PREDICTING STEM VOLUME TO ANY HEIGHT LIMIT FOR NATIVE TREE SPECIES IN SOUTHERN NEW SOUTH WALES AND VICTORIA

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(Received for publication 11 January 1999; revision 2 June 1999)

ABSTRACT

Trigonometric volume ratio equations, which predict stem volume to any specified top height limit as a percentage of the total stem volume, were developed for 25 native tree species in southern New South Wales and Victoria. When used together with companion total stem volume equations, they allow direct predictions of volume to any height limit. Validation statistics obtained through a resampling approach showed little local bias over any range of relative height and the 95% confidence limits of prediction error were mostly within 10% over the entire range of relative height for all species. These equations will be useful for forest managers when estimating the volumes of multiple logs from the same stem during integrated logging, and in forest inventory where yield estimates by product categories are required.

Keywords: trigonometric volume-ratio equations; *Eucalyptus* spp.; *Corymbia* spp.

INTRODUCTION

Predicting the merchantable volume of a tree is an integral part of forest management. Stems are usually cut to specified log lengths during harvesting, and more than one log may be cut from the same stem for different products. Defect sections, often found in native forest trees of Australia, need to be cut off and discarded before merchantable material is extracted from the stem. With such variations, the ability to predict stem volume to any specified height limit is highly desirable.

Two approaches have been commonly used to predict stem volume to any height limit. The first is to construct taper functions and obtain indirect estimates of sectional volume through integration. The second approach is to develop volume ratio equations which predict the volume to any specified top height limit as a percentage of the total stem volume (e.g., Honer 1964; Cao & Burkhart 1980; Cao *et al.* 1980; Van Deusen *et al.* 1981; Knoebel *et al.* 1984; Avery & Burkhart 1994). When used together with companion total stem volume equations, volume ratio equations allow direct predictions of volume to any height limit. Because the volume ratio equations are fitted to the data to optimise volume estimates, they provide more accurate predictions of volume than taper equations (Matney & Sullivan 1982;

Cao *et al.* 1980). Indeed, a taper equation may predict stem diameters accurately, but still have a large bias in estimating merchantable volume (Newnham 1992).

Recently, total stem volume equations have been developed for 25 native tree species in southern New South Wales and Victoria (Bi & Hamilton 1998). This paper reports the development of volume ratio equations that are to be used together with these total stem volume equations for predicting stem volume to any height limit for these species.

DATA

The data set for this work included taper measurements of 5699 trees from more than 25 species of *Eucalyptus* and *Corymbia* sampled from 24 forest types in southern New South Wales and Victoria. Most of the samples were regrowth trees with a distinguishable top. The sample size ranged from more than 1000 trees to less than 20 trees among the species. For most major commercial species, the sample size was more than 100. For a number of minor species including *E. consideniana* Maiden, *E. maidenii* F. Muell., *E. rubida* Dean & Maiden, *E. angophoroides* R. Baker, and *E. bosistoana* F. Muell. the sample size was less than 20, and so these species were grouped together and coded as *Eucalyptus* spp. The number of sample trees for each species, the diameter at breast height (dbh) and height distribution of samples, the geographical areas and forest types where the samples were taken, and the field measurements of stem taper have been described in detail by Bi & Hamilton (1998).

METHODS

Stem Volume and Volume Ratio Calculations

Smalian's formula was used to calculate underbark sectional volumes, apart from the tip volume which was calculated as a cone (as described by Bi & Hamilton 1998). The sum of sectional volumes of each tree was then taken as the "true" stem volume of the tree. Because the lower stem of most tree species is neiloidal, the sectional volume calculated by Smalian's formula is likely to be higher than the actual volume. Dargavel & Ditchburne (1971) showed an example of over-estimation of less than 3% by Smalian's formula over an approximately 1.5-m butt section of some *Pinus radiata* D. Don trees in comparison with true sectional volume obtained through water displacement methods. However, most of the sample trees in this study were measured at an interval less than 1.5 m over the butt and 1.5 m elsewhere, and so the bias in the sectional volume calculations was minimal.

Since the taper data comprise diameter measurements taken at a number of heights along the stem of each tree, underbark stem volume from ground to each height was calculated by summing the volume of all stem sections below that height, which was then divided by the total underbark stem volume to derive the corresponding volume ratio.

