reforestation in Period j, $i < j$.
- w_{iT} are the units of area reforested in Period i and left part of the ending inventory in Period T.
- A_i are the units of area present in Period 1 that were either afforested or reforested in Period i (prior to the commencement of the planning horizon).
- M is the number of periods before Period 0 in which the oldest age-class in Period 1 was regenerated or established.
- T is the number of periods in the planning horizon.
- z is the minimum periods between regeneration harvests.

Removal of the premature harvest possibilities in (1) gives (3); (2) and (3), then, are the revised set of Model II constraints.

$$\sum_{j=\max[1,i+z]}^{T} x_{ij} \quad + w_{iT} \quad = A_i \tag{3}$$

Gunn & Rai (1987) recognised the acyclic network structure associated with Model II constraints. An area regulation network corresponding to constraints (2) and (3) is illustrated in Fig. 1.

It is clear that, since arcs are directed in the sense of increasing time, it is not possible to create a cycle. This has ramifications when considering implementations of Dantzig-Wolfe decomposition procedures in that it allows a simple version of that algorithm to be used. It is possible to consider forestwide area transfers using a constraint set (2) and (3) for each crop type. In this situation the forestwide area conservation network is the same as (isomorphic to) a network of the regeneration form of a multi-item replacement problem (*see* Wagner 1969, Fig. 10.4) having the following attributes:

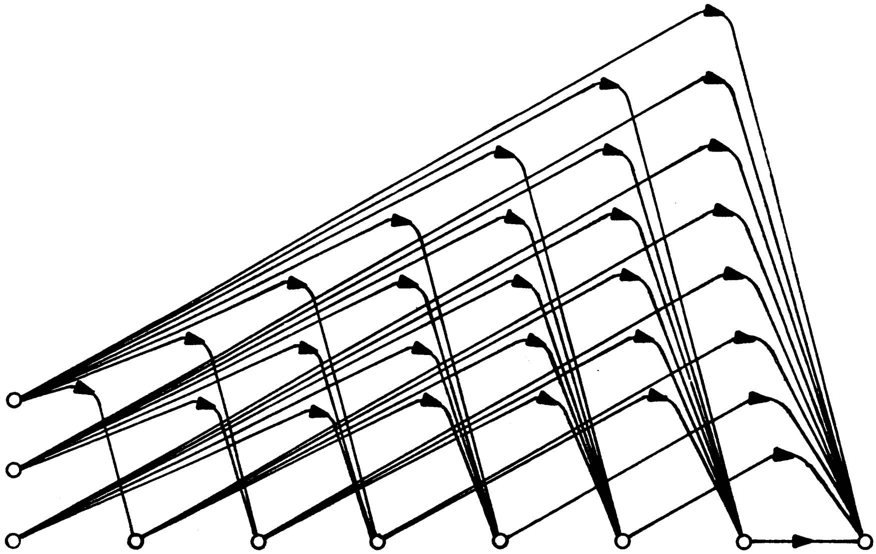

FIG. 1–An area network corresponding to constraints (2) and (3) with the following attributes. The minimum rotation length is three periods and the area was established three periods before the initial model period. The alternatives consider the harvest-reforestation of the same crop type. Arc flows at the leftmost nodes satisfy (3) while arc flows at the other nodes, excepting the sink, satisfy (2).

(a) Items may be initially aged (cf. crop types existing at the commencement of the planning horizon);

(b) Renewal of equipment items can be made only by replacement with a new item of the same type (cf. reforestation of the same crop type after harvest).

Representation of multiple crop types using a Model II formulation requires that a separate set of area constraints be used for each crop type. Using such an approach it is not possible to transfer area between crop types either prior to the planning horizon or at regeneration time. Thus, area initially available in some age-class of some specified crop type will be retained in that crop type at the end of the planning horizon although it may have cycled through a number of rotations. Note that this is the approach adopted by Berck & Bible (1984) to represent a multiple crop type (timber classes) situation where regulatory constraints that specify forestwide nondeclining yield must also be specified.

The formulation of Garcia (1984) differs from that of Johnson & Scheurman (1977) in that it permits the transfer of area between crop types at the commencement of the initial model period and after harvest/reforestation during model periods. Additionally, Garcia's formulation allows greater control over harvest possibilities in that it is possible to specify the maximum age of clearfelling for a crop type, thereby possibly restricting the number of arcs in the network. Garcia's set of structural constraints is given by (4) to (7) below. In this form they are cyclic since (6) permits the cyclic transfer of area between crop types via the z_{ijk}'s.

$$\sum_{k} r_{tik} \quad - \sum_{j} y_{tij} = 0 \qquad \forall_{i,t} \tag{4}$$

$$\sum_{s=t+1}^{T+1} y_{s,i,s-t} \quad - \sum_{k} r_{tki} = 0 \qquad \forall_{i,t} \tag{5}$$

$$\sum_{s=1}^{T+1} y_{s,i,j+s-1} + \sum_{k} z_{ijk} - \sum_{k} z_{kji} = a_{ij} \qquad \forall_{i,j} \tag{6}$$

$$y_{tij} \geq 0, \; r_{tij} \geq 0, \; z_{ijk} \geq 0 \tag{7}$$

where

y_{tij} is the area cut in Period t, crop type i, age class j.

r_{tik} is the area harvested in Period t from crop type i, and immediately replanted into crop type k.

z_{ijk} is the area transferred from crop type i, age class j, to crop type k (transfers are made at the commencement of the initial planning period).

a_{ij} is the initial area in crop type i, age class j.

