SECTIONAL MEASUREMENT OF TREES: A RATIONALISED METHOD

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ABSTRACT

Hypothetical diameters of geometric solids are used to compare volume estimates when frusta are considered in turn to be paraboloidal, conoidal, or neiloidal. Comparison shows that only very small differences exist between estimates, that the traditional concept of a generalised tree shape has no bearing in this context, and that it is better to choose representative and repeatable measuring points along a stem at regular drops in diameter, if a reproducible and realistic index of volume is to be obtained. A procedure for taking sectional measurements is suggested.

INTRODUCTION

Much has been written about the approximate shape of trees or logs, and the merits of various formulae for calculating volume. Curiously, no recent text has emphasised the height accumulation approach which Grosenbaugh has expounded from time to time (e.g., in 1954 and 1966). The following comments in his 1966 paper (p. 449) certainly deserve mention, if not discussion, in text books published since 1966 (e.g., Avery, 1967 and Carron, 1968).

"Thus the traditional one-parameter conoid, paraboloid, and neiloid are merely convenient instances in a continuum of short monotonic shapes. More or less convex or concave shapes are common, but differences are imperceptible when frusta are short, or when the two terminal diameters differ by 20 percent or less."

If Grosenbaugh's claims can be substantiated, much more efficient and more representative methods of making sectional measurements could be evolved.

One can consider a stem as a solid of revolution defined by a curve or portion of a curve revolving 360° about its own longitudinal axis. A general expression for the curves that define such tapering bodies is

$$y^2 = p\,x^r$$

where p = a constant
y = radius of the stem or log at x
x = distance measured from the top.

The exponent r governs the shape of the curve and p its taper. For a cylinder, $r = 0$; for a quadratic paraboloid, $r = 1$; for a cone, $r = 2$; and for a neiloid, $r = 3$.

The volume of a solid may be calculated by integrating the curve of the required shape. If one further considers an individual frustum with large-end diameter D, small-end diameter d, and length, L, the volume v is

$\pi/4 \cdot L \cdot (Dd + (D + (D - d)^2/(r + 1)))$ (See Grosenbaugh 1966, fig. 4, page 450).

This general formula reduces as follows, for a

paraboloid, $v = \pi/8 \cdot L \cdot (D^2 + d^2)$;

cone, $v = \pi/12 \cdot L \cdot (D^2 + d^2 + Dd)$;

neiloid $v = \pi/16 \cdot L \cdot (D^2 + d^2 + 2\,Dd)$.

It can easily be shown that both Smalian's and Huber's formulae are equivalent to that of the paraboloid. It can also be deduced from the generalised curve that

$$d_i^2/(D - d)^2 = ((L - l_i)/L)^r$$

where d_i = diameter intermediate between D and d, at a point l_i distant from D.

Experiment 1

A simulation routine was run to examine differences in estimating volume from frusta of three curves in turn, a paraboloid, a conoid, and a neiloid. The values examined ranged from small-end diameters of 3in. (76.2mm) to 40in. (1016mm) and log lengths of multiples of 4ft (1.2m) between 4ft (1.2m) and 40ft (12.2m). Each small-end diameter was coupled with a large-end diameter from 1in. (25.4mm) to 5in. (127mm) greater than itself. A small sample of the results has been converted to metric dimensions and is presented in Table 1, from which the salient features of the simulation can be seen.

Table 1 shows that both the absolute and the percentage differences between a paraboloid and a conoid, and between a neiloid and a conoid are negligible when the end diameters differ by only 25mm, but steadily increase with successively greater differences in end diameters. The larger the small-end diameter, however, the smaller the difference in volume estimates from the various curves for the same difference in end diameters.

TABLE 1—Differences in volume with paraboloidal, conoidal, and neiloidal shapes

L	d	D	Volume of a Solid			Differences*			
			Paraboloid	Conoid	Neiloid	P — C		N — C	
m	mm	mm	m³	m³	m³	m³	%	m³	%
	100	125	0.02013	0.01996	0.01988	0.00016	0.82	—0.00008	—0.41
	100	150	0.02553	0.02487	0.02454	0.00065	2.63	—0.00033	—1.32
2	100	175	0.03191	0.03043	0.02970	0.00147	4.84	—0.00074	—2.42
	100	200	0.03927	0.03665	0.03534	0.00262	7.14	—0.00131	—3.57
	100	225	0.04761	0.04352	0.04148	0.00409	9.40	—0.00205	—4.70
	100	125	0.10063	0.09981	0.09940	0.00082	0.82	—0.00041	—0.41
	100	150	0.12763	0.12435	0.12272	0.00327	2.63	—0.00164	—1.32
10	100	175	0.15953	0.15217	0.14849	0.00736	4.84	—0.00368	—2.42
	100	200	0.19635	0.18326	0.17671	0.01309	7.14	—0.00654	—3.57
	100	225	0.23807	0.21762	0.20739	0.02045	9.40	—0.01023	—4.70
	500	525	0.41282	0.41266	0.41258	0.00016	0.04	—0.00008	—0.02
	500	550	0.43353	0.43328	0.43295	0.00065	0.15	—0.00033	—0.08
2	500	575	0.45602	0.45455	0.45381	0.00147	0.32	—0.00074	—0.16
	500	600	0.47909	0.47647	0.47517	0.00262	0.55	—0.00131	—0.27
	500	625	0.50315	0.49906	0.49701	0.00409	0.82	—0.00205	—0.41
	500	525	2.00412	2.06331	2.06290	0.00082	0.04	—0.00041	—0.02
	500	550	2.16966	2.16639	2.16475	0.00327	0.15	—0.00164	—0.08
10	500	575	2.28011	2.27275	2.26906	0.00736	0.32	—0.00368	—0.16
	500	600	2,39545	2.38237	2.37583	0.01309	0.55	—0.00654	—0.27
	500	625	2.51573	2.49528	2.48505	0.02045	0.82	—0.01023	—0.41

