ANALYSIS AND SIMULATION OF A LOGGING WEIGHBRIDGE INSTALLATION

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ABSTRACT

Queuing theory and simulation methods were used to quickly and inexpensively estimate waiting times at a forest industry weighbridge system. Arrival, beginning, and finishing times of trucks were measured over 2 days. Inter-arrival times were exponentially distributed, allowing the system to be approximated by a simple queuing model. Estimates of queue waiting times agreed well with observed values. The models suggested that if a second weighbridge were installed it could save over 50 hours of truck time per day. They were also used to demonstrate that the system then in use was operating at close to its maximum capacity. A simulation model written in GPSS validated the approximations that had been used to fit the queuing formulas. The simulation also demonstrated the difficulty of estimating mean waiting times accurately by observation, in systems where the traffic intensity is high.

There may be other areas in forestry research which would benefit from similar analyses, probably leading to increased efficiency.

Keywords: weighbridge models; queuing; simulation models.

INTRODUCTION

Each year some 2.5 million m^3 of wood materials, mainly *Pinus radiata* D. Don, are transported to the NZFP Ltd complex at Kinleith, near Tokoroa. The materials, mostly logs, are either processed on the site or are in transit for export. Cartage is by truck, operated by contractors. All trucks, without exception, are required to enter the site via one throughway where each vehicle is weighed. The tare weight of each truck (and trailer, if applicable) is catalogued, so that the weight of the wood material can be estimated. These data are critically important for contractual payments, while the system acts as a monitor of the fellings from the Company's forests. On average, more than 400 loads per day cross the weighbridge. The actual number on any particular day is determined partially by the operational requirements of the complex.

In early 1989, activity at the weighbridge increased to the extent that the facility sometimes became congested, with a queue of vehicles forming over several hundred metres. In this contribution we report results from a quantitative analysis of the installation, and give details of a simulation study. The analysis and the simulation were designed in part to estimate the effects of building a second weighbridge in parallel with the existing bridge.

QUEUING PROCESSES

In general, queues or waiting lines can be visualised as consisting of three components:

(a) One or several sources of arrivals;

(b) Queue(s);

(c) A service facility, consisting of one or several parts.

It is useful to study these components in terms of probability distribution functions, and to trace transactions passing through a queuing system in terms of arrival, waiting, and service times. For brevity, a notation initiated by Kendall (1953) is available to describe basic types of models, with

$$I/F/S/N \qquad (1)$$

where I = the input process (distribution)

F = the service time distribution

S = the number of service units (in parallel)

N = the number of customers allowed in the system

(When N is assumed to be infinite, the fourth letter is usually omitted.)

Some standard symbols are:

M = negative exponential time distribution (Markovian)

D = deterministic or constant times

G = general service time distribution

GI = general independent inter-arrival time distribution.

The most basic queuing model is M/M/1—that is, where inter-arrival times come from a Poisson process, service times are assumed to be exponential, arrivals join a queue unlimited in size, wait in line on a first-come-first-served basis, and then are serviced by a single server. For such a system, it is relatively easy to derive a system of balance equations (Daellenbach *et al.* 1983, Chapter 15) which gives rise to analytical solutions for statistics of interest. Important system characteristics include:

L, the average number of units in the system

L_q, the average number of units waiting in the queue

W, the average time in the system

W_q, the average waiting time in the queue.

For example, for an M/M/1 model

$$L = (\lambda/\mu) / (1 - \lambda/\mu)$$

and

$$L_q = (\lambda/\mu)^2 / (1 - \lambda/\mu) \qquad (2)$$

where λ and μ are the inter-arrival and service time rates.

Unfortunately, simple formulas such as (2) can be derived only for a small number of queuing models. When the inter-arrival or service times are not exponentially distributed, equivalent formulas become more complex or must be replaced by numerical approximations. In real life, departures from an M/M/1 process can be frequent. For example, service times often have truncated general distributions, or are better described by normal or uniform distributions. Nevertheless, luckily, for the class of M/G/1 queues the mean characteristics follow from the application of the so-called Pollaczek-Khinchin formula (Daellenbach *et al.* 1983, p.448). Provided inter-arrival times are exponential, then for any service time distribution, with mean s and variance σ_s^2

then $$L_q = (\lambda^2\sigma_s^2 + (\lambda s)^2) / 2(1-\lambda s) \quad (3)$$

and other statistics can be calculated from

$$\begin{aligned} L &= L_q + \lambda s \\ W &= L/\lambda \text{ (Little's formula)} \quad (4) \\ W_q &= W - s. \end{aligned}$$

Extensions to multiserver systems are even more limited. Analytical solutions exist for an M/M/S model, viz

$$P_o = 1/[\Sigma_{n=0}^{S-1} (\lambda/\mu)^n/n! + \{(\lambda/\mu)^S/S!(1-\lambda/S\mu)\}] \quad (5)$$

The probability that a customer must wait is

$$P_o(\lambda/\mu)^S/S!(1-(\lambda/S\mu))$$

and $$W_q = L_q/\lambda \quad (6)$$

P_o is the probability of no transaction in the system (Daellenbach *et al.* 1983, p.440)

but otherwise, simple solutions are difficult to obtain.