Model Specification

Since volume ratios are proportions of total stem volume, they range from 0 to 1. These proportions are nondecreasing from the base to the top of the tree. Similarly, any height limit (H) can be expressed as relative to the total tree height (TH). Because $0 \leq H \leq TH$, the corresponding relative height, h, being the ratio of H over TH, also ranges from 0 to 1. When expressed as a function of relative height, volume ratio increases from 0 to 1 as relative height increases from 0 to 1. These characteristics of this function are shared by the sine curve between 0 and $\pi/2$, which enables the use of this trigonometric function to construct volume ratio equations in a similar manner to that used to describe relative stem profiles (Bi & Turner 1994).

A number of models were constructed using combinations of four basic components in the form of $\sin(^{\pi}/_{2}\, h^{A})$, by taking A as 1, 1/2, 1/3, and 1/4 respectively. The model involving all four components was

$$R_h = \sin^{a_1}(^{\pi}/_{2}\, h)\, \sin^{a_2}(^{\pi}/_{2}\, h^{1/2})\, \sin^{a_3}(^{\pi}/_{2}\, h^{1/3})\, \sin^{a_4}(^{\pi}/_{2}\, h^{1/4}) \tag{1}$$

where R_h is the volume ratio at relative height h, and a_1, a_2, a_3, and a_4 are coefficients.

These models were linearised through logarithmic transformation and were fitted to the data using least squares regression.

In addition, several other models were fitted to the data using nonlinear least squares regression with multiple initial values to ensure global minimum. These models included the nonlinear model form of Cao & Burkhart (1980),

$$R_h = 1 + a_1 \frac{(1-h)^{a_2}}{TH^{a_3}} \tag{2}$$

the power function of Forslund (1991),

$$R_h = [1-(1-h)^{a_1}]^{a_2} \tag{3}$$

and three other nonlinear model forms from those reviewed by Ratkowsky (1990) but modified for R_h to be between 0 and 1

$$R_h = h^{a_1 + a_2 h} \tag{4}$$

$$R_h = 2^{h^{a_1 + a_2 h}} - 1 \tag{5}$$

and

$$R_h = \frac{2h^{a_1}}{1 + h^{a_2}} \tag{6}$$

To select the best equation form, the six models were compared in terms of their logical consistency, local bias and precision, and overall fit statistics. Volume ratio predicted from Equation 4 was greater than 1 for 25 out of the 26 species over the top 3% to 20% of the stem, an illogical behaviour for a volume ratio model. Equation 5, modified from Equation 4, showed no improvement in the logical behaviour. These two equations were excluded from further model comparisons. The residuals from the remaining four models were divided into 10 even intervals over the entire range of relative height, and within each interval the mean and variance of residuals were calculated for each model and species to evaluate their local bias and precision of volume ratio estimation. Then the squared local bias and the local variance were averaged over the 10 relative height intervals for each model and species for further model comparisons. Fitted values from Equation 1 were back-transformed from logarithm and residuals on the real scale were obtained for this purpose. Finally, a generalised form of R^2 was calculated to show how well the models fitted the data overall:

$$R^2 = 1 - \frac{\sum_{i=1}^{n} (R_{hi} - \hat{R}_{hi})^2}{\sum_{i=1}^{n} (R_{hi} - \bar{R}_h)^2} \tag{7}$$

where R_{hi} and $\hat{R}_{hi}$ are the observed and fitted volume ratios of point i and $\hat{R}_h$ is the mean volume ratio of all data points for each species. Equation 1 had the best performance in terms of local bias and overall fit (Fig. 1). Equation 2 was a close second, but it consistently overestimated volume ratio when relative height approaches zero. Equation 6 was the distant last. Since Equation 1 performed the best among all equation forms evaluated, it was chosen as the equation form for all species.

Regression Analysis

After logarithmic transformation, Equation 1 was fitted to the data for each species using least squares regression to obtain initial parameter estimates and diagnostic plots. Residual plots showed an uneven spread of residuals for all species, with the variance of residuals decreasing with the predicted values. In addition, outliers were clearly present and often several of them were correlated because they were from the same outlying tree. In the presence of heteroscedasticity, studentised residuals and other diagnostic statistics based on homogeneous error variance cannot be employed to help detect outliers. To detect outlying trees in this situation, the ratio between the sum of residuals and the sum of predicted values were calculated for each tree. The distribution of the ratio was examined for each species and two particular values was calculated for discriminating outlying trees: the lower quartile minus 1.5 times the inter-quartile range and the upper quartile plus 1.5 times the inter-quartile range. Trees