MODELLING OF PINUS RADIATA WOOD PROPERTIES.
PART 2: BASIC DENSITY

XIN TIAN, D. J. COWN, and D. L. McCONCHIE
New Zealand Forest Research Institute,
Private Bag 3020, Rotorua, New Zealand

(Received for publication 6 April 1995; revision 20 February 1996)

ABSTRACT

Increment cores (5 mm diameter) and wood discs (50 mm thick) were used to study wood basic density patterns in *Pinus radiata* D. Don in relation to tree age, ring number from pith, position in the stem, and site in about 1000 stems from the North Island of New Zealand. Based on the relationships determined within trees, between trees, and between sites, a model was constructed which can satisfactorily predict wood basic density according to tree age, ring number from pith, and height in the stem at any location, and can explain the main contributions to variations in the wood density of individual trees. Furthermore, the model predicts the average density of given log height classes from data collected from breast height (1.3 m) outerwood increment cores.

The detailed variation of wood density within the tree revealed by the simulation software can be directly applied by industry, e.g., in “growing forward” density predictions for stands assessed prior to harvest, and allocation of stands and log types to alternative processes.

Keywords: wood properties; basic density; modelling; simulation software; *Pinus radiata*.

INTRODUCTION

Wood is characteristically a highly variable material, exhibiting differences in anatomy and physical properties between wood cells, tree rings, individual trees, sites, and geographic regions (Cown *et al.* 1991). Variation in basic wood density is of particular interest to the forest industry as this property is related to a number of solid wood utilisation characteristics (e.g., timber strength and stiffness, machinability, and drying) and manufacture and performance of reconstituted products (e.g., raw material consumption, uptake of chemicals, pulp yields, and paper properties). Wood density is therefore one of the most important qualities for wood and wood products. For *Pinus radiata*, wood density varies not only from tree to tree (between-trees), but also within trees (within-trees). The within-tree variation constitutes a major part of the overall variability that can be expected for one species (Kandeel & Bensend 1969). Other properties (e.g., spiral grain, tracheid length, compression wood, microfibril angle, presence of knots) can be significant for specific end-uses and are dealt with in other reports (Tian *et al.* 1995; Tian & Cown 1995).
Many studies have shown that there is a large within-tree variation in wood quality (Zobel & van Buijtenen 1989), and the variation of wood density in particular makes it difficult to predict its performance precisely (Zhang et al. 1993). Therefore, establishing a model to predict the inherent within-tree variation in wood density is desirable for both forest management and efficient wood utilisation. Development of a software package to simulate within-tree variation in wood density would enable a better understanding of the within-tree variation in wood density and provide a basis for studies on wood quality improvement. Although much research has been carried out in New Zealand and elsewhere on the within-tree variation in wood density (Cown 1992), modelling approaches, particularly those aimed at practical application, have rarely been explored. It is well documented that wood density is correlated with tree age, ring number from the pith, stem height, and site (Cown & Parker 1978; Cown 1980, 1992), and so it is logical to model those relationships.

MATERIALS AND METHODS

For the present study, data from breast height increment core samples from hundreds of sites throughout New Zealand (Cown et al. 1984, 1991) were re-analysed to generate mathematical functions. In addition, density values from about 1000 individual sample trees from previous studies, mainly from the North Island in New Zealand (e.g., Cown & McConchie 1982), were analysed for within-tree patterns. In particular, a large set of detailed data on 40 trees (23 and 43 years old) from Kaingaroa Forest was used to build the whole-tree model.

ANALYSIS OF VARIATION AND CONSTRUCTION OF WOOD DENSITY MODEL

Density Variation with Ring Number from the Pith

The typical pattern of radial wood density variation for *P. radiata* is a rapid increase with increasing ring number from the pith in the early years, followed by a levelling-off later in life (Fig. 1 and 2). Numerous studies have described this relationship between density and
FIG. 2—Radial wood density variation by ring number from the pith, 43-year-old trees.

Ring number for *P. radiata* and determined that a linear regression equation (Fig. 3a) is not appropriate. The developmental trend of wood density with ring number is a curve approximating the Logistic curve, and so a Logistic non-linear regression model was selected to simulate the law of density change with ring number from the pith.

Logistic curve model:

\[ y = \frac{k}{1 + ae^{-bx}} \]  

where \( y \) is wood density; \( k \) is the maximum value of wood density and it can be estimated by using the three-point form of equal difference:

\[ k = \frac{y_2^2(y_1 + y_3) - 2y_1y_2y_3}{y_2^2 - y_1y_3} \]  

\( a \) and \( b \) can be estimated by using the least square method; then the minimal value (initial value) of the wood density is equal to \( k/(1+a) \).