An acyclic formulation requires that (6) be replaced with (8) and (9). Constraints (4), (5), (7), (8), and (9) constitute the modified Garcia area conservation constraint set.

$$\sum_{k} z_{ijk} = a_{ij} \qquad \forall_{i,j} \tag{8}$$

$$\sum_{s=1}^{T+1} y_{s,i,j+s-1} \quad - \sum_{k} z_{kji} = 0 \qquad \forall_{i,j} \tag{9}$$

The corresponding acyclic area conservation network is illustrated in Fig. 2.

In the general situation, when consideration is given to representing multiple crop-types using the modified Garcia formulation, then the area conservation network will be isomorphic to that of the regeneration form of a multi-item replacement problem having the following attributes:

(a) Items may be initially aged;

(b) Renewal of items may be made by replacement with a new equipment item of a possibly differing type (cf. crop type transfers after a harvest);

(c) Replacement may be initially made by exchanging items at the commencement of the initial model period (cf. initial crop type transfers).

Garcia initially develops area conservation constraints in terms of y_{tij} and x_{tij}, where x_{tij} is the residual area after cutting of crop type i, age class j, in period t (*see* Garcia 1984, Fig. 2). In this form, area conservation is analagous to the alternative representation of a replacement problem (*see* Wagner 1969, Fig. 10.5).

NETWORK STRUCTURES AND OPTIMISATION

The implementation of Dantzig-Wolfe decomposition techniques to FMPs possessing either Model II or modified Garcia constraint structures is simplified in that

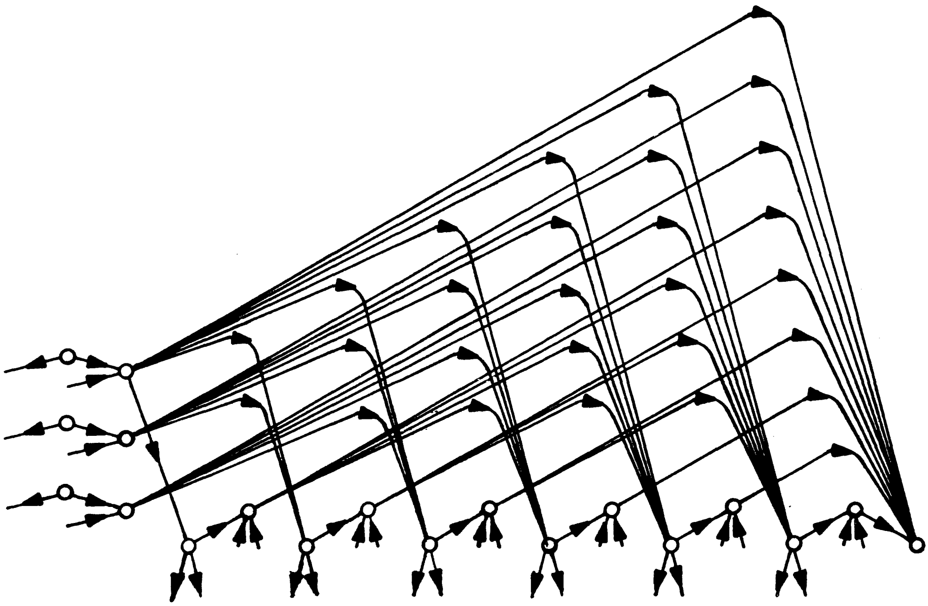

FIG. 2–An area conservation network corresponding to constraints (4), (5), (7), (8), and (9) with the following attributes. A crop type with a minimum rotation length of three periods is represented over a planning horizon of six periods; area transfers to and from other crop types are represented by dangling arcs. Arc flows at the leftmost nodes satisfy (8), and arc flows at nodes immediately to the right satisfy (9). Arc flows at nodes with dangling outward arcs satisfy (4), and arc flows at nodes with dangling inward arcs satisfy (5).
Source: García (1984)

these systems are acyclic. The implication of this is that the subproblem(s) always give rise to a bounded polyhedral set which facilitates the solution of the subproblem since extreme rays do not have to be dealt with in the decomposition (*see* Bazaraa & Jarvis 1977). It is clear that the Model II system gives rise to a bounded polyhedral set since an implied upper bound on the flow in each arc is area available in the crop type (timber class) under consideration; similarly, for the acyclic Garcia system, an implied upper bound on the flow in each arc is the total of the area available in all crop types.

