ESTIMATING THE ABOVE-GROUND WEIGHT OF FOREST PLOTS USING THE BASAL AREA RATIO METHOD

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ABSTRACT

Simulated sampling of nine forest plots was used to compare estimated plot component weights with known weights based on complete harvest. On average, estimates based on the relationship plot weight = (Σ sample tree weight)/(Σ sample tree basal area) $\times$ (plot basal area) showed a negligible negative bias and were approximately as variable as those found using logarithmic estimating equations. The minimum estimates using the basal area ratio method were slightly poorer, but the means and maxima consistently better than those found using logarithmic regressions. Stratified random sampling using five diameter strata was only slightly better than random sampling.

INTRODUCTION

A burgeoning literature on the weights of tree stands reflects a growing interest in the use of weight to describe forest growth (Pardé 1980). Various computational methods have been evolved to estimate plot weights from sample tree data and several studies have reported comparisons of resultant estimates (Ovington & Madgwick 1959; Ando 1962; Crow 1971; Madgwick 1971; Swank & Schreuder 1974; Madgwick & Satoo 1975). Such studies yield interesting information on the relative magnitudes of plot weight as affected by computational methodology. However, few report the measured weight of the plot as an absolute basis for comparison. In previous papers the results of simulated sampling of plots using logarithmic estimating equations have been described (Madgwick 1971; Madgwick & Satoo 1975). This paper describes the results of a similar sampling procedure but using the basal area ratio method to estimate plot weight (W) from the formula

$$W = \frac{\Sigma w_i}{\Sigma b_i} . B \qquad 1$$

where w_i and b_i are the weight and basal area of the i^{th} sample tree and B is the total plot basal area.

The major advantage of this method is its computational simplicity which allows ready estimation of plot weight with the minimum of facilities. Ando (1962) found that this method yielded results which were numerically similar to those based on logarithmic regressions.

The variance of the estimate of W is given approximately by

$$\text{variance} = \frac{N(N\text{-}n)}{n(N\text{-}1)} \;.\; \Sigma\,(w_i - \hat{R}\;.\;b_i)^2 \qquad 2$$

where N is the number of trees in the plot, n is the number of sample trees, and $\hat{R}$ is the estimated ratio of sample tree weight to sample tree basal area (Cochran 1963). The estimation of variance using Equation 2 assumes a "large" value of n.

METHODS

Nine sample plots used by Madgwick & Satoo (1975) were the basis for the present paper (Table 1). The dry weights of foliage, branches, and stems of each tree were known. In the five plots with at least 25 trees, 500 replicate samples of five trees were taken both on a random and on a stratified random basis. In plots with fewer trees only 100 replicates were used. Stratification was based on diameter.

Thus trees were ordered by increasing diameter and grouped into five diameter classes with approximately equal numbers of trees in each class. One tree was chosen at random from each class so that the total number of different samples was $\Pi^{5} n_i$ where n_i was the number of trees in the i^{th} class. The total number of different samples ranged from 108 for the plot with fewest trees to 14.9 million for the plot with most trees.

To confirm the effects of sample size on variability of estimates, additional sets of 500 replicates of random samples of 10, 20, and 40 trees were taken from the two plots with at least 100 trees. The total number of possible different samples here was ${}^{N}C_n$ where N is the number of trees in the plot and n the number of sample trees.

Variability of estimates was recorded as the coefficient of variation and results were expressed as percentages of measured plot weights. For each estimate confidence intervals were calculated and the percentage of results was determined in which the actual plot value was more than two standard deviations from the estimated value.

RESULTS

The overall average estimated plot weights ranged from an under-estimate of 2.3 to an over-estimate of 0.1% of the measured plot values (Table 2). Variability of estimated plot weights increased in the order stems < foliage < branches for seven or eight plots, depending on whether random or stratified random sampling was used (Table 3). Stratification had only a small effect on the variability of estimates of canopy components but made a fairly consistent improvement on estimates of stem weight.

Confidence intervals were under-estimated but were improved using stratified random sampling (Table 4).

TABLE 1—Summary of sample plot data (from Madgwick & Satoo 1975)

Species	Age (yr)	Plot area (m^2)	Stems (N)	Mean diam. (cm)	Mean ht (m)	Origin
Abies sachalinensis[*]	9-30	1.5	45	1.66†	1.12	natural regeneration
Abies sachalinensis[*]	17-30	2	34	2.27†	1.38	natural regeneration
Betula ermanii[*]	18	24	25	4.93	7.00	natural regeneration
Cryptomeria japonica[*]	10	37.2	16	7.97	5.24	plantation
Cryptomeria japonica[*]	43	32	14	15.18	14.85	plantation
Larix leptolepis[**]	18	100	14	11.05	9.11	plantation
Pinus densiflora	15	20	13	7.13	6.61	natural regeneration
Pinus radiata [***]	8	810	100	13.28	7.91	plantation
Pinus virginiana	19	237	136	7.54	8.65	natural regeneration

[*] Data collected by the joint study group on forest productivity of four universities, Japan.

