SITE INDEX CURVES FOR *PINUS NIGRA* GROWN IN THE SOUTH ISLAND HIGH COUNTRY, NEW ZEALAND

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ABSTRACT

Height/age equations were derived for *Pinus nigra* Arn. subsp. *laricio* (Poiret) Maire (Corsican pine) growing in the South Island high country of New Zealand. These equations can be used to predict height growth, including site index (dominant height at age 40), when provided with a measure of height and age. The equations were derived from dominant height data obtained from 30 stands covering the range of rainfall zones typical of the region. Because of the species' monocyclic growth pattern it was possible to measure annual heights for each tree. A variety of sigmoidal height/age curves were tested, with the Hossfeld and Chapman-Richards equations performing best. Anamorphic and polymorphic forms of these models were tested using cross-validation. The simpler anamorphic forms of the equations gave better results than the more complex polymorphic forms. The two methods of fitting the equations that were compared—one treating site index as a fixed effect, the other as a random effect—gave almost identical results. The shape parameters of the growth curves were not related to altitude or rainfall, but site index was positively correlated with rainfall, thus allowing formulation of a predictive equation.

Keywords: height/age curves; height growth; site index; rainfall; *Pinus nigra.*

INTRODUCTION

The long-term sustainability of traditional extensive pastoralism in the South Island high country is being questioned (Hughes 1991). Some introduced forest plantation species grow well in parts of the high country (Ledgard & Belton 1985) and forestry is increasingly being regarded as a viable land-use for both commercial and sustainable land-use reasons (Belton 1991). The main commercial forest species are *Pseudotsuga menziesii* (Mirb.) Franco (Douglas-fir) in the moister areas (>800 mm rainfall) and *Pinus nigra* and *P. ponderosa* P. et C. Lawson (ponderosa pine) in the drier areas (<800 mm) (Ledgard 1994).

A requirement for evaluating the potential of forestry in any region is an ability to estimate productivity. Site index, defined as the average height of dominant or codominant trees in a stand at a specified age, is an important indicator of site productivity. For *P. nigra* growing in New Zealand, site index is defined as the predominant mean height (PMH) at age 40. Goulding (1995) defined PMH as the average height of the tallest tree, free of malformation, in each 0.01-ha plot within a stand. Height/age equations, which can be used to predict height growth and site index when provided with a measure of height and age, have been derived for *P. nigra* and are reported here.

METHODS

Data Collection

The height/age data used in this paper were originally obtained for the purpose of seeing whether internode length could be used as an index of growth for *P. nigra* (H. Spiering & N. Ledgard, pers. comm.). Thirty Corsican pine stands were selected for sampling. Although the number of stands available was limited, an attempt was made to cover as wide a range of rainfall zones and tree stockings as possible (Table 1). Annual rainfall varied from less than 400 mm to 1600 mm, and altitude ranged from 350 to 820 m. Stockings were from 520 to 3360 stems/ha, and the age range was 13 to 54 years with 73% of the samples aged between 25 and 40 years. Within each stand, five 0.01-ha plots were established in areas of trees considered to have "average" stocking. The tallest straight-stemmed tree in each plot was selected for detailed measurement of height growth. Each sample tree was climbed and the distance (internode) between every whorl of branches was measured. In this way, every year's height growth from time of planting could be determined, as *P. nigra* has a monocyclic growth pattern (producing one internode, or whorl of branches, annually). Internode lengths from the terminal bud to ground level were summed to give tree height. The average of the five trees in each stand was used as an estimate of PMH. A total of 146 trees were measured.

TABLE 1–Summary statistics of the 30 *Pinus nigra* stands sampled.

