RESIDUAL WITHIN-TREE VARIATION IN STIFFNESS OF SMALL CLEAR SPECIMENS FROM *PINUS RADIATA* AND *PSEUDOTSUGA MENZIESII*

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ABSTRACT

Static bending of small clear specimens is one of the most commonly used methods for assessing the stiffness (modulus of elasticity, MoE) of sawn timber and trees. Small clear specimens have traditionally been cut at breast height from the same growth rings on opposing radii, thus seeking to minimise the radial and longitudinal variation. The remaining (residual) variation between small clear specimens determines the precision of the estimate of the tree mean MoE, but has rarely been analysed in detail because the method originally was not intended for tree-level analyses. To investigate this, axial stiffness measurements previously collected from small clear specimens taken from opposing radii at breast height on New Zealand-grown *Pinus radiata* Don and *Pseudotsuga menziesii* (Mirb.) Franco (Douglas fir) were reanalysed to ascertain the magnitude of the residual variation at breast height. Expressed as coefficient of variance between small clear specimens from the same radial position (growth ring), the variation ranged from 8% to 32% for *P. radiata* and from 7% to 13% for Douglas fir. Using two small clear specimens, the associated margin of error for estimates of mean stiffness of individual trees ranged from 40% to 140% for *P. radiata* and 40% to 60% for Douglas fir. It is recommended that at least four small clear specimens are used (margins of error of 10–40%) when estimating the mean MoE of individual trees from small clear specimens extracted at the same height from the same growth ring.

**Keywords**: small clear specimens; sample size; modulus of elasticity; stiffness; *Pseudotsuga menziesii*; *Pinus radiata*.

INTRODUCTION

As more of the world’s supply of wood comes from fast-growing tree species managed in shorter rotations, there is an increasing need to investigate and evaluate the physical properties of the wood produced (Mamdy et al. 1999; Jayawickrama 2001; Huang et al. 2003). In particular, if the wood is to be used for structural purposes it is important to investigate the structural properties, particularly stiffness (MoE) (Madsen 1995). There are many ways of measuring/estimating stiffness of wood, including ultrasound (Sandoz & Lorin 1994), piece matching (Noren 1994), the modulometre (Mamdy et al. 1999; Rozenberg et al. 1999), acoustic methods (Wang et al. 2000; Huang et al. 2003),...
near-infrared reflectance spectrometry (So et al. 2002), and X-ray densitometric and diffractometric methods (Evans & Ilic 2001). See also the Handbook of the Forest Products Laboratory (1999), which provides a comprehensive review of methods.

Standard in-grade testing of structural sawn timber in bending is considered to be the best and most direct method of measuring stiffness, but it is expensive and not always practical. An alternative method is bend testing of small clear specimens (British Standards BS 373:1956(1986); American Society for Testing and Materials ASTM D143-94(2000)e1; or the French Norm NF B 51-016). This method was originally devised to provide strength data for engineering design purposes. Such sampling was intended to provide data at the species and forest level as the minimum. Due to its early standardisation and relative ease of use, it has become one of the most widely applied methods — see, e.g., Mack (1979), Okstad & Karstad (1985), Ishengoma & Nagoda (1987), Okstad (1987), Kliger et al. 1998, Bier & Britton (1999), Flaete & Kucera (1999), Evans et al. (2000), Burdon et al. (2001). Recent advances in technology have allowed for small clear specimens to be extracted from living trees without the need to fell them, thus making this method available also to tree breeders (Jayawickrama 2001).

Despite its status as a de facto standard, both Madsen (1995) and Jayawickrama (2001) raised concerns about the small clear specimens method. This concern revolves mainly around the fact that small clear specimens are considered to be samples of the whole tree and standard sampling theory applied, i.e., the mean stiffness of a set of small clear specimens is assumed to be an unbiased estimator for the mean of the tree or the sawn board. The problem with this approach is that the small clear specimens reflect wood properties at a much smaller scale than at the level of the tree or the board. An estimate of the mean for the clearwood of an individual tree or board is therefore influenced by the within-tree variation at the level of the small clear specimens. Finally, using the method at the individual-tree level is a significant extension of the purpose for which it was originally intended.