[**] Data collected by the joint study group on forest productivity of five universities, Japan.

[***] Data of Ovington **et al.** (1968) supplied by Dr J. D. Ovington and Dr W. G. Forrest.

† At base of stem.

TABLE 2—Summary of replicated sampling - percentage error of the mean estimated plot weights using both random and stratified sampling

		Stems	Branches	Foliage
Random	Maximum	2.6	0.5	1.1
	Average	0.1	—2.3	—1.2
	Minimum	—2.0	—4.7	—6.1
Stratified random	Maximum	3.4	2.0	1.6
	Average	0.0	—1.6	—0.6
	Minimum	—1.1	—8.9	—1.6

TABLE 3—The coefficients of variation (%) of estimated plot weight based on replicated random and stratified random sampling

Species	Random			Stratified random		
	Stems	Branches	Foliage	Stems	Branches	Foliage
Abies sachalinensis	9.0	36.9	22.0	7.6	35.0	23.4
Abies sachalinensis	11.3	15.2	14.3	7.8	13.7	13.0
Betula ermanii	3.9	13.7	18.7	2.7	9.3	18.0
Cryptomeria japonica	2.6	4.5	4.0	2.7	4.8	3.9
Cryptomeria japonica	2.9	3.9	4.1	2.1	3.1	2.9
Larix leptolepis	4.9	12.4	10.9	3.5	15.4	13.4
Pinus densiflora	4.5	15.3	8.1	4.3	17.8	7.6
Pinus radiata	6.2	12.4	8.7	5.8	12.6	8.9
Pinus virginiana	10.1	37.5	18.9	8.3	37.3	14.7

TABLE 4—The fraction of replicated samples for which the range of estimated weight ± 2 standard deviations included the true weight

		Stems	Branches	Foliage
Random	Maximum	0.90	0.94	0.89
	Average	0.80	0.78	0.80
	Minimum	0.69	0.64	0.71
Stratified random	Maximum	0.93	0.93	0.93
	Average	0.88	0.82	0.82
	Minimum	0.79	0.72	0.74

DISCUSSION

The coefficient of variation of estimates was proportional to

$$\sqrt{\frac{(N-n)}{n\,.\,(N-1)}} \qquad 3$$

as sample size changed (cf. Cochran 1963). This is illustrated in Fig. 1 using the *Pinus radiata* D. Don plot as an example and varying the sample size from 5 to 40 trees taken at random.

Similarly, the coefficients of variation for the nine plots using a fixed sample size n=5 were correlated with $\sqrt{(N-5)/(N-1)}$ (Fig. 2). The correlation coefficients were

