SAMPLING PROCEDURES FOR ESTIMATING FOREST BIOMASS IN THE PURUKI WATERSHED

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ABSTRACT

Assuming that there are no biases in the selection of bounded plots or trees to be biomassed, the error associated with estimates of stand biomass consists of the error associated with sampling trees for biomass and the error associated with the variability of stand characteristics between bounded plots. At Puruki, the error associated with sampling trees for biomass is generally the larger. Post-stratification of plots according to altitude had little effect on the percentage error associated with estimates of biomass per hectare, partly because post-stratification reduces only one component of the error and partly because it was often reducing the smaller of the two errors. The percentage error associated with estimates of mean net stem weight increment depended on whether it is appropriate to combine data from both years in deriving a relationship between biomass and diameter over bark at breast height. Cost-effective schemes for sampling trees for biomass assume a linear relationship between a function of biomass and a function of diameter. The trees chosen are generally at the extreme ends of the diameter distribution. This means the assumption of linearity cannot be checked.

Keywords: sampling; forest biomass; ***Pinus radiata***.

INTRODUCTION

The biomass of a forest stand is generally estimated using a sampling procedure consisting of two phases. Firstly, a large sample of trees, usually all the trees within a number of bounded plots, is measured for diameter over bark at breast height (dbhob). Secondly, a small sample of trees, usually not a sub-sample of the first sample, is measured for dbhob and biomass. An equation relating biomass to dbhob, derived from the second sample, is used to estimate the biomass of the first sample of trees, providing an estimate of biomass for the stand. Measurement errors, biases in the selection of plot locations and trees biomassed, together with random sampling errors, control the accuracy of this estimate. Assuming that measurement errors and biases are absent, the accuracy of the biomass estimate for each individual plot depends on the accuracy of the biomass equation. However, if we require an estimate of biomass per hectare for a whole stand, then the accuracy also depends on the variability of basal area per hectare within the first-stage sample.

Data collected at Puruki (38° 26′ S, 176° 13′ E), a 34.4-ha experimental forest about 30 km south-west of Rotorua, are valuable for examining the errors associated with estimates of biomass. Puruki was planted in *Pinus radiata* D. Don in 1973 with a nominal stocking of 2200 stems/ha. The area is divided into three subcatchments which have been subjected to different thinning regimes since 1979 (Table 1). From 1974 till 1979 inclusive, the sampling procedure was to measure the biomass of 15 trees selected throughout the whole catchment, and to measure dbhob on all trees within 30, 0.01-ha, assessment plots. Since 1979, between five and seven trees from each subcatchment have been measured for biomass, and dbhob has been measured on all trees within 30 to 50 assessment plots.

TABLE 1—Nominal stockings for the Puruki subcatchments

Subcatchment	Data (month.year)	Stocking (stems/ha)
Tahi	.73 – 5.79	2200
	5.79 – 8.83	550
	8.83 –	137
Rua	.73 – 4.80	2200
	4.80 –	550
Toru	.73 – 11.81	2200
	11.81 – 9.84	550
	9.84 –	275

The original 30, 0.01-ha, assessment plots were distributed between the three subcatchments in proportion to their area, but randomly located within the subcatchment. When a subcatchment was first thinned, the area of each assessment plot within that subcatchment was increased to 0.04 ha. Subsequently, the total number of assessment plots at Puruki was increased to 50. These extra plots were also randomly located within the subcatchments. Further details on Puruki have been provided by Beets & Brownlie (1987).

Using data collected at Puruki, we examined how the errors associated with estimates of stand biomass varied with stand age and stocking, and how the error estimates can be reduced by post-inventory stratification. We then examined how the errors associated with estimates of annual net biomass increment compared with the errors associated with estimates of biomass. We also used these data to calculate sample sizes needed to estimate stand biomass to within a given accuracy. Finally, we examined the cost-effectiveness of different schemes for sampling trees to be biomassed. In the discussion we present some suggestions for future sampling schemes.