	Mean	Standard deviation	Minimum	Maximum
Age (years)	30.4	9.6	13	54
Site index (H_{40}) (m)	17.2	3.1	12.8	23.6
Stocking (stems/ha)	1676	760	520	3360
Altitude (m)	600	115	350	820
Mean annual rainfall (mm)	876	368	380	1600

Height/Age Growth Functions

A number of yield equations were tested for their suitability for modelling the height growth of *P. nigra* (Table 2). All the equations were three-parameter sigmoidal curves which predicted mean height in metres, H, as a function of T, the age in years. All had zero intercept, an upper asymptote, *c*, and two other parameters, *a* and *b*, controlling the shape and slope. The logistic equation is one of the most widely used sigmoidal curves. In the current analyses, it was modified to have a zero intercept. The Hossfeld equation was originally proposed for describing tree growth as early as 1822 (Peschel 1938). The Weibull equation is simply the cumulative form of a widely used probability distribution function that has proved to be a good model of tree growth (Yang *et al.* 1978). The Chapman-Richards equation (Richards

1959) is one of the most commonly used forestry growth equations, as is the Schumacher equation (Schumacher 1939). Woollons & Wood (1992) compared most of these equations on height measurements for a variety of species growing in New Zealand, and found that the Schumacher and Weibull equations generally performed best.

TABLE 2–Standard forms of the height/age equations tested.

Logistic	$H = \dfrac{c(1 - e^{-aT})}{1 + e^{-b-aT}}$
Hossfeld	$H = \dfrac{cT^b}{a + T^b}$
Weibull	$H = c(1 - e^{-aT^b})$
Chapman-Richards	$H = c(1 - e^{-aT})^{1/b}$
Schumacher	$H = ce^{(-aT^b)}$

Methods of Fitting Equations

One commonly used method of fitting height/age equations is the parameter prediction method (Clutter *et al.* 1983). This involves several steps. Firstly, a height/age function is fitted to each individual tree or plot, and a site index is obtained using the fitted function. Regression equations relating the function parameters to site index are then derived. These are substituted back into the height/age function, resulting in an equation for predicting height as a function of site index and age. An obvious improvement of this method is to fit the complete model with parameters expressed as functions of site index, directly to the data, rather than using a multi-step process. This approach has been used by, for example, García (1983) who used a differential equation form of the Chapman-Richards model in which one or more parameters were specific to each plot (local parameters) while others were common to all plots (global parameters). He developed a maximum likelihood procedure for estimating the parameters. The disadvantage of this general approach is that the parameter estimation procedure is complex, and involves large numners of local parameters. However, with modern regression software, this is not necessarily a major problem, and the approach was adopted for this study.

The following steps were performed to fit the equations in Table 2. The equations were firstly reparameterised by using site index (H_{40}, PMH at age 40) instead of the asymptote parameter, *c*. This had two advantages. Firstly, expected-value parameters such as H_{40}, which correspond to the fitted value of the response variable at a particular age, generally have good estimation properties (Ratkowsky 1990). Secondly, the parameter prediction method required expressing one or more parameters as a function of H_{40}, and the simplest means of achieving this was to use H_{40} as one of the parameters. It was also felt desirable for all parameters to have approximately symmetrical distributions. The equations were therefore fitted to each individual tree, and the distributions of the parameters examined. For all models, the parameter *b* was reasonably symmetrically distributed, but the parameter *a* was highly skewed, and was therefore replaced by *a′*, its logarithm, or for the Schumacher equation, its inverse. The reparameterisations are shown in Table 3.

The next step was to determine whether anamorphic or polymorphic forms of the equations should be used. In anamorphic equations, the height of any tree at one age is a