Although there are a vast number of theoretical results for queuing systems (*see*, for example, Gross & Harris 1985), many of them do not lead to simple formulas for the operating characteristics. The situation we faced, where the total service time depends on whether or not a transaction waits, is one of these. Theoretical results are known (Posner 1973); however, they involve solving complex integral equations which our data could not really support. We found, however, we could produce simple bounds on the expected reduction in delay by using (4) and (6).

An attractive and viable alternative when queuing theory fails is to build a simulation model of the system. Most of the discrete simulation packages handle queuing models very well. SIMSCRIPT and GPSS are examples of two packages that are available for microcomputers. Neelamkavil (1987) has given a good comparison of their characteristics. In this contribution we do not attempt a description of these packages; full details have been published by Schriber (1974) and Russell (1983).

METHODS AND QUEUING ANALYSES

On two working days, the following data were obtained for each vehicle arriving at the weighbridge entrance:

(a) Time of arrival;
(b) Time of entering the bridge mechanism;
(c) Time of leaving the weighbridge;
(d) Number of vehicles in the queue.

All timings were recorded to the nearest second and are summarised in Table 1. The moving-up times in Table 1 refer to the fact that, unless the weighbridge is vacant, there is an appreciable time between one truck vacating the bridge and the next one entering it.

TABLE 1–Summary statistics for the system

Day*	Mean	s.d.	Min.	Max.	p
			Inter-arrival times (min)		
1	1.81	1.69	0.02	8.80	0.17
2	1.61	1.77	0	13.57	
			Service times (min)		
1	1.19	0.66	0.20	4.08	0.73
2	1.21	0.76	0.08	6.87	
			Moving-up times (min)		
1	0.35	0.41	0	4.61	0.50
2	0.33	0.18	0.07	2.75	
			Queuing-times (min)		
Day	Mean	s.d.	Min.	Max.	Utilisation (%)
1	4.84	4.90	0	17.85	79
2	8.94	6.14	0	23.70	96

* Day 1: 223 trucks, observed 0820 to 1500 hours
Day 2: 431 trucks, observed 0530 to 1700 hours

The arrival, move-up, and service times of the two datasets were compared for common population means, using two-sample t-tests. This hypothesis was accepted for all three variables. The p-values are given in Table 1. The data were therefore pooled, giving total parameters (in minutes):

	Mean	s.d.
Inter-arrival times	1.68	1.74
Move-up times	0.33	0.27
Weighing times	1.21	0.73

The coefficient of variation of the inter-arrival times (1.07) was very encouraging since this agreed well with that which would occur if the inter-arrival times came from a Poisson process. The cumulative distribution of the inter-arrival times was compared with that of a negative exponential distribution with the same mean. The observed and fitted distributions are plotted in Fig. 1.

Goodness of fit was evaluated by the Kolmogorov-Smirnov test (*see*, for example, Gross & Harris 1985, p. 398). The hypothesis that the marginal distribution was negative exponential was accepted at a level of significance larger than 20%. To be a Poisson process the inter-arrival times would also have to be independent and stochastically stationary. An autocorrelation analysis showed no evidence of dependence between inter-arrival times, while the graph of the length of the intervals against time of day showed neither correlation nor trend. There were no significant differences between morning and afternoon arrival rates.