Finally, the fit of the regression equation can be tested using the coefficient of correlation \( (R) \) and mean square error \( (\sigma^2) \).

\[ R = \sqrt{1 - \frac{\sum (y_i - \hat{y_i})^2}{\sum (y_i - \bar{y})^2}} \]  

FIG. 3—Comparison of linear and non-linear models for the relationship between wood density and ring number from the pith.
Tian et al.—*Pinus radiata* wood properties. 2: Basic density

\[
\sigma^2 = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 / n} \quad [4]
\]

According to Equations 1–4, a model relating average wood density and ring number for 25-year-old *P. radiata* from the central North Island was created using mean values [Equation 5] and plotted in Fig. 4.

\[
WD = \frac{455.9883}{[1 + 0.4988e^{-0.1443\cdot RN}]} \quad [5]
\]

where WD symbolises wood basic density and RN represents ring number from the pith. The mean square error \( \sigma^2 = 5.4931 \) and the coefficient of correlation \( R = 0.9896 \).

Similarly, a model between average wood density and ring number is illustrated for 43-year-old *P. radiata* stems from another stand in Kaingaroa Forest [Equation 6] (Fig. 5):

\[
WD = \frac{469.5485}{[1 + 0.3508e^{-0.0489\cdot RN}]} \quad [6]
\]

![FIG. 4-Measured and predicted wood density based on the Logistic model [Equation 5] for 25-year-old trees.](image1)

![FIG. 5-Measured and predicted wood density based on the Logistic model [Equation 6] for 43-year-old trees.](image2)
The mean square error $\sigma^2 = 4.1488$ and the coefficient of correlation $R = 0.9903$.

These examples [Equations 5 and 6] indicate that the model of wood density by ring number from the pith conforms to the Logistic curve.

**Vertical Density Variation with Height in the Stem**

Average wood density appears to be high at the base of tree, increases slightly with increasing stem height, then decreases non-linearly to the top of the tree. Differences have been noticed between individual trees, but for most trees studied there is a non-linear (quadratic parabola) relationship (Fig. 6 and 7). A quadratic parabola model was selected to simulate the law of density change with stem height from the butt.

The quadratic parabola model:

$$Y = ax^2 + bx + c$$

[7]

where $a$, $b$, and $c$ can be estimated by using non-linear regression analysis.

---

FIG. 6–Vertical variation in average wood density with tree height, 25-year-old trees.

FIG. 7–Vertical variation in average wood density with tree height, 43-year-old trees.
A quadratic parabola model for average wood density and tree height for 25-year-old *P. radiata* from Kaingaroa Forest was created (Fig. 8) using tree mean values and with the form:

\[
WD = -0.0766H^2 + 0.2757H + 410.1516
\]  \[8\]

where \(WD\) symbolises wood basic density and \(H\) represents tree height (m) from the butt. The mean square error \(\sigma^2\) = 4.69 and the coefficient of variation \(R = 0.92\).

Similarly, a model of average wood density and tree height for 43-year-old *P. radiata* from Kaingaroa Forest was created (Fig. 9):

\[
WD = -0.07497H^2 + 2.2244H + 393.5021
\]  \[9\]

The mean square error \(\sigma^2\) = 9.43 and the correlation coefficient \(R = 0.89\).

**Geographic Density Variation**

The environment has a strong effect on wood density values in New Zealand. Extensive research has demonstrated that average mean annual temperature is the main influencing
factor, with contributions from other site factors such as latitude, rainfall, and soil nutrient status (Cown et al. 1991).

Regional average breast height wood density patterns of sample trees from several hundred sites are plotted in Fig. 10. Regional models were developed to fit the data: \( WD = \frac{G1}{1 + G2 \cdot e^{G3 \cdot RN}} \) where \( WD \) = wood density, \( RN \) = ring number from the pith, \( G1 \)–\( G3 \) = model parameters (Table 1, Fig. 11). These regional models clearly confirmed the following:

- Regional averages appeared to fall into three wood density groups—high (Auckland), medium (Rotorua, Nelson), and low (Wellington, Westland, Canterbury, and Southland).
- Average regional differences were more pronounced in the outer rings than the inner rings (juvenile wood).

![FIG. 10–Regional breast height average wood density models.](image)

![FIG. 11–Regional wood density breast height models.](image)
TABLE 1—Regional wood density models.