TABLE 3–Reparamaterisations of the height/age equations

Logistic	$H = H_{40}\left[\frac{1+e^{-b-e^{a'}40}}{1+e^{-b-e^{a'}T}}\right]\left[\frac{1-e^{-e^{a'}T}}{1-e^{-e^{a'}40}}\right]$
Hossfeld	$H = H_{40}\left[\frac{T}{40}\right]^{b}\frac{e^{a'}+40^{b}}{e^{a'}+T^{b}}$
Weibull	$H = H_{40}\frac{1-e^{-e^{a'}T^{b}}}{1-e^{-e^{a'}40^{b}}}$
Chapman-Richards	$H = H_{40}\left[\frac{1-e^{-e^{a'}T}}{1-e^{-e^{a'}40}}\right]^{1/b}$
Schumacher	$H = H_{40}e^{\frac{1}{a'}\left[\frac{1}{40^{b}}-\frac{1}{T^{b}}\right]}$

constant proportion of the height of another at the same age (Clutter *et al.* 1983). If this is not so, the equations are termed polymorphic. A number of studies have suggested that polymorphic curves are better suited to modelling height growth than anamorphic curves (Bailey & Clutter 1974; Devan & Burkhart 1982). The equations in Table 3 are anamorphic if the shape and slope parameters (a' and b) are held constant, and polymorphic if they are varied with H_{40}. One method of obtaining a polymorphic family of curves is to allow each parameter to vary monotonically with site index (Rennolls 1995). Given the symmetrical distributions of the parameters, expressing them as linear functions of site index was considered an appropriate way of obtaining polymorphic curves. For example, the polymorphic form of the Hossfeld equation was fitted using:

$$H = H_{40}\left[\frac{T}{40}\right]^{b+qH_{40}}\frac{e^{a'+pH_{40}}+40^{b+qH_{40}}}{e^{a'+pH_{40}}+T^{b+qH_{40}}}$$

The two additional parameters, p and q, allow the original a' and b to vary linearly with site index.

Two methods of fitting the equations were tested. The first method treated site index as a fixed effect estimated for each tree. The SAS nonlinear regression procedure NLIN (SAS Institute Inc. 1989) was used with dummy variables representing each individual tree. Each regression involved estimating 147 site index dummy parameters (one for each tree) plus the additional two (anamorphic equations) or four (polymorphic equations) shape and slope parameters. The second method utilised recently developed algorithms for fitting random coefficient nonlinear models (e.g., Sheiner & Beal 1985; Lindstrom & Bates 1990) as implemented in the SAS macro NLINMIX (Littell *et al.* 1996). Site index, H_{40}, was treated as a random coefficient, varying between trees, while the remaining parameters were treated as fixed effects.

The accuracy of the five models, the two fitting methods, and the anamorphic/polymorphic forms, were assessed using the mean square error, and "leave-out-one" cross-validation which is a standard tool for estimating prediction error (e.g., Stone 1974; Allen 1974). This involved excluding an individual tree from the data, fitting the model to the remaining trees, and comparing height predictions using the resulting parameter estimates, with actual heights for the excluded tree. This procedure was performed for each tree in turn, and the prediction errors were examined to assess bias and to measure the precision of each method. Because cross-validation compares predictions against data not utilised in obtaining them, it gives a more realistic measure of prediction error than do the model residuals.

Obtaining Site Index from Height/Age Equations

In the anamorphic forms of the equations, the input variable site index, H_{40}, is allowed to vary between sites, while the shape and slope parameters a' and b remain constant. Thus, by algebraic rearrangement of the equations, site index can be expressed as a function of height, H, at a given age, T. For example, for the Hossfeld equation, the site index can be found using:

$$H_{40} = H \left(\frac{40}{T} \right)^{b} \frac{e^{a'} + T^{b}}{e^{a'} + 40^{b}}$$

Heights at any other age can be readily predicted by replacing 40 in this equation with the desired age.

For the polymorphic equations, an algebraic rearrangement for obtaining site index from a given H and T is not possible, and an iterative procedure is required. For example, for the Hossfeld equation, a starting value for H_{40} of say, 17, is inserted into the right-hand side of the following expression to obtain an improved value of H_{40}, and the procedure is repeated until sufficient accuracy is achieved:

$$H_{40} := H \left(\frac{40}{T} \right)^{b + qH_{40}} \frac{e^{a' + pH_{40}} + T^{b + qH_{40}}}{e^{a' + pH_{40}} + 40^{b + qH_{40}}}$$

Generally only three or four iterations are required. Again, heights at ages other than 40 can be predicted by inserting the appropriate age into this equation.