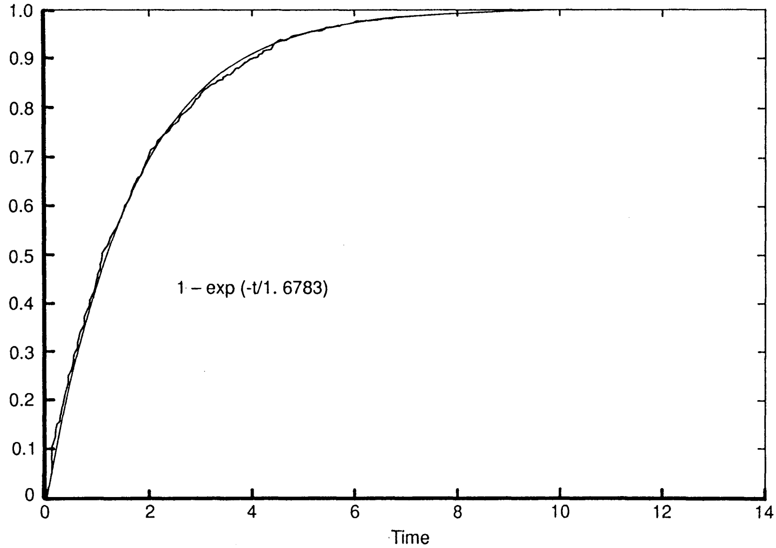

FIG. 1 — Inter-arrival times

In fact, it is extremely unlikely that the inter-arrival times were generated by a Poisson process. Studies of arrival processes for theoretical models (*see* Disney *et al.* 1980) where customers cycle through the same server a number of times have shown that they are not. However, our data showed no obvious deviations from that produced by a Poisson process, over a range of statistical tests.

The fact that it was reasonable to model the arrival process by a Poisson process gave us two great advantages: firstly, at least the arrival process resembled that required by the theoretical formulas (4) and (6); secondly, the stream of arriving trucks could be generated in the simulation by sampling from a probability distribution. This made it easier to compare the performance of a one-weighbridge and a two-weighbridge system, since exactly the same stream of arrivals could be used for the two models.

Because the first truck in any busy period did not have to wait for the moving-up time, the bridge queue did not satisfy the independent service time assumption of the M/G/1 model. Two variations which did fit the assumption were postulated:

(1) What if all the trucks were assumed to wait the moving-up times? This would provide an upper bound on the delay.

(2) What if the moving-up times were arbitrarily assigned to trucks in the correct proportion, regardless of whether the weighbridge was vacant? Since the present system is adapting in the worst possible way, in the sense that the total service times are longer when the bridge is busy, this should provide a lower bound on the delay. Using a formula that will under-estimate the steady-state delay is also attractive because in practice the weighbridge starts, each morning, with a small queue or a single truck. A formula that under-estimates steady-state delay will allow, in part, for this transient effect.

From the pooled data and Equations (3) and (4), we get for the two models:

Situation	Traffic intensity (λ/μ)	Mean No. in queue	Mean waiting time (min)
(1)	0.917	6.38	10.72
(2)	0.897	5.00	8.38

What if a second (parallel) weighbridge were installed? The coefficient of variation of the service times is well below that of a negative exponential distribution. It is a reasonably safe rule-of-thumb (*see*, for example, Gross & Harris 1985, section 7.1.2) that less variable service (or inter-arrival) distributions lead to reduced delay; hence, an upper bound can be obtained from Equations (5) and (6), which gives:

Probability of waiting = 0.19
Mean number in queue = 0.11
Mean waiting time = 0.18 minutes

Here we have assumed that very few trucks will now wait, and so the moving-up times can be ignored. The small percentage (19%) who wait confirm that this assumption was reasonable.

Taking the best predictions from the one- and two-bridge systems, the savings in waiting time per truck will be about 8.38 - 0.18 = 8.20 minutes.

SIMULATION OF WEIGHBRIDGE SYSTEM

Simple queuing models could be fitted to the system only by making some quite substantial approximations. To see if those approximations would stand up, it was decided to also simulate the system. GPSS is a somewhat dated simulation system which, however, handles most queuing models very efficiently. A simulation model of the system was written in GPSSR/PC (Richards 1983). Arrival times were generated by an exponential distribution, and the move-up and service times were represented by the observed empirical distributions. If a truck found the bridge engaged, it waited in the queue, and then moved up when vacant. If the bridge was empty, a vehicle proceeded directly to the bridge. Simulations were run over 1000 working days (250 × 4 years), initially with one bridge, and then with a two-bridge single-queue system. The program required no more than 37 lines of code, and used 11 hours of CPU time to finish a simulation on an inexpensive micro-computer. Three random number generators were employed, using the same initial seeds with each run, to help variance-reduction (Neelamkavil 1987, Chapter 10). A summary of output statistics is given in Table 2.