<table>
<thead>
<tr>
<th>Region</th>
<th>Model</th>
<th>(\sigma^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Auckland</td>
<td>(\text{WD} = 515.6212 / [1 + 0.8015 \cdot e^{-0.1539 \cdot \text{RN}}])</td>
<td>4.1488</td>
</tr>
<tr>
<td>Nelson</td>
<td>(\text{WD} = 487.4623 / [1 + 0.4979 \cdot e^{-0.1132 \cdot \text{RN}}])</td>
<td>3.1325</td>
</tr>
<tr>
<td>Rotorua</td>
<td>(\text{WD} = 484.7812 / [1 + 0.5723 \cdot e^{-0.1194 \cdot \text{RN}}])</td>
<td>5.3262</td>
</tr>
<tr>
<td>Wellington</td>
<td>(\text{WD} = 457.3690 / [1 + 0.4414 \cdot e^{-0.1110 \cdot \text{RN}}])</td>
<td>4.2556</td>
</tr>
<tr>
<td>Westland</td>
<td>(\text{WD} = 465.4959 / [1 + 0.4488 \cdot e^{-0.1050 \cdot \text{RN}}])</td>
<td>2.9275</td>
</tr>
<tr>
<td>Canterbury</td>
<td>(\text{WD} = 452.1364 / [1 + 0.4299 \cdot e^{-0.1313 \cdot \text{RN}}])</td>
<td>10.5088</td>
</tr>
<tr>
<td>Southland</td>
<td>(\text{WD} = 459.5329 / [1 + 0.4449 \cdot e^{-0.0817 \cdot \text{RN}}])</td>
<td>3.6003</td>
</tr>
</tbody>
</table>

\(\sigma^2 = \text{mean square error}\)

- If juvenile wood were to be defined as a density level (e.g., <400 kg/m\(^3\)) rather than a number of rings (e.g., 10 rings from the pith), there would be very significant regional differences.

**Variation of Wood Density with Tree Age**

Many *Pinus radiata* wood quality studies have yielded information on wood density at various positions up the stem and attempted to relate the data to log position (based on nominal 5.5-m log lengths) from the butt log to the top of the tree (pulpwood).

Using a modified exponent curve model \((y = ax^b)\) and a logarithmic curve model \((a + b \cdot \log(x))\), a group of models for stand average tree and log densities were created from 1674 records describing individual tree densities from 241 sites throughout New Zealand (Cown et al. 1991) (Table 2, Fig. 12 and 13).

**WITHIN-TREE DENSITY MODEL**

To construct the overall pattern of variation in wood density within trees, existing wood sample data from trees in the age range 23–46 years were examined according to height in the stem and radial position. Data from a typical sample tree are given in Table 3.

The two data sets (25- and 43-year-old stands) were used to generate the overall pattern of variation in wood density within stems (Fig. 14a, b). The values in each cell represent the
average wood density (adjusted for section width) for each five-ring section at a particular sample height and growth period.

Large differences in wood density were observed between the sampling positions (Fig. 14a), with a minimum average value of 329 kg/m³ at 32.7 m (rings 0–5) and a maximum average wood density of 459 kg/m³ at 1.2 m (rings 40–45), a difference of 130 kg/m³ at age 43 years. Similarly, for the 25-year-old trees (Fig. 14b) the minimum value was 332 kg/m³ and the maximum value 435 kg/m³, a difference of just over 100 kg/m³. The data indicated similar overall vertical trends in the two stands (Table 4).

This again confirms the importance of both tree age and position in the stem.

Theoretically, a conical tree exhibiting cylindrical symmetry will have the same average trends in wood properties as those along a radius at the base of the tree. Therefore, for any
TABLE 3—Wood density data for Tree No. 15-01 (42 years old, Wairoa Forest)

<table>
<thead>
<tr>
<th>Height (m)</th>
<th>Diam. (mm)</th>
<th>No. of rings</th>
<th>Wood density (kg/m²) at rings from pith</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.15</td>
<td>94</td>
<td>40</td>
<td>319 353 375 394 397 419 430 445</td>
</tr>
<tr>
<td>1.22</td>
<td>78</td>
<td>37</td>
<td>329 357 366 392 401 402 415 410</td>
</tr>
<tr>
<td>3.04</td>
<td>75</td>
<td>35</td>
<td>311 361 388 380 393 405 406</td>
</tr>
<tr>
<td>8.69</td>
<td>64</td>
<td>32</td>
<td>344 379 400 403 414 409 392</td>
</tr>
<tr>
<td>12.3</td>
<td>61</td>
<td>30</td>
<td>360 380 400 409 418 407</td>
</tr>
<tr>
<td>16.9</td>
<td>54</td>
<td>27</td>
<td>370 379 403 416 425 404</td>
</tr>
<tr>
<td>20.4</td>
<td>48</td>
<td>25</td>
<td>359 385 413 430 423</td>
</tr>
<tr>
<td>26.8</td>
<td>36</td>
<td>20</td>
<td>355 391 427 436</td>
</tr>
<tr>
<td>33.2</td>
<td>24</td>
<td>15</td>
<td>367 412 437</td>
</tr>
<tr>
<td>38.4</td>
<td>16</td>
<td>10</td>
<td>351 380</td>
</tr>
<tr>
<td>41.5</td>
<td>5</td>
<td>5</td>
<td>317</td>
</tr>
</tbody>
</table>