METHODS

Catchment Estimates of Biomass

To investigate the effect of stand age and stocking on the accuracy of stand estimates of biomass, the standard error of per hectare estimates of stem biomass (wood plus bark), branch biomass (live wood plus bark), and foliage biomass was calculated in the following years:

1977 – representing a young stand prior to canopy closure
1979 – representing a young stand where the canopy is generally closed
1982 – (by subcatchment) same stocking in each subcatchment but varying numbers of years since thinning
1985 – (by subcatchment) stands at different stockings.

Both simple random sampling and post-inventory stratification of assessment plots were investigated.

Simple random sampling: Assuming that the biomass of the weighed sample of trees can be predicted from dbhob by the equation:

$$y = b_{0i} + b_{1i}\, d^2 \qquad (1)$$

where d is dbhob at 1.4 m
y is the biomass per tree
b_{0i} and b_{1i} are the least-squares regression coefficients
i is the year of measurement

then the variance of the mean biomass per hectare (Cunia 1985, pp. 16–9) is given approximately by:

$$S_w = \mathbf{b}^t\, \mathbf{S}_{zz}\, \mathbf{b} + \mathbf{z}^t\, \mathbf{S}_{bb}\, \mathbf{z} \qquad (2)$$

where S_w is the variance of mean biomass per hectare
$\mathbf{b}^t = (b_{0i}, b_{1i})$, a vector containing the regression coefficients of Eqn 1
$\mathbf{z}^t = (z_0, z_1)$
z_0 is the mean (over all plots) number of stems per hectare
z_1 is the mean (over all plots) sum of squared diameters per hectare
$\mathbf{S}_{bb}$ is the variance-covariance matrix of $\mathbf{b}$
$\mathbf{S}_{zz}$ is the variance-covariance matrix of $\mathbf{z}$

$\mathbf{z}^t\, \mathbf{S}_{bb}\, \mathbf{z}$ can be thought of as the variance due to errors in sampling trees for biomass, while $\mathbf{b}^t\, \mathbf{S}_{zz}\, \mathbf{b}$ can be thought of as the error involved in sampling the plots.

Post-inventory stratification: Pre-inventory stratification of sample plots according to age, species, treatments, etc., is common but post-inventory stratification is rare, even though it can substantially reduce error. For example, Whyte & Tennent (1975) found that the standard error of basal area estimates could be reduced by a factor of about two by post-stratifying the area according to one or more criteria such as stocking, position on slope, soil type, or year of establishment.

Part of the Puruki catchment was replanted after 1 year because of poor establishment. The poorest establishment was generally at the higher altitudes so it was expected that altitude would be a suitable variable for post-stratifying the area. The high correlation between basal area per hectare and altitude prior to thinning (Table 2) confirmed this expectation.

At Puruki, the altitude of assessment plots ranges from 546 m to 639 m a.s.l. The area was split into 15-m altitude zones using a survey map. Taking into account the number of assessment plots within in each altitude zone and the variation in mean basal area per hectare within a zone, it was decided that suitable zones for post-stratifying

TABLE 2—Correlation coefficients, r, between plot basal area per hectare and altitude. Number of plots, n, is given in parentheses

Year	Whole catchment		Tahi		Rua		Toru	
	r	n	r	n	r	n	r	n
1975	–0.58	(30)**						
1976	–0.66	(30)**						
1977	–0.70	(30)**						
1978	–0.73	(30)**						
1979	–0.69	(30)**						
1980			0.07	(10)	0.07	(7)	–0.76	(13)**
1981			0.11	(10)	–0.24	(7)	–0.67	(13)*
1982			0.13	(10)	–0.23	(7)	–0.04	(13)
1983			0.32	(20)	–0.17	(7)	0.06	(13)
1984			0.55	(20)*	–0.05	(10)	0.17	(20)
1985			0.57	(20)**	0.02	(10)	0.46	(20)*
1986			0.53	(20)**	0.07	(10)	0.55	(20)*

* r is significantly different from zero at $p \leq 0.05$
** r is significantly different from zero at $p \leq 0.01$

were 525–585 m, 585–600 m, 600–615 m, and 615–645 m. Using these zones, the errors associated with estimates of biomass per hectare were re-calculated for 1977 and 1979 using the method of Cunia (1986, pp. 67–70).