TABLE 2–Summary of simulation results

Attribute	One bridge		Two bridges	
	Mean	s.d.	Mean	s.d.
Average No. trucks/day	411	19	411	19
Average waiting time/truck (min)	8.05	5.45	0.14	0.05
Total waiting time/day (h)	56.2	40.3	0.96	0.37
Proportion of trucks not waiting	15%		81%	

DISCUSSION

The summary statistics given in Table 1 confirm that the weighbridge system was working at close to its maximum capacity, with the bridge busy either weighing trucks, or waiting for them to move up 96% of the time on Day 2. Actual waiting times of up to 17 minutes on Day 1 and 23 minutes on Day 2 were observed. The Day 1 data would be equivalent to 385 trucks on a full working day, 46 fewer than on Day 2, which is reflected in an 11% lower arrival rate [0.552 and 0.621, respectively]. This partly accounts for the increase in the average waiting time in the queue from 4.84 minutes on Day 1 to 8.94 minutes on Day 2.

The pooled inter-arrival times did not show any substantial deviations from those produced by a Poisson process. This allowed us to use simple queuing formulas to estimate average waiting times. Once the results from the formulas had been validated, they could be used to estimate average waiting times under different assumptions about the number of loads per day that must use the bridge. For example, we were able to show how close the bridge was to its maximum capacity. A variation on Equation (4) was used to plot graphs of average delay against number of loads per 12-hour day. These suggested that if 5% more trips per day were to be attempted, the average waiting time would increase to over 20 minutes. It turned out that this agreed very well with what had been observed on one day on which an abnormally high number of trips had been attempted. Plotting such a graph by simulation would have required at least 11 hours of computer time for each point.

The queuing formulas (Situation 2) give an estimate of 8.38 minutes for the mean waiting time, which compares remarkably well with the observed values. Waiting times in queues show very high variation, especially when the traffic intensity is high, since successive times are highly correlated. Thus, although the sample sizes on both days are large by most forestry criteria, it can be shown that sample sizes of over 100 000 are necessary to confidently detect small differences in average waiting times (*see*, for example, Fishman 1978). The closeness of the observed value on Day 2 to the theoretical estimate, and even the fact that the observed average delay increased with the increased arrival rate from Day 1 to Day 2, has to be ascribed to good fortune rather than accurate measurement. The subsequent simulation study was the only practical way to gather enough observations to validate the queuing formulas.

When a second weighbridge is (hypothetically) added to the system, an appreciable drop in average waiting time and in the number of trucks which must wait occurs, to the extent that only 19% of trucks now are estimated to wait. The size of this gain in efficiency may be surprising, but it is to be expected on two counts:

(a) Availability of a second bridge, *per se*;

(b) Virtual abolition of moving-up time.

With only one bridge, if a truck is very slow in service (often this is due to an inexperienced driver or incorrect documents) the system is brought to a halt, except for the arrival of additional trucks. The chance of two trucks simultaneously causing such a delay is much lower, so generally one bridge will keep moving. Point (b) is perhaps less obvious; even though the average moving-up time is only 0.3 minutes, it comes into play at the worst possible time, in the sense that the over-all service time is longer when trucks are waiting and shorter when the queue is empty. With two bridges the chance of an arrival moving straight on to a bridge is much higher, and hence this lost time is virtually eliminated.

Simulation of the system with GPSS produces results very comparable to the queuing equation analysis. With one bridge the estimated waiting time is 8.05 minutes (cf. 8.38), decreasing to 0.14 minutes (cf. 0.18) when a second weighbridge is added. this gives some validation for the approximations that were used to derive the queuing formulas. The simulation study also demonstrates the variability of the average waiting time that we could expect to measure over a single day. Since the simulation was started from an empty queue on each of the 1000 days, the average waiting times per day are independent. From their standard deviation (5.45 minutes) it can be seen that the queuing equation value just falls within a 95% confidence interval around the average simulation result.

The results discussed above, and the option to install a second bridge in parallel, are naturally dependent on the prevailing sytem and the number of trucks passing through the mechanism each day. Subsequent to this study, service times have been significantly decreased by the installation of an enhanced computer system, while plans to introduce a night-shift are scheduled. Thus, while a second bridge remains an attractive proposition, appreciable gains in efficiency have been achieved in other ways.

Nevertheless, it would seem there are several areas in forestry research where applications of queuing formulas and simulation models would be useful. Flows of logs through milling yards, or mill procedures, frequently incorporate subsystems that are amenable to such studies. Gains in efficiency, as demonstrated here, could lead to appreciable saving in running costs.

CONCLUSIONS

A forest industry weighbridge system was found to be easily and inexpensively modelled by standard queuing models. Inter-arrival times of trucks fitted a negative-exponential distribution remarkably well, allowing simple analytical formulas to be used to estimate average waiting times. For the weighbridge in question, a dramatic decrease in waiting time is predicted if another bridge is installed in parallel.

A simulation programme was used to validate the queuing formulas. The simulation demonstrates that average waiting times are often extremely expensive to measure, and may require thousands of transactions to estimate with even moderate accuracy.

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