Berck & Bible (1984) implemented a decomposition procedure based around the Model II structural constraints. The problem addressed was to schedule wood from a number, S, of distinct crop types (termed timber classes). The crop types are considered disjoint in that area initially held in a crop type will remain in that crop type for the duration of the planning horizon—that is, subsequent rotations will always involve reforestation of the same crop type. The forestwide area digraph for the S crop types consists of S disconnected digraphs each representing a single crop type and having a structure similar to that of Fig. 1. Forestwide, non-declining, harvest flow requirements must be satisfied, while the objective is to maximise the present value of wood from the S crop types. The decomposition results in a master with S convexity rows and a candidate column is constructed by solving each of the S subproblems having the form of problem (P1) below.

$$\max. \quad \sum_{j=1}^{T} \sum_{i=-M}^{j-z} D^s_{ij} x^s_{ij} \quad + \quad \sum_{i=-M}^{T} E^s_{iT} w^s_{iT} \qquad \text{(P1)}$$

s.t.

$$\sum_{j=\max.[1,i+z^s]}^{T} x^s_{ij} \quad + \quad w^s_{iN} \quad = A^s_i \qquad \forall_i$$

$$\sum_{k=j+z^s}^{T} x^s_{jk} \quad + \quad w^s_{jT} \quad = \sum_{i=-M}^{j-z^s} x_{ij} \qquad \forall_j$$

$$x^s_{ij} \geq 0, \quad w^s_{iT} \geq 0, \quad w^s_{jT} \geq 0$$

$$i = -M, \ldots, 0; \; j = 1, \ldots, T; \; s = 1, \ldots, S$$

where

- z^s is the minimum number of periods between regeneration harvest for crop type s.
- D_{ij} is discounted net revenue per hectare for hectares regenerated in Period i and regeneration harvested in Period j.
- E_{iT} is discounted net revenue during the planning horizon from hectares regenerated in Period i and left as ending inventory in Period N plus discounted net value per hectare of leaving these hectares as ending inventory.

Considering the structure of Fig. 1 along with (2) and (3), problem (P1) is recognisable as a network programming problem. Although P1 is not formulated as a shortest path problem, a shortest path algorithm may be used to solve it; such an implementation is given below.

Step 1 — Initialise all arc flows to zero.

Step 2 — Given a source (supply) vertex in the network use a shortest path algorithm to determine arcs on an optimal route from the source vertex to the unique sink vertex.

Step 3 — Increment all flows along the path determined in Step 2 by an amount equal to the supply at the source (i.e., the units of area for crop type s).

Garcia (1984) has pointed to the application of an algorithm given by Wagner (1969 Section 7.7) to conduct Step 2. Note that in an acyclic area conservation network the recursions that arise through application of Wagner's algorithm are identical to those that occur when the problem is regarded as a backwardly formulated dynamic program (*see* Wagner 1969, Section 10.6, Equations (2) and (3)). It is always possible to modify the flows in Step 3 without violating the constraints on primal feasibility since there are no upper bounds on arc flows. The Berck & Bible formulation involves a maximisation at each subproblem and hence the optimal route obtained in Step 2 is the longest route through the network. The utilisation of the network structure within the subproblem is arguably a more tractable approach to the development of the subproblem solution than that presented by Berck & Bible (1984) which involves Lagrange multipliers.

With the acyclic Garcia system it is permissible to transfer area between crop types either initially or after some harvest event. Thus, the (sub)networks representing different crop types may be connected and can no longer be treated as separate entities as in the Model II situation. Having regard to this, a general decomposition procedure would involve only one subproblem (i.e., one convexity row at the master level). The subproblem would once again be a network programming problem and, as suggested by Garcia (1984), a shortest path algorithm could be used to solve it. It is of interest to note that Garcia's approach allows a wide variety of regulatory constraints to be given consideration in FMPs.

An alternative to utilising column generation Dantzig-Wolfe decomposition procedures to solve FMPs that involve networks and side constraints is to use algorithms within which the network structure is used for maintaining the basis in the simplex procedure. Such methods that function by partitioning the primal basis are known as primal partitioning methods (Kennington & Helgason 1980). The SAS institute (SAS 1989) has re-released its primal partitioning algorithm that solves network problems with side constraints (NPSC), now called Proc NETFLOW, in SAS/OR version 6.0. Attempts by the author in 1987 to solve medium- to large-scale FMPs, generated under FOLPI (*see* Garcia 1984), using the NPSC algorithm in version 5.16 SAS/OR led to the identification of numeric problems associated with scaling of the side constraints. This problem has been remedied in version 6.0 so that Proc NETFLOW may offer computational advantage over currently used linear programming codes in the solution of FMPs that have network structural constraints.

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