Results

Comparison of Growth Functions

The fixed and random coefficient methods of fitting the equations produced almost identical fitted coefficients. Only the results of the fixed effect method are therefore presented. Using the mean square error to compare the fit of the equations, it was apparent that the Hossfeld and the Chapman-Richards were the best fitting equations (Table 4). These were followed by the Weibull and the Schumacher equations, with the logistic equation having by far the poorest fit.

An examination of the mean error in prediction of H_{30}, available for 84 of the trees in the data set, against the age of the height measurement used to predict H_{30} indicated that none

TABLE 4–Residual error mean squares of anamorphic and polymorphic forms of the growth equations.

Model	Anamorphic	Polymorphic
Logistic	0.1897	0.1896
Hossfeld	0.1291	0.1251
Weibull	0.1373	0.1353
Chapman-Richards	0.1293	0.1258
Schumacher	0.1390	0.1281

of the models was completely free of bias across all ages (Fig. 1). The Chapman-Richards and the Hossfeld models had an almost identical pattern of bias, with a tendency to under-predict H_{30} from measurements below age 10 years, but virtually no bias for measurement ages greater than 10 years. The behaviour of all the other models was generally inferior to that of these two models.

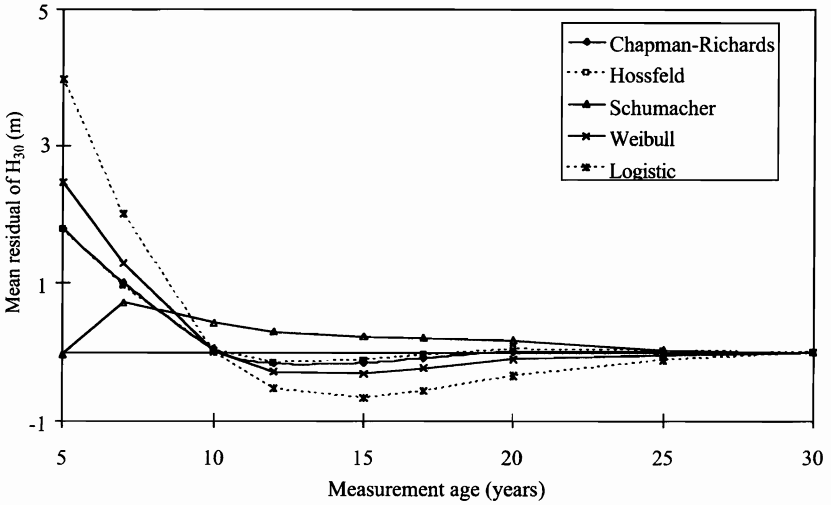

FIG. 1–Mean residual (actual – predicted) of H_{30} *v.* age of height measurement used for prediction (anamorphic models).

The polymorphic forms of the equations produced some reduction in mean square error compared with the anamorphic forms (Table 4). However, because the anamorphic and polymorphic forms have different numbers of parameters, the mean square error does not provide a reliable comparison. A more realistic comparison of the two forms was provided by cross-validation which is a standard tool for estimating prediction errors. Cross-validation was applied to the two best growth equations (Table 5). Somewhat surprisingly,

TABLE 5–Cross-validation tree height root mean square errors for the Hossfeld and Chapman-Richards equations.