Within-tree variation of wood properties (including stiffness) can be grouped into three components: (1) radial, (2) longitudinal (height), and (3) tangential (circumferential) (Tsoumis 1991). The radial variations represent the within- and between-year changes in wood properties. These changes are reasonably well understood and described (e.g., Zobel & Buijtenen 1989; Tsoumis 1991; Walker et al. 1993; Lausberg et al. 1995; So et al. 2002; Knowles et al. 2003). It is also generally accepted that stiffness within stems of conifers more or less increases with height above the ground (So et al. 2002; Knowles et al. 2003). The tangential variation, on the other hand, shows no consistent pattern (except for tree lean) either across or within species (Tsoumis 1991; Walker et al. 1993), or is traditionally considered to be insignificant, and is thus ignored (Nicholls 1986).

In estimating individual-tree mean MoE the within-tree variation at the level of the small clear is minimised by extracting small clear specimens at a fixed height (e.g., breast height), and by assigning each small clear to a specific ring number relative to the pith. This, however, does not account for the circumferential variation, which together with measurement errors makes up the remaining (residual) within-tree variation. The precision of the estimates of the mean is determined by the amount of residual within-tree variance, but the magnitude of this variation has rarely been analysed in detail.
The purpose of this study was to:

1. Examine and describe the residual (partly circumferential) within-tree variation in stiffness of small clear specimens using historical records of small clear specimens extracted from New Zealand-grown *P. radiata* and Douglas fir;

2. Based on sampling theory and the results from the first part of the study, to ascertain the minimum number of small clear specimens required for estimating the mean stiffness of individual trees, with a given precision.

**MATERIAL**

The data consisted of static bending stiffness measurements (MoE) of small clear specimens extracted from New Zealand-grown *P. radiata* and Douglas fir, and are summarised in Table 1.

<table>
<thead>
<tr>
<th>Species</th>
<th>Location</th>
<th>Age (yr)</th>
<th>Shipments</th>
<th>Trees</th>
<th>Radial positions per tree</th>
<th>Height positions per tree</th>
<th>Small clears per tree</th>
<th>Pairs of small clears in total</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Pinus radiata</em></td>
<td>New Zealand</td>
<td>9–41</td>
<td>33</td>
<td>161</td>
<td>1–9</td>
<td>1</td>
<td>2–18</td>
<td>841</td>
</tr>
<tr>
<td>Douglas fir</td>
<td>Tapanui</td>
<td>18</td>
<td>1</td>
<td>60</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>101</td>
</tr>
<tr>
<td>Douglas fir</td>
<td>Rotoehu</td>
<td>41</td>
<td>1</td>
<td>18</td>
<td>2–3</td>
<td>3–5</td>
<td>6–14</td>
<td>176</td>
</tr>
</tbody>
</table>

A total of 33 shipments of small clear specimens from *P. radiata* were available, each representing one stand, i.e., one age, one location, one genotype. Each shipment consisted of small clear specimens from four or five trees, with 2–18 small clear specimens from each tree. All small clear specimens had been cut in pairs at similar ring numbers from the pith along two opposing radii centred at breast height (1.4 m). The data have been described fully by Walford (1985).

Only two Douglas fir data sets of small clear specimens were available. The first set was from 60 trees in an 18-year-old stand in Cpt 202, West Tapanui Forest. In each tree, two pairs of small clear specimens had been cut from opposing radii, centred on the fifth and tenth growth ring at breast height. The second shipment originated from a 41-year-old stand in Cpt 55, Rotoehu Forest, where a total of 18 trees had been sampled. Four to six small clear specimens had been cut in pairs at similar ring positions (five-ring intervals) from the pith on opposing radii at each of five heights up each stem.

**METHODS**

**Background and Assumptions**

The confidence limits for an estimate of a single mean (for a normal population) are defined as:

$$\mu \pm t_{\alpha,n-1} \frac{s}{\sqrt{n}}$$  \hspace{1cm} (1)

where $\mu$ is the mean,
\[ t_{\alpha, n-1} \] is the value of the cumulated \( t \)-distribution with \((n-1)\) degrees of freedom at confidence level \( \alpha \),
\( s \) is the standard deviation, and
\( n \) is the number of observations (Weisstein 2002).