Mean 344 378 402 408 410 408 417 445

Note: Wood density = resin-extracted

FIG. 14—Wood density variation within stems of *Pinus radiata* (a) 43 years old, (b) 25 years old. Least-squares means were derived from 13 and 15 trees respectively, adjusted for average widths of 5-year (five-ring) sections.

piece of wood within the tree, its volume-weighted average property can be expressed in terms of:

ring number (RN), diameter (D), stem height (H), and tree age (AGE) (Fig. 15).
TABLE 4—Comparison of wood density in two stands

<table>
<thead>
<tr>
<th>Tree age</th>
<th>Weighted density (kg/m^3) by disc height</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.2 m</td>
</tr>
<tr>
<td>25</td>
<td>393</td>
</tr>
<tr>
<td>43</td>
<td>415</td>
</tr>
</tbody>
</table>

FIG. 15—Relationship between wood density and position within the tree.

According to Equation [1], the overall model for average wood basic density within the tree should be given by the multivariate non-linear formula:

\[ WD = F(RN, H) + \varepsilon \]

\[ = \frac{\alpha}{[\beta + e^{\phi}]} + \varepsilon \]  

[10]

There \(\alpha, \beta, \phi\) are undetermined model parameters,

and \(\alpha = f_1(H)\)

\(\beta = f_2(H)\)

\(f = f_3(RN, H), f_1 \sim f_3\) are undetermined functions,

\(F\) is the multi variate non-linear function,

\(e\) is random error and \(e \sim N(0, \sigma^2)\) normal distribution.

In the model [Equation 10], parameters can be calculated with the improved Gauss-Newton iterative method (Tian et al. 1995).

Generally, construction of the multivariate non-linear model includes three steps: (1) analysis of sample data (Fig. 16a), (2) local two-element model building (Fig. 16b), (3) multivariate model building (Fig. 16c).

Final solution:

\[ WD = \frac{a_0 - a_1 \cdot H}{1 + (a_2 - a_3 \cdot H) \cdot e^{-RN \cdot (a_4 + a_5 \cdot H)}} \]  

[11]
where $a_0$, $a_1$, ..., $a_5$ are parameters. The parameters of the model [Equation 11] were estimated from 25- and 43-year-old stands in Wairoa Forest using non-linear regression. They were $a_0 = 482.89$, $a_1 = 2.97$, $a_2 = 0.42$, $a_3 = 0.0046$, $a_4 = 0.040$, $a_5 = 0.0024$.

This analysis was performed using the proc NLIN from SYSTAT. The main results of the statistical analysis are presented in Table 5.
Residuals were examined for the effects of ring number from the pith and height in the tree on wood density (WD) for this model.

\[
\text{Residuals} = 100 \times \frac{\text{measured WD} - \text{predicted WD}}{\text{measured WD}}
\]

The residuals, when represented v. ring number and height in the stem, were randomly distributed and unbiased (Fig. 17 and 18) indicating that the regression coefficients are highly significant.

The overall pattern of variation in wood density in a 45-year-old stand can be calculated by this model (Table 6).

![Graph](image1.png)

**FIG. 17**—Residuals between the measured and predicted wood density (%) v. ring number from the pith.

![Graph](image2.png)

**FIG. 18**—Residuals between the measured and predicted wood density (%) v. height in the tree.

Generally, the model [Equation 11] predicts density within ±8 kg/m\(^3\). The results (Table 6) show that the model fits the trees from the 43-year-old stand quite well, and the
model is able to account for a significant part of the variation in wood density of different stands.

**Application of the Model**

Having determined that average tree wood density patterns are largely predictable, based on tree age and location, the model can be used to:

1. Predict log and tree densities from small non-destructive breast height samples
2. Predict future log wood density levels from field samples.

**Modelling Within-tree Density Patterns**

The average distribution of wood density within the whole tree at different tree ages was calculated by using model [11] (Fig. 19) since it has been established that there is a strong pattern of density development, characteristic of the species and dependent on tree age and height.