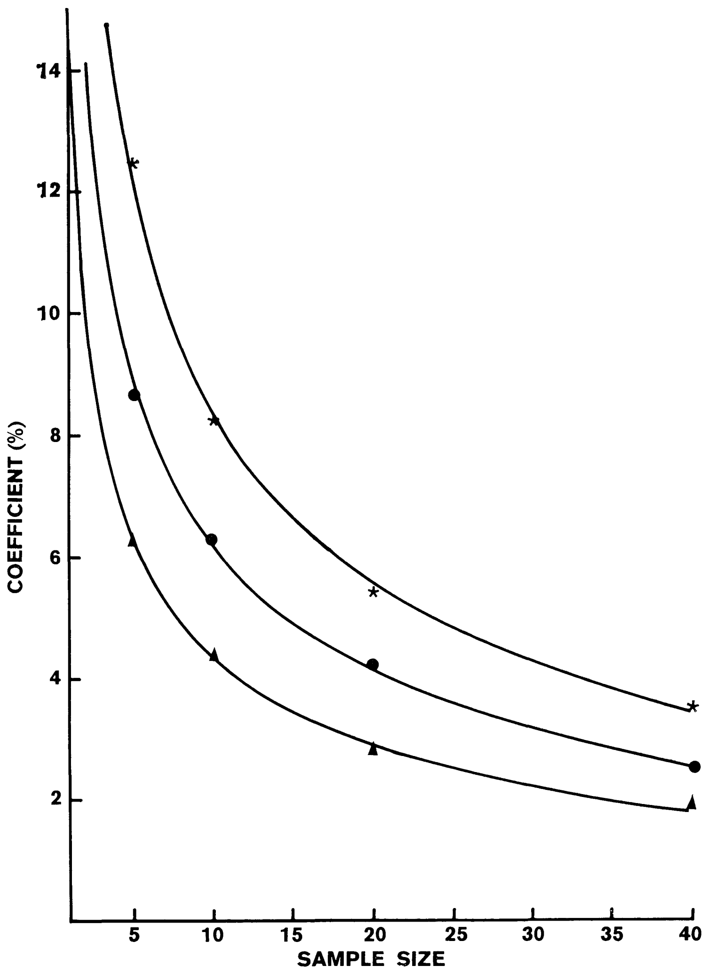

FIG. 1—The relationship between the coefficient of variation of estimated plot weight and sample size based on simulated sampling of the **Pinus radiata** plot (* branches; ● foliage; ▲ stems).

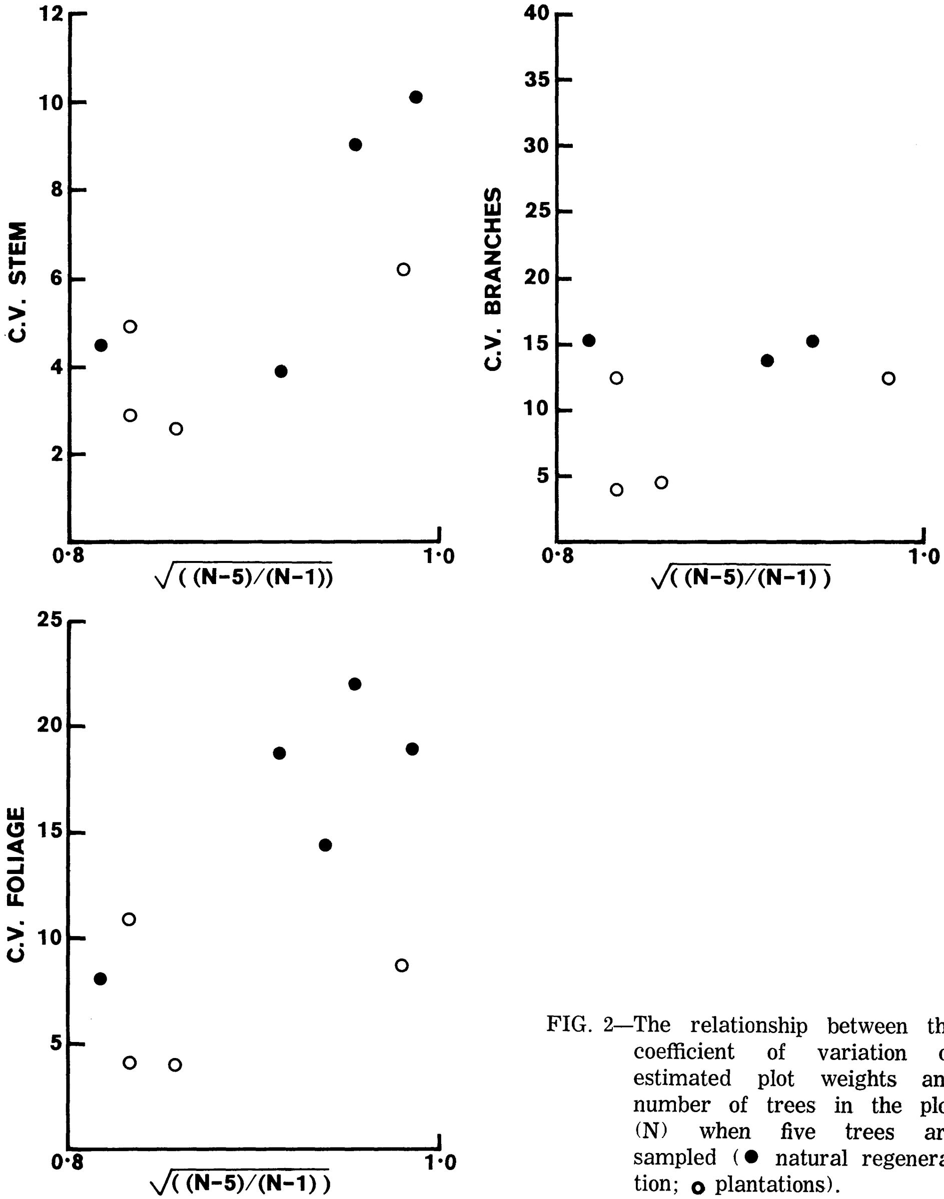

FIG. 2—The relationship between the coefficient of variation of estimated plot weights and number of trees in the plot (N) when five trees are sampled (● natural regeneration; o plantations).

0.73, 0.64, and 0.65 for stems, branches, and foliage, respectively, using random sampling. There was a suggestion that variability of estimates from plantations was smaller, on average, than those from naturally regenerated stands.