Knowledge of the standard deviation \( s \) thus allows for estimation of the number of samples \( n \) required to obtain an estimate within a certain margin of error. The problem is that the standard deviation for a particular population is rarely known prior to a study. By studying similar historical data it is, however, possible to get a useful estimate of its range.

The data consisted entirely of pairs of stiffness measurements. Because the observations in each pair were extracted at the same height and the same ring number from the pith, it was assumed that the difference between them was independent of the radial and longitudinal within-tree trends in stiffness — thus reflecting the residual variation only (part of which is circumferential variation). To analyse the data it was further assumed that each measurement in a pair was drawn from a tree-specific normal distribution with a mean \( m \) and standard deviation \( \sigma_x \). The difference \( z \) between the measurements was thus normally distributed with a mean of zero and a standard deviation of \( \sigma_z = \sqrt{\sigma_x^2 + \sigma_x^2} = \sigma_x \sqrt{2} \). Iterating this for all trees achieved a \( z \)-value for each individual tree. The population of individual-tree-level \( z \)-values could then be analysed by calculating the mean and the standard deviation \( \sigma_z \), i.e., estimating the average residual within-tree variation.

**Analyses**

The \( z \)-values were plotted against pair mean stiffness, radial position, shipment, and, where possible, height. The graphs were visually analysed in order to ascertain any trends.

The \( z \)-values from \( P. \ radiata \) were partitioned into groups by radial position (nominal positions), mean stiffness (in intervals of 1 GPa), shipment, tree, or radial position within shipment. For each group the mean \( z \)-value was calculated, and the normality assumption was tested, using the Shapiro-Wilks test, as implemented in PROC UNIVARIATE in SAS. The distribution of \( z \)-values across groups was summarised by descriptive statistics, i.e., mean, standard deviation, minimum and maximum values. For Douglas fir the \( z \)-values were partitioned into groups based on height (nominal groups and only for the Rotoehu data), radial position (intervals of five annual rings), mean stiffness (in intervals of 1 GPa), or tree, and the standard deviation \( \sigma_z \) was calculated for each group.

The residual within-tree standard deviation was then estimated from \( \sigma_z \) using
\[
\sigma_x = \sigma_z / \sqrt{2} \tag{2}
\]
and expressed in terms of coefficient of variation, by division by mean stiffness.

Finally, the margin of error \( R \) was calculated using Equation 3 (StatSoft 2004)
\[
R = t_{\alpha, n-1} \frac{\sigma_x}{\mu \sqrt{n}} \tag{3}
\]
and iterated for \( \alpha = [0.05], n = [2, 4, 6, 8, 10, 12, 14, 16] \), and coefficients of variation \( (\sigma_x / \mu) \) between 0 and 30%. The margin of error thus expresses the 95% confidence limits of the estimate for the mean, e.g., a margin of error of 50% means that the estimated mean with 95% confidence is within \( \pm 50\% \) of the true mean.
RESULTS

Trends in z-values

The z-values are plotted against mean stiffness and radial position for *P. radiata* in Fig. 1 and 3, and for Douglas fir in Fig. 2 and 4. There were no trends in z-values with height for the Douglas fir data from Rotoehu. The *P. radiata* z-values are plotted by shipment in Fig. 5.

FIG. 1–*Pinus radiata* z-values (GPa) against mean stiffness

FIG. 2–Douglas fir z-values (GPa) against mean stiffness
Standard Deviation of z-values

The standard deviations of z-values for *Pinus radiata* are presented in Table 2 and for Douglas fir in Tables 3 and 4.

The average within-group standard deviation of z-values for *P. radiata* varied between 1.54 and 2.10, which converted to residual within-tree standard deviations of 1.09 and 1.48. For Douglas fir the similar values were 1.30 for Rotoehu and 1.50 for Tapanui, which converted to residual within-tree standard deviations of 0.92 and 1.06, respectively.
The average stiffness by shipment for *P. radiata* varied from 4.61 to 13.06 GPa, which gave estimates of residual within-tree coefficients of variance between 1.09/13.06 = 8% and 1.48/4.61 = 32%. In Douglas fir the mean stiffness by tree ranged from 8.2 to 13.07 GPa. This gave residual within-tree coefficients of variance between 7% and 13%.