Age		Hossfeld		Chapman-Richards	
Measurement	Prediction	Anamorphic	Polymorphic	Anamorphic	Polymorphic
5	20	2.46	3.01	2.47	3.39
	30	3.85	3.92	3.86	4.23
	40	3.17	3.55	3.17	3.98
10	20	1.10	1.23	1.09	1.26
	30	2.14	2.14	2.14	2.15
	40	2.22	1.86	2.21	1.88
15	20	0.47	0.46	0.47	0.47
	30	1.15	1.27	1.16	1.24
	40	1.50	1.57	1.52	1.57
20	30	0.62	0.82	0.62	0.78
	40	0.90	1.31	0.91	1.30
Average		1.78	1.92	1.78	2.02

the anamorphic forms of the equations generally gave lower prediction errors than the more complex polymorphic forms. The cross-validation also confirmed that the anamorphic forms of the Hossfeld and Chapman-Richards models gave almost equally good results.

In summary, the anamorphic forms of the Hossfeld and the Chapman-Richards equations gave the best results. These equations are shown below in the form in which they will most often be used, for predicting height H_2 at age T_2 from a measure of height H_1 at age T_1. Their coefficients are given in Table 6. Hossfeld height/age curves for a range of site indices are shown superimposed on the actual height data in Fig. 2.

Hossfeld: $$H_{T_2} = H_{T_1} \left[\frac{T_2}{T_1}\right]^b \frac{e^{a'} + T_1^{\,b}}{e^{a'} + T_2^{\,b}}$$

Chapman-Richards: $$H_{T_2} = H_{T_1} \left[\frac{1 - e^{-e^{a'} T_2}}{1 - e^{-e^{a'} T_1}}\right]^{1/b}$$

TABLE 6–Fitted coefficients for the Hossfeld and Chapman-Richards equations. Standard errors of coefficients are in parentheses.

	a′	b
Hossfeld	5.532 (0.011)	1.640 (0.007)
Chapman-Richards	–3.061 (0.009)	5.532 (0.003)

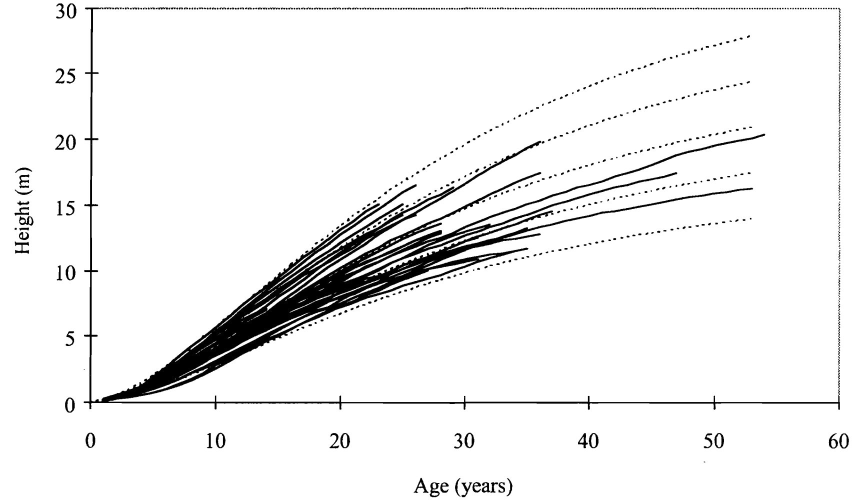

FIG. 2–Hossfeld height/age equations superimposed on actual data. Each data line consists of the mean height of an individual stand of five trees plotted up to the age of the youngest tree.

Accuracy of Predictions

The results of the cross-validation of the anamorphic Hossfeld model were further examined to assess the bias in more detail, and to estimate the precision of height predictions across a range of ages. Mean prediction errors, and standard deviations of prediction errors, were obtained for H_{20}, H_{30}, and H_{40}. The mean errors of prediction (Fig. 3) again showed the tendency to under-predict from height measurements below age 10 years. There appeared to be a tendency to over-predict H_{40} for measurement ages between 8 and 25 years. However, as only 21 trees were old enough to provide H_{40} values, compared with 84 trees for H_{30} and 131 trees for H_{20}, not too much weight should be placed on the H_{40} prediction errors. Very similar results were obtained for the Chapman-Richards model.