Confidence limits calculated using Equation 2 proved under-estimates because a sample size of five trees was small (Cochran 1963).

The ratios of weight to diameter-squared were usually positively correlated with stem diameter (Table 5) indicating that stratified sampling, based on stem diameter, should be better than strictly random sampling. In practice the effect of stratification was variable though overall it reduced coefficients of variation by 18, 2, and 5% for stems, branches, and foliage, respectively.

The mean estimates of plot weight using the basal area ratio method were consistently closer to the measured values than were those from logarithmic estimating equations (Madgwick & Satoo 1975). Further comparisons of the methods are presented in Table 6. In all comparisons 500 replicates of a stratified sample of five trees have been used and only those plots with at least 25 trees have been considered. The coefficients of variation for the two methods are broadly comparable. The minimum estimates obtained using logarithmic regressions were consistently slightly better than those from the basal area ratio method. However, the maximum estimates were

TABLE 5—Correlation coefficients of weight/(diameter squared) against diameter for individual trees

	N	Stems	Branches	Foliage
Abies sachalinensis	45	0.39 **	0.61 **	0.63 **
Abies sachalinensis	34	0.50 **	0.73 **	0.37 **
Betula ermanii	25	0.68 **	0.84 **	0.22 ns
Cryptomeria japonica	16	—0.57 *	0.54 *	0.10 ns
Cryptomeria japonica	14	0.29 ns	—0.04 ns	—0.28 ns
Larix leptolepis	14	0.73 **	0.46 ns	0.31 ns
Pinus densiflora	13	0.17 ns	0.64 **	0.55 **
Pinus radiata	100	0.51 **	0.68 **	0.66 **
Pinus virginiana	136	0.64 **	0.68 **	0.61 **

** $p < 0.01$
* $p < 0.05$
ns $p > 0.05$

TABLE 6—Minimum, maximum, and coefficients of variation of weight estimates as a percentage of measured stand weight, using 500 replicates of a stratified random sample of five trees with basal area ratio (B.A.) and logarithmic (Reg.) estimating procedures

		Stems			Branches			Foliage		
		Min.	Max.	C.V.	Min.	Max.	C.V.	Min.	Max.	C.V.
Abies sachalinensis	B.A.	80	122	7.6	27	168	35.0	53	164	23.4
	Reg.	76	132	9.9	34	170	26.1	74	302	26.5
Abies sachalinensis	B.A.	83	126	7.8	73	131	13.7	70	139	13.0
	Reg.	88	181	15.6	72	181	17.1	73	198	20.0
Betula ermanii	B.A.	91	107	2.7	75	120	9.3	64	141	18.0
	Reg.	92	130	5.8	79	146	9.3	66	195	22.5
Pinus radiata	B.A.	72	115	5.8	57	131	12.6	64	131	8.9
	Reg.	85	117	5.9	71	149	13.2	83	126	7.5
Pinus virginiana	B.A.	75	124	8.3	48	140	37.3	54	144	14.7
	Reg.	84	122	7.2	58	213	26.4	69	227	18.3

frequently much larger using the logarithmic method than the maxima using the basal area ratio estimates.

In prediction equations relating tree weight (w) to tree diameter (d) by equations of the form

$$w = a.d^b \qquad 4$$

where a and b are constants, the value of the constant b is usually greater than 2. Thus for the nine sample plots, mean values of b based on regressions using all trees in each plot were 2.3, 2.7, and 2.5 for stems, branches and foliage, respectively. This suggests that a better method of prediction would be

$$W = \frac{\Sigma w_i}{\Sigma d_i^{2.5}} . D \qquad 5$$

where w_i and d_i are the weight and diameter of the i^{th} sample tree, and

$$D = \sum^{N} d_j^{2.5}$$

where d_j is the diameter of the j^{th} tree in the sample plot.

Recalculating results using Equation 5 decreased the small average biases in foliage and branch estimates but led to an over-estimate of stems and poorer estimates of confidence intervals. Any theoretical improvement of Equation 5 over Equation 1 appears negligible in practice.

The advantage of random sampling is that it allows sequential sampling to the point where error estimates are within desired bounds. In practice, this would be approximate as the control of sample size would be likely to be defined by fresh weight, say of the tree crowns, rather than component dry weight. The necessary calculations are easily performed using a simple pocket calculator. In contrast, stratified sampling requires that the number of sample trees be predetermined but yields slightly better estimates.

The basal area ratio method yields satisfactory estimates of plot weight and compares well for accuracy and precision with the use of logarithmic estimating equations. The basal area method has the added advantage of computational simplicity and a potential for sequential sampling which could be readily applied in the field.

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