Catchment Estimates of Annual Net Biomass Increment

Estimates of net biomass increment were calculated from estimates of biomass at two different times. To examine how errors associated with estimates of net biomass increment compared with the errors associated with estimates of biomass, we calculated the mean net stem weight increment per hectare for two periods, 1977–78 and 1982–83, using the method of Cunia (1986, pp. 24–30). We assumed simple random sampling of bounded plots, and considered two different assumptions about the biomass equation, namely,

- That different biomass equations of the form of Eqn 1 apply to each year, and that these equations are independent,
- That one biomass equation can be used for both years.

Assuming that different biomass equations apply, the mean net stem weight increment per hectare is given by:

$$W_g = \mathbf{B}^t \mathbf{Z} \qquad (3)$$

and the variance, S_g, of the estimate of mean net stem weight increment per hectare, is given by:

$$S_g = \mathbf{B}^t \mathbf{S}_{ZZ} \mathbf{B} + \mathbf{Z}^t \mathbf{S}_{BB} \mathbf{Z} \qquad (4)$$

where $\mathbf{B}^t = (b_{02}, b_{12}, -b_{01}, -b_{11})$

$\mathbf{Z}^t = (z_{02}, z_{12}, z_{01}\ z_{11})$

b_{01} and b_{11} are the coefficients of the biomass equation at time 1

b_{02} and b_{12} are the coefficients of the biomass equation at time 2

z_{01} and z_{02} are the mean number of stems per hectare at times 1 and 2 respectively.

z_{11} and z_{12} are the mean per hectare values of d^2 at times 1 and 2 respectively

d is dbhob

$\mathbf{S}_{BB}$ is the variance-covariance of $\mathbf{B}$

$\mathbf{S}_{ZZ}$ is the variance-covariance of $\mathbf{Z}$

Cunia (1986) commented that using two regression equations yields poor estimates and that it is better to use one regression equation if possible.
In this case:

$$\mathbf{B}^t = (b_0, b_1)$$

where b_0, b_1 are the coefficients of the combined regression equation

$$\mathbf{Z}^t = (z_0, z_1)$$

where $z_0 = z_{02} - z_{01}$

$z_1 = z_{12} - z_{11}$

Number of Plots Needed to Estimate Basal Area per Hectare Within a Given Accuracy

The error associated with catchment estimates of biomass can be improved by reducing the errors associated with either or both sampling stages. In this section we consider methods for improving the accuracy of estimates of basal area per hectare.

Assuming the size of assessment plots is not changed, we estimated the number of assessment plots which should have been measured each year within the whole catchment between 1975 and 1979, or within each subcatchment between 1980 and 1985 so that there is a 95% probability of the true mean being within 5% of the estimated mean basal area per hectare. Between 1975 and 1979 we calculated the number of plots needed, assuming simple random sampling and stratified random sampling with both proportional allocation and optimum allocation. Stratification was by altitude, the strata being 525–570 m, 570–585 m, 585–600 m, 600–615 m, 615–645 m.

Between 1980 and 1985 we considered only simple random sampling. Formulae used are given by Freese (1962, p. 26 and p. 34).

Cost-efficient Sampling Scheme for Sampling Trees for Biomass

Marshall & Demaerschalk (1986) presented a methodology for determining cost-efficient sampling distributions for simple linear regression problems. We used their computer program to determine the number of trees which should have been sampled for foliage biomass in 1979 and 1985 (by subcatchments) in order to estimate the foliage biomass per hectare on a plot with a 95% probability that the true value was within ± 5% of the estimated value, assuming that plot diameters were measured without error. For each data set, the linear regression considered was

$$\ln w_i = b_0 + b_1 \ln d_i$$

where w_i is the foliage biomass for tree i
d_i is dbhob for tree i
b_0 and b_1 are the regression coefficients

as this gave reasonably uniform variances. The range of sample tree diameters was divided into seven classes of equal width (on a log-log scale) and the cost of sampling a tree within a class was assumed to be proportional to $d^{1.5}$. Data collected in 1986 indicated that this cost function was realistic.