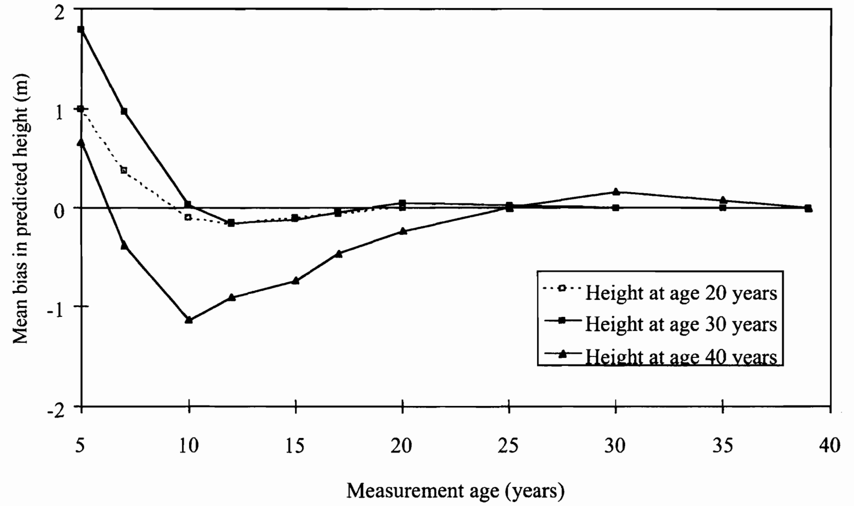

FIG. 3–Mean bias of cross-validation predictions of H_{20}, H_{30}, and H_{40}, plotted against the age of the height measurement used to obtain the predictions, using the anamorphic Hossfeld model.

The standard deviation in prediction errors (Fig. 4) provided an indication of the likely precision of height predictions using the Hossfeld equation. Cross-validation was performed for individual trees, and also for stands. In the latter, the model was fitted using stand mean heights up to the age of the youngest sample tree in the stand, rather than individual tree values. The standard deviations of prediction errors were somewhat lower for stand predictions than for individual tree predictions, particularly for early prediction ages. Site index for stands predicted from height at age 10 years had a standard deviation of 1.3 m, and at 20 years the standard deviation was 1.0 m. The figures for individual trees were 2.0 and 0.9 m, respectively.

Effects of Rainfall, Stocking, and Altitude on Growth Curves

To determine whether site index, or the shapes of the height/age curves, could be related to any site factor, correlations were obtained between the three parameters of the Hossfeld equations fitted to each stand, and rainfall, stocking, and altitude (Table 7). Only the correlation between rainfall and site index was significant ($r = 0.68$, $p < 0.01$). This demonstrated that the shapes of the growth curves were not influenced by any of these factors, but that site index increased with rainfall. The following linear regression equation

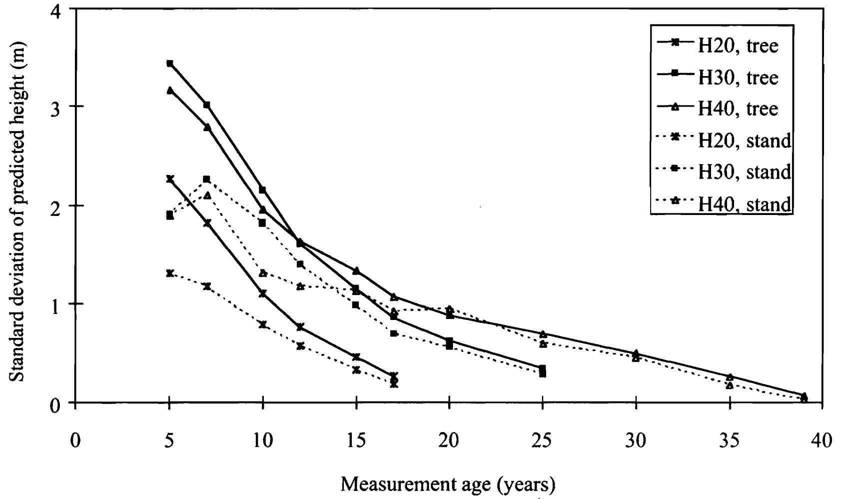

FIG. 4—Standard deviation in individual tree and stand cross-validation predictions of H_{20}, H_{30}, and H_{40}, plotted against the age of the height measurement used to obtain the predictions, using the anamorphic Hossfeld model.