**TABLE 2—Summary statistics of the standard deviation for *z*-values for *P. radiata***

<table>
<thead>
<tr>
<th>Group</th>
<th>All</th>
<th>Radial position</th>
<th>Mean stiffness</th>
<th>Shipment</th>
<th>Tree</th>
<th>Shipment and radial position</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of groups</td>
<td>1</td>
<td>10</td>
<td>15</td>
<td>33</td>
<td>161</td>
<td>196</td>
</tr>
<tr>
<td>Average number per group</td>
<td>841</td>
<td>84.1</td>
<td>56.07</td>
<td>25.48</td>
<td>5.22</td>
<td>3.95</td>
</tr>
<tr>
<td>Mean of $\sigma_z$</td>
<td>2.1030</td>
<td>2.1912</td>
<td>2.0803</td>
<td>1.9344</td>
<td>1.5462</td>
<td>1.7850</td>
</tr>
<tr>
<td>Standard deviation of $\sigma_z$</td>
<td>–</td>
<td>0.3804</td>
<td>0.5576</td>
<td>0.5124</td>
<td>0.8561</td>
<td>1.0520</td>
</tr>
<tr>
<td>Maximum of $\sigma_z$</td>
<td>–</td>
<td>2.5776</td>
<td>2.6454</td>
<td>2.8870</td>
<td>4.2350</td>
<td>7.1849</td>
</tr>
<tr>
<td>Minimum of $\sigma_z$</td>
<td>–</td>
<td>1.2873</td>
<td>0.6941</td>
<td>0.9600</td>
<td>0.2156</td>
<td>0.3111</td>
</tr>
<tr>
<td>Failed Shapiro-Wilk test (95% confidence)</td>
<td>–</td>
<td>None</td>
<td>None</td>
<td>2%</td>
<td>10%</td>
<td>15%</td>
</tr>
</tbody>
</table>

**TABLE 3—Summary statistics of the standard deviation for *z*-values for Douglas fir from Rotoehu***

<table>
<thead>
<tr>
<th>Group</th>
<th>All</th>
<th>Height</th>
<th>Radial position</th>
<th>Mean stiffness</th>
<th>Tree</th>
<th>Tree and radial position</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of groups</td>
<td>1</td>
<td>5</td>
<td>5</td>
<td>8</td>
<td>18</td>
<td>18</td>
</tr>
<tr>
<td>Average number per group</td>
<td>176</td>
<td>35.20</td>
<td>35.20</td>
<td>22.13</td>
<td>9.78</td>
<td>9.78</td>
</tr>
<tr>
<td>Mean of $\sigma_z$</td>
<td>1.3106</td>
<td>1.2952</td>
<td>1.3330</td>
<td>1.3008</td>
<td>1.2150</td>
<td>1.2150</td>
</tr>
<tr>
<td>Standard deviation of $\sigma_z$</td>
<td>–</td>
<td>0.1589</td>
<td>0.1372</td>
<td>0.3626</td>
<td>0.3708</td>
<td>0.3708</td>
</tr>
<tr>
<td>Maximum of $\sigma_z$</td>
<td>–</td>
<td>1.5378</td>
<td>1.5372</td>
<td>1.9959</td>
<td>1.9100</td>
<td>1.9100</td>
</tr>
<tr>
<td>Minimum of $\sigma_z$</td>
<td>–</td>
<td>1.1516</td>
<td>1.2039</td>
<td>0.8644</td>
<td>0.7600</td>
<td>0.7600</td>
</tr>
</tbody>
</table>

![FIG. 5–*Pinus radiata* $z$-values (GPa) by shipment](image)
The margins of error (Equation 1) for different sample sizes, using coefficients of variance of the order of those found for small clear specimens (i.e., irrespective of species), are plotted in Fig. 6 and 7.