The number of trees required was calculated for two different sampling schemes –

- A uniform distribution of sample trees
- The most cost-efficient sampling distribution.

RESULTS

Basal area was well correlated with altitude prior to thinning, but negligibly after thinning (Table 2). The decrease in correlation after thinning was due in part to small altitude range within each subcatchment. The correlation increased again in subcatchments Tahi and Toru after extra assessment plots were added. The increased correlation could be attributed to the fact that there were more trees in the new plots prior to thinning. In Tahi, in 1983, the original plots contained 17 to 23 trees while the new plots contained 18 to 37 trees. In Toru, in 1984, the original plots contained 16 to 27 trees while the new plots contained 23 to 35 trees.

The percentage error (standard error / mean) × 100, associated with estimates of mean per hectare biomass of foliage, stem (wood + bark), and branches (live wood + bark) varied between 5% and 25% when simple random sampling was assumed (Tables 3 and 4). There was a tendency for the percentage error to decrease with

TABLE 3—Estimates of variance and percentage error associated with estimates of mean biomass per hectare for the Puruki catchment, assuming simple random sampling and post-inventory stratification of sample plots

Year	Simple random sampling				Post-inventory stratification			
	Mean tonnes /ha	Variance due to biomass trees	Variance due to plots	Error (%)	Mean tonnes /ha	Variance due to biomass trees	Variance due to plots	Error (%)
Foliage								
1977	5.02	0.26	0.14	12.6	4.99	0.26	0.087	11.7
1979	14.67	3.47	0.99	14.4	14.56	3.47	0.55	13.8
Stem								
1977	5.83	0.055	0.26	9.7	5.79	0.055	0.16	8.0
1979	23.33	0.75	2.26	7.4	23.16	0.76	1.25	6.1
Branches								
1977	4.14	0.26	0.12	15.0	4.11	0.26	0.076	14.2
1979	18.11	3.68	2.30	13.5	17.93	3.68	1.29	12.4

Note: Percentage error = (standard error/mean) x 100.

TABLE 4—Estimates of variance and percentage error associated with estimates of mean biomass per hectare for Puruki subcatchments, assuming simple random sampling

Subcatchment	Year	Mean tonnes/ha	Variance due to biomass trees	Variance due to plots	Error (%)
Foliage					
Tahi	1982	12.19	1.68	0.88	13.1
Rua	1982	9.81	3.46	0.40	20.0
Toru	1982	5.79	1.00	0.14	18.4
Tahi	1985	6.88	0.17	0.08	7.3
Rua	1985	13.31	2.98	0.31	13.6
Toru	1985	8.50	0.80	0.08	11.0
Stem					
Tahi	1982	50.50	6.24	8.37	7.6
Rua	1982	38.43	88.20	4.54	25.0
Toru	1982	29.80	5.02	3.08	9.5
Tahi	1985	42.89	0.33	1.24	2.9
Rua	1985	105.06	33.93	16.71	6.8
Toru	1985	52.92	7.26	0.31	5.2
Branches					
Tahi	1982	21.50	1.24	2.26	8.7
Rua	1982	18.75	14.28	1.62	21.3
Toru	1982	9.91	2.89	0.38	18.2
Tahi	1985	18.74	1.46	1.06	8.5
Rua	1985	44.97	82.34	6.63	21.0
Toru	1985	15.22	1.98	0.26	9.8

Note: Percentage error = (standard error/mean) x 100

increasing age but there were no obvious trends with stocking. Any trends are likely to have been masked by the odd high variance due to the inclusion of an "odd" tree in the sample biomassed. This certainly happened in subcatchment Rua, for stems and branches in 1982, and for stems and branches in 1985. In general, the greater proportion of the error was due to sampling trees for biomass, the exceptions being stem biomass prior to thinning and stem biomass in subcatchment Tahi after thinning. While post-inventory stratification of assessment plots by altitude reduced the variance due to sampling of assessment plots by about 40%, the effect on percentage error associated with mean biomass per hectare was much smaller because we have reduced only one component of the error which, for foliage and branches, was the smaller of two components of error.