TABLE 7—Correlations between the parameters of the Hossfeld model fitted to individual stands, and rainfall, stocking, and altitude.

Parameter	Rainfall	Stocking	Altitude
H_{40}	0.68*	–0.08	0.06
a′	0.12	0.05	–0.25
b	0.17	0.28	–0.24

* significant at p=0.01

linking site index to rainfall was derived (standard errors of coefficients are given in brackets; residual standard deviation of equation is 2.1 m):

$$H_{40} = 11.8 + 0.0061 \times \text{rainfall}$$
$$(1.0) \quad (0.0011)$$

DISCUSSION

In this study, a number of steps were followed to obtain height/age equations for South Island high country *P. nigra*. These included identifying the best form of growth function, and determining whether anamorphic or polymorphic equation forms should be used. Two methods of estimating the model coefficients were also compared.

The choice of growth function is of some importance if biased height predictions are to be avoided (Fig. 1). All five sigmoidal growth functions tested tended to under-predict future height from a height measurement made at less than about 10 years. This bias was smallest for the Schumacher function, but it remained evident at a greater age than happened with the Hossfeld and Chapman-Richards curves which overall gave the best results. Both were free from bias for measurements at 10 years of age or greater. Although both equations gave equally good results, it is suggested that to avoid future confusion, the Hossfeld equation (which had a marginally lower mean square error) be adopted as the standard height/age equation for South Island high country *P. nigra*.

Although stands can undoubtedly follow different-shaped growth trajectories, no advantage was found in using polymorphic rather than simpler anamorphic growth curves. Polymorphic curves were accommodated in this study by expressing the shape and slope parameters as linear functions of site index. The results indicated that the shapes of individual growth curves were not related to site index. It is possible that a different method of introducing polymorphism, e.g., by relating shape parameters to the asymptote, would give a superior result. However, as an important use of this height/age curve is to predict future stand height at ages similar to the site index base age of 40 years, it appears that anamorphic equations are the most suitable in practice.

The two methods of parameter estimation used in this study, one treating site index as a fixed effect and the other as a random effect, gave virtually identical results. This may have been because of the unusually complete nature of the data, with annual height measurements available for each tree from planting. As the fixed effects method can be fitted using any standard nonlinear regression programme, this would be the recommended procedure for similar data. The main problem with this approach is that it requires fitting an equation with parameters for each individual tree or stand. In this case, nearly 150 parameters were required. Nevertheless, using the parameterisations in Table 3, all equations converged rapidly.

The significant correlation between rainfall and site index was expected, as in Ledgard & Belton's (1985) study of the influence of site factors on exotic tree growth in the Canterbury high country the only consistently positive relationship was with rainfall.

Using branch whorl heights to provide height/age data proved to be straightforward, and the method could be applied to other species with similar monocyclic growth patterns. Suitable species growing in the region would include *P. ponderosa*, *P. contorta* Loudon, and possibly *Pseudotsuga menziesii*.

Previous analysis has shown a good relationship between sections of internodes and PMH at age 40 (H_{40}) (Spiering & Ledgard, pers. comm.). For example, the correlation between the sum of five internodes above a starting point 2.0 m above ground level and H_{40} was 0.88, whilst the relationship with three internodes starting at 1.4 m was 0.80. The idea of a site growth index based on periodic height increments (an "internode index") measured in the lower portions of a tree is not new (Economou 1990), and it could be a much simpler and more readily obtained measure for comparing forest productivity between sites, than the accepted site index of height at age 40.

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