**TABLE 4—Summary statistics of the standard deviation for $z$-values for Douglas fir from Tapanui**

<table>
<thead>
<tr>
<th>Number of groups</th>
<th>All</th>
<th>Radial position</th>
<th>Mean stiffness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average number per group</td>
<td>101</td>
<td>50.5</td>
<td>16.83</td>
</tr>
<tr>
<td>Mean of $\sigma_z$</td>
<td>1.4978</td>
<td>1.4646</td>
<td>1.5983</td>
</tr>
</tbody>
</table>

**DISCUSSION**

From Fig. 1 it was evident that the distribution of differences between pair-wise stiffness measurements ($z$) in *P. radiata* was independent of mean stiffness for values from 4 to 14 GPa. A similar result (Fig. 2) seemed valid for Douglas fir even though there was a slight tendency for decreasing differences with increasing mean stiffness. This was most likely an effect of the few data available for Douglas fir, which was also indicated by the overall mean difference being negative. In effect, the data did not comply with the normality assumption, or there was some sort of consistent lean in the investigated stands (both stands were situated on quite steep slopes). With respect to radial position there seemed to be no effect on the $z$-values for *P. radiata* or for Douglas fir (Fig. 3 and 4). In combination, the independence of $z$-values from radial position and mean stiffness indicated that it was fair to assume that the $z$-values were also independent of individual tree effects (within groups).
It was evident that in *P. radiata* there was variation between shipments (Fig. 5), with some varying widely, and again others with mostly negative differences (similar to the Douglas fir shipments). Making *a priori* assumptions of standard deviations less than the average found in this study therefore seems to be unwise, as some trees and stands showed considerably more residual within-tree variation.

From Tables 2, 3, and 4 it was evident that the variation in $z$-values, regardless of how the observations were grouped, seemed to be quite stable within species. For *P. radiata* the mean standard deviation was around 2 GPa with a standard deviation around the mean of 0.5 GPa, except when the observations were extensively divided into groups, i.e., few observations in each group. The latter corresponded with an increasing number of failed Shapiro-Wilk tests, indicating that the intensive grouping to some extent violated the normality assumption upon which the analyses were based. For Douglas fir there was also a difference between the shipments, with a standard deviation at Rotoehu of 1.3 and Tapanui of 1.5. However, within shipments the amount of variation was very similar across the different groups. The *P. radiata* data were also analysed through a standard analysis of variance (PROC GLM of SAS 8.2) with shipment, tree, and ring position as independent variables (not reported here). This analysis gave a residual standard deviation of 2.11 GPa. However, this must be examined with some caution, as the data did not comply fully with the assumptions of an ANOVA.

The residual within-tree variation in stiffness of small clear specimens for both species expressed as coefficient of variance varied from 7% to 32%. The margin of error for these values using two small clear specimens, for example, was between 40% and 140% (Fig. 6 and 7). In other words, estimating the mean stiffness of an individual tree based on two outer-wood small clear specimens extracted at breast height may give very faulty estimates.
— typically from 40% difference from the actual mean. However, simply by using four small clear specimens (e.g., cruciform sampling) the margin of error under the best conditions was reduced to under 10%. Increasing the number of samples even further to 16 brings the margin of error to between 5% and 12%. Note that all the above assume that the small clear specimens were extracted at the same growth ring from the pith and at the same height.

**CONCLUSION**

The within-tree variation of stiffness measured by small clear specimens recovered from the same growth ring on opposing radii at breast height, expressed as coefficients of variance, ranged from 8% to 32% for *P. radiata* and from 7% to 13% for Douglas fir. Using two small clear specimens per tree, the associated margin of error for estimates of individual tree mean stiffness (with 95% confidence) ranged from 40% to 140% for *P. radiata* and 40% to 60% for Douglas fir. Using four small clear specimens per tree, the margin of error reduced to between 10% and 40% for *P. radiata* and 10% to 20% for Douglas fir. The number of small clear specimens required to achieve a reliable estimate of the stiffness of individual trees depends on the allowable margin of error. It seems untenable to use fewer than four small clear specimens extracted cruciformly at the same ring and longitudinal position, but using more than eight seems excessive.

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