The percentage error associated with estimates of mean net stem weight increment per hectare depended on whether the biomass data were combined to give one equation or whether separate biomass equations were used for each year. The percentage error was three times larger if two equations were used as opposed to one equation. If one equation was used, the percentage error was comparable with those associated with estimates of mean biomass per hectare (Table 5).

TABLE 5—Estimates of variance and percentage error associated with estimates of mean net stem weight increment per hectare at Puruki, assuming simple random sampling

Year and area	Two biomass equations				One biomass equation			
	Mean tonnes /ha	Variance due to biomass trees	Variance due to plots	Error (%)	Mean tonnes /ha	Variance due to biomass trees	Variance due to plots	Error (%)
1977–78								
Puruki	7.36	1.36	2.30	26.0	9.03	0.15	0.36	7.9
1982–83								
Tahi	10.06	11.33	0.26	33.8	11.24	0.85	0.22	9.2
Rua	23.58	102.81	2.00	43.4	13.46	1.96	0.41	11.4
Toru	15.86	12.22	0.75	27.7	9.80	2.78	0.41	18.2

Note: Percentage error = (standard error/mean) x 100.

The number of assessment plots needed to obtain 95% confidence limits within ± 5% of the estimated mean basal area per hectare (Tables 6 and 7) decreased with increasing age. Stratified random sampling reduced the number which needed to be measured in all but one area. The high number of plots needed with proportional allocation was due to a high variance in the lowest altitude zone. Optimum allocation required far fewer plots, particularly in the first few years. The large number of plots needed in 1975 and 1976 was probably due to the fact that diameters were small and many trees had not reached 1.4 m in height. The increase in the number of assessment plots needed in subcatchment Tahi from 1984 onwards appeared to be related to the change in estimated stand variability after the increase in the number of assessment plots.

The computer program of Marshall & Demaerschalk (1986) predicted that with sample trees distributed uniformly over seven diameter classes, between seven and 21

TABLE 6—Number of assessment plots needed to estimate 95% confidence limits for basal area per hectare to within 5% of mean basal area per hectare, assuming simple random and stratified random sampling, and a t value of 1.96. In each year 30 assessment plots were actually measured

Year	Number of plots which should have been measured using:		
	Simple random sampling	Stratified random sampling	
		Proportional allocation	Optimum allocation
1975	1145	1183	684
1976	803	718	449
1977	504	387	291
1978	294	172	157
1979	148	90	88

TABLE 7—Number of assessment plots which needed to be measured in each subcatchment to estimate 95% confidence limits for basal area per hectare to within 5% of mean basal area per hectare

Year	Number of plots by subcatchment		
	Tahi	Rua	Toru
1980	61	30	136
1981	54	28	92
1982	49	33	59
1983	43	30	54
1984	76	26	55
1985	75	23	33
1986	38	22	32

Note: Extra assessment plots included from 1984 (Tahi) and 1985 (Rua and Toru).

trees should have been sampled (Table 8). The number depended on the standard error of the log-log equation. The most cost-efficient sampling strategy did not necessarily reduce the total number of trees which needed to be sampled, but reduced the costs by between 14% and 50%. These cost-efficient samples were weighted towards the smallest diameter class, with the remaining trees allocated to one or two of the three largest diameter classes.

TABLE 8—Relative costs and numbers of sample trees needed for estimating foliage mass of a plot with a 95% probability of being within ±5% of the true value, using a uniform and the most cost-efficient sampling distribution given by the method of Marshall & Demaerschalk (1986)

Year	Catchment	Sampling procedure	Diameter class							Cost ($)
			1 Small	2	3	4	5	6	7 Large	
1979		Uniform	2	2	2	2	2	2	2	1.00
		Cost eff.	9	0	0	0	8	0	0	0.50
1985	Tahi	Uniform	1	1	1	1	1	1	1	1.00
		Cost eff.	3	0	0	0	1	1	0	0.58
1985	Rua	Uniform	2	2	2	2	2	2	2	1.00
		Cost eff.	5	0	0	0	0	3	3	0.86
1985	Toru	Uniform	3	3	3	3	3	3	3	1.00
		Cost eff.	6	0	0	0	0	3	4	0.65

DISCUSSION

Assuming no biases in the selection of sample plots and trees biomassed, and no measurement error, the error associated with estimates of mean biomass per hectare has two components – the error associated with sampling trees for biomass, and the error associated with the variability in basal area between plots within the forest.