**Prediction of Tree Density and Tree Component Densities**

The model [Equation 11] is capable of accurately calculating the average wood density values of any section of wood within the tree. Given that accurate estimates of the outerwood density can be obtained efficiently from breast height increment cores (5-mm diameter), the
model can be used to predict average whole-tree densities and tree component densities from age and outerwood basic density (Fig. 20).

To validate the model, the average breast-height outerwood data (wood density of the outer five growth rings at 1.4 m height) from the 25 year-old stand (410 kg/m³) was used to predict whole-tree densities and tree component densities (Fig. 20). The predicted values were compared with measured data and the results showed that the predicted values were very close to the actual measured data (Table 7).

Of more general interest is the ability to predict whole-tree and tree component densities from increment core outerwood basic density and to predict density into the future, e.g., to the anticipated rotation age.

**TABLE 7**—Comparison of predicted and measured wood density values in a 25-year-old stand.

<table>
<thead>
<tr>
<th>Tree component</th>
<th>Predicted values (kg/m³)</th>
<th>Measured values (kg/m³)</th>
<th>Difference (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Butt log</td>
<td>392</td>
<td>393</td>
<td>1</td>
</tr>
<tr>
<td>Second log</td>
<td>375</td>
<td>379</td>
<td>4</td>
</tr>
<tr>
<td>Third log</td>
<td>365</td>
<td>368</td>
<td>3</td>
</tr>
<tr>
<td>Fourth log</td>
<td>357</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>All sawlogs</td>
<td>370</td>
<td>373</td>
<td>3</td>
</tr>
<tr>
<td>Pulplogs</td>
<td>350</td>
<td>356</td>
<td>6</td>
</tr>
<tr>
<td>Whole tree</td>
<td>369</td>
<td>371</td>
<td>2</td>
</tr>
</tbody>
</table>

Breast height outerwood density = 410 kg/m³ at 25 years.
The model was used with the above sample data (outerwood density 410 kg/m$^3$ at 25 years) to predict tree and log densities 18 years into the future (Fig. 21). The predicted values were compared with those measured in the 43-year-old sample trees. The results (Table 8) show that the predicted and measured wood density values are extremely close.

The model performs very well to predict both current and future wood density (Tables 7 and 8).

![Prediction of tree density and tree component densities using the model.](image)

**TABLE 8—Comparison between predicted values and measured values**

<table>
<thead>
<tr>
<th>Whole-tree and tree component densities at 43 years</th>
<th>Predicted values (kg/m$^3$)</th>
<th>Measured values (kg/m$^3$)</th>
<th>Difference (kg/m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Butt log</td>
<td>422</td>
<td>413</td>
<td>9</td>
</tr>
<tr>
<td>Second log</td>
<td>409</td>
<td>400</td>
<td>9</td>
</tr>
<tr>
<td>Third log</td>
<td>403</td>
<td>391</td>
<td>12</td>
</tr>
<tr>
<td>Fourth log</td>
<td>392</td>
<td>389</td>
<td>3</td>
</tr>
<tr>
<td>All sawlogs</td>
<td>403</td>
<td>398</td>
<td>5</td>
</tr>
<tr>
<td>Pulplogs</td>
<td>378</td>
<td>372</td>
<td>4</td>
</tr>
<tr>
<td>Whole tree</td>
<td>397</td>
<td>389</td>
<td>8</td>
</tr>
</tbody>
</table>

Breast height outerwood density = 410 kg/m$^3$ at 25 years

**DISCUSSION AND CONCLUSION**

Sawing studies with *P. radiata* logs have consistently shown that wood density is a critical property for production of structural lumber (Cown *et al.* 1986). It is therefore
important to understand the sources of variation in this wood property and to anticipate any changes.

Mathematical techniques have been applied to describe the patterns of variation in wood density of *P. radiata* in New Zealand, and a general model for average within-tree wood basic density was constructed. The model reveals the variation of wood density within the tree in detail, based on a very large dataset collected over several decades. The key variables have been confirmed as stand location (region) and tree age.

Prediction of tree component density from samples conveniently collected from the breast height position has been used before (Cown *et al.* 1984; Cown 1992) but the use of a model creates the possibility of incorporating wood property data into routine field assessment procedures. The wood density of the whole tree or parts of the stem (sawlogs, pulplogs) at a given age can then be projected forward to the anticipated rotation length, allowing foresters to influence felling and/or allocation decisions using wood property information.

Further refinement of the model could include linking the data to individual tree or stand models so that the influence of silvicultural decisions can be more accurately predicted for different growing conditions.

ACKNOWLEDGMENTS

The authors wish to thank Mr Graeme Young for his assistance with sample data and Mr Owen Cox for his valued opinions and help during the project.

REFERENCES