* Differences in volumes in m³ may appear to be in error by ±0.00001 m³ because of rounding off both volumes and differences to the nearest fifth decimal place.

Experiment 2

Tables 2 and 3 show respectively interpolated diameters for the three curves at $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{3}{4}$ the length between the same small- and large-end diameters as in Table 1, and the absolute and percentage differences in the various estimates. For the three intermediate points chosen, the greater the distance from D, the larger are the differences between the interpolated diameter for the three curves. The absolute difference in interpolated diameter at a given point is identical, for a given difference between small- and large-end diameter, irrespective of small-end diameter; hence, the greater the small-end diameter, the smaller is the percentage difference in such a situation.

TABLE 2—Interpolated diameters for paraboloidal, conoidal, and neiloidal shapes

		Interpolated diameters in mm at:								
		¼ L			½ L			¾ L		
d mm	D mm	Par.	Con.	Neil.	Par.	Con.	Neil.	Par.	Con.	Neil.
100	125	112.5	106.2	103.1	117.7	112.5	108.8	121.6	118.8	116.2
100	150	125.0	112.5	106.2	135.4	125.0	117.7	143.3	137.5	132.5
100	175	137.5	118.8	109.4	153.0	137.5	126.5	165.0	156.2	148.8
100	200	150.0	125.0	112.5	170.7	150.0	135.4	186.6	175.0	165.0
100	225	162.5	131.2	115.6	188.4	162.5	144.2	208.2	193.8	181.2
500	525	512.5	506.2	503.1	517.7	512.5	508.8	521.6	518.8	516.2
500	550	525.0	512.5	506.2	535.4	525.0	517.7	543.3	537.5	532.5
500	575	537.5	518.8	509.4	553.0	537.5	526.5	565.0	556.2	548.8
500	600	550.0	525.0	512.5	570.7	550.0	535.4	586.6	575.0	565.0
500	625	562.5	531.2	515.6	588.4	562.5	544.2	608.2	593.8	581.2

TABLE 3—Differences in interpolated diameters for paraboloidal, conoidal, and neiloidal shapes

		Differences in estimated diameters in mm at:											
		¼ L				½ L				¾ L			
d mm	D mm	P - C	%	N - C	%	P - C	%	N - C	%	P - C	%	N - C	%
100	125	6.2	5.88	—3.1	—2.94	5.2	4.60	—3.7	—3.26	2.9	2.40	—2.5	—2.10
100	150	12.5	11.11	—6.2	—5.56	10.4	8.28	—7.3	—5.86	5.8	4.22	—5.0	—3.64
100	175	18.8	15.79	—9.4	—7.89	15.5	11.29	—11.0	—7.99	8.7	5.57	—7.5	—4.80
100	200	25.0	20.00	—12.5	—10.00	20.7	13.80	—14.6	—9.77	11.6	6.63	—10.0	—5.71
100	225	31.2	23.81	—15.6	—11.90	25.9	15.92	—18.3	—11.27	14.5	7.48	—12.5	—6.45
500	525	6.2	1.23	—3.1	—0.62	5.2	1.01	—3.7	—0.71	2.9	0.56	—2.5	—0.48
500	550	12.5	2.44	—6.2	—1.22	10.4	1.97	—7.3	—1.40	5.8	1.08	—5.0	—0.93
500	575	18.8	3.61	—9.4	—1.81	15.6	2.89	—11.0	—2.04	8.7	1.56	—7.5	—1.35
500	600	25.0	4.76	—12.5	—2.38	20.7	3.76	—14.6	—2.66	11.6	2.02	—10.0	—1.74
500	625	31.2	5.88	—15.6	—2.94	25.9	4.60	—18.3	—3.26	14.5	2.44	—12.5	—2.11

DISCUSSION

The simulation routines have shown that if a measure of total volume alone is required there is less than 1% difference, whichever formula is used, provided that the difference in end diameters of a frustum is less than 75mm for small-end diameters greater than 300mm, the greater the small-end diameter and vice-versa, successively less and less constraint need be imposed.

If it is also necessary to accurately predict intermediate diameters, greater stringency is required. Thus, below a small-end diameter of 250mm, end diameter should not differ by more than 25mm, below 500mm by 50mm, below 750mm by 75mm, and so on. This recommendation is twice as stringent as Grosenbaugh's figure of 20% quoted earlier, and ensures that errors in interpolating diameter are less than ±6% down to a 100mm small-end diameter.