The percentage error associated with estimates of mean per hectare biomass of foliage, stems, and branches at Puruki varied between 5% and 25% (Tables 3 and 4). The greater proportion of the error was generally due to sampling trees for biomass, rather than the variability between plots. Consequently, post-stratification according to altitude reduced the percentage error associated with estimates of foliage biomass only slightly (Table 3). Even for stems the drop in the percentage error was small, suggesting that while post-stratification is excellent for reducing the error associated with estimates of basal area per hectare, it is unlikely to be a useful tool in estimating forest biomass.

The percentage error associated with estimates of mean net stem weight increment per hectare depended on whether one or two equations were used in predicting stem biomass from dbhob. If two equations were used the percentage error associated with the estimate of stem biomass increment was approximately three times larger than if one equation had been used (Table 5). If one equation was used the percentage error was of a comparable size to that obtained in estimating biomass. The use of one equation assumes comparability of two sets of biomass. It is necessary to determine, *a priori,* whether data may be combined. For instance, if live branch biomass per hectare is expected to remain constant in a closed stand and no mortality of trees occurs, then combining samples would be inappropriate. On the other hand, in open stands, with increasing live branch biomass over time, combining data may be appropriate.

The sampling scheme at Puruki could have been improved by sampling more trees for biomass as this component generally had the larger variance (Tables 3–5). However, the cost of a sampling scheme is also important. We need to choose an optimum allocation of effort between plot and tree measurements to obtain realistic estimates of stand biomass per hectare. The cost of estimating biomass is highly sensitive to the desired reliability. Doubling the confidence interval would cut sampling sizes and costs by about 75%. Consequently care must be exercised in determining acceptable levels of error in deciding on sampling intensity.

The method of Marshall & Demaerschalk (1986) is an ideal tool for helping to reduce the cost. Cost-efficient sampling schemes can reduce costs without compromising accuracy. However, the most cost-efficient sampling strategy raises problems. Firstly, in closed stands trees in the small suppressed category may be atypical for crown components (e.g., Madgwick 1971). Secondly, the most cost-efficient strategy concentrates sample trees at the extremes of the diameter distribution and it would be impossible to test the hypothesis of linearity between size and biomass. If the relationship was not linear, the estimate of biomass would be biased. This problem could be overcome, at an increased cost, by forcing sampling into small, intermediate, and large diameter classes. Thirdly, repeated sampling could change the stand structure. As can be seen in Table 8, at least 10 trees per subcatchment are required to estimate biomass for a plot consistently to within ± 5% of its true value at the 95% probability level.

The error associated with sampling plots is related to both number and size of plots. Numbers of plots which should have been measured to achieve an estimate of basal area per hectare within ± 5% are indicated in Tables 6 and 7. We do not know how numbers would be affected by a change in plot size. Whyte (1969) examined the variability in basal area per hectare with plot areas of 0.04 ha, 0.1 ha, and 0.16 ha in a thinned (96 stems/ha) and an unthinned (185 stems/ha) stand of *P. radiata* (aged 35 years). In the thinned stand, there was no difference in the standard error of basal area per hectare with plot size. In the unthinned stand there was more variability in the 0.04-ha plots than in the other two sizes. For *P. banksiana* Lamb., Heygi (1973) showed that the coefficient of variation between plots decreases to an asymptote with increasing plot size. He chose the optimum plot size to be the point where the curve of coefficient of variation *v.* plot size tails off. A preliminary field study like Heygi's would give cost-effective cost areas. An alternative is to examine plot sizes/number of trees per plot recommended in the literature. For stem volume, Lees (1967) suggested that, with effective stratification, approximately 30 bounded plots each containing 20–60 trees should be measured in each forest type in order for the 95% confidence limits to be within 10% of the mean volume. Deadman & Goulding (1979) suggested that the plot should contain between 15 and 40 trees. The cost of forest inventory indicates a need for research on efficient plot size.

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