The obvious choice of formula is that for a cone, the intermediate one. More important, however, is the need to choose representative points of measurement, so that different operators can take a set of sectional measurements on any one tree which will not alter the estimate of its total stem volume, and can maintain consistency from tree to tree. Fixed lengths for sectional measurement are unsatisfactory because nodal swellings, sudden reductions in diameter and changes in tree shape upset the order of things, and these variations will be given different weightings from tree to tree, with inconsistent results.

Traditional methods of sectional measurements have aimed at simplifying calculations on desk calculators rather than obtaining representative estimates of actual volume, by having one fixed length of section. Possible dangers in this approach have been indicated previously when volume alone is under consideration (Whyte, 1968). When there is also a need to interpolate diameters between successive points of measurement, the importance of representative data is even greater. The irrelevance of this simplification for desk calculators when computer processing is envisaged and the need for a consistent standard for total stem volume based on measurements at unequivocal points has led to the evolution of the following technique for sectional measurement:

1. measure over-bark diameter at the nearest mid-internode to 0.75m and four bark thicknesses at that height on the north, south, east and west aspects of the tree;
2. mark breast-height (say 1.5m above ground) and measure diameter and bark thicknesses as in (1);
3. fell tree;
4. using breast height as a reference point, measure diameter over-bark and four bark thicknesses at mid-internodes, in taper steps of approximately 25mm diameter over-bark if the large-end diameter is 250mm or less, of 50mm if the large-end diameter is between 500mm and 250mm, of 75mm if the large-end diameter is between 750mm and 500mm, and so on down to about 50mm, diameter over-bark;
5. record diameters, bark thicknesses, and heights of measurement above ground sequentially from butt to tip, as shown in Table 4;
6. measure all heights to the nearest 0.05m, all diameters and bark thicknesses to the nearest millimetre;
7. head each tree with plot and tree reference numbers and the number of diameter measuring points.

TABLE 4—Example of sectional measurement

FIELD SHEET "D"			COMPUTER PROGRAMME		FRI	105
P No. 99	Tree No.		1	No. of diam. meas.		18
No.	Ht above ground (m)	Diam. O.B. (mm)	Bark thickness (mm)			
			(1)	(2)	(3)	(4)
1	0.75	798	51	51	53	50
2	1.50	718	48	50	50	46
3	4.70	650	29	28	28	27
4	10.70	574	20	20	20	20
5	12.95	519	13	15	15	14
6	20.60	465	13	13	13	13
7	23.40	422	11	11	13	13
8	28.30	363	11	15	10	11
9	30.85	312	10	11	10	13
10	33.20	262	10	8	8	8
11	35.00	244	8	8	8	8
12	36.35	216	8	8	8	8
13	37.40	198	8	8	8	8
14	38.40	147	9	8	8	8
15	39.60	119	5	5	5	5
16	40.80	99	5	5	5	5
17	42.35	70	4	4	5	5
18	44.10	38	3	3	3	3
19	45.10					

Mid-internodal sampling points are recommended, because their use gives excellent consistency in measurement by different operators and in volume estimate from tree to tree. Volume so determined will invariably be lower than that by traditional methods, but as between tree variance is reduced and as total stem volume is simply a standard from which recoverable and merchantable out-turns are deduced, this negative bias is of no consequence. Taper steps need not be exactly 25mm, 50mm, or 75mm, a tolerance of up to ±10mm being permissible. Whenever a sharp reduction in diameter occurs from one mid-internode to the next (say a drop of 15mm), it is recommended that diameter and bark measurements be made at both in order to minimise the volume to which such a frustum contributes. Because of butt-swell, it has been found that, one measurement below breast height is usually sufficient to characterise the taper in that region. In practice, diameters (if they can be measured) rarely differ between breast height and stump, in plantation trees, by amounts sufficient to yield significant errors in interpolating and extrapolating (*see* Tables, 1, 2, and 3).

This technique has been evolved for felled sample trees, but there is no reason why it cannot also be used when the tree is climbed or when a dendrometer is used to measure over-bark volume.

In certain circumstances it may be just as acceptable to measure all diameters in 50mm or even 100mm taper steps, depending upon the use to which the data are put. For example, in constructing volume and taper functions for a valuable crop, narrow taper steps should be used, and great attention paid to the presence of sudden reductions in diameter from one internode to the next, because—

1. consistency in estimating individual tree volume is very important;

2. interpolation of diameters is often required;
3. more time is spent in choosing and then felling or climbing sample trees or setting up a dendrometer than is spent on taking detailed measurements.

On the other hand, in a 3-P sampling procedure, when a dendrometer is used to measure the volumes of sample trees, it may not be possible to measure so intensively; in that case a larger step may be employed, and more trees sampled.

The needs of a particular situation can be assessed by examining the full results of the simulation routines (they are available on request at the Forest Research Institute, Rotorua), and relating them to the given set of circumstances. In doing so, it is important to allocate minimum requirements for separate parts of the tree, and to note what changes in the shape occur along a profile, and where, so that efficient sampling can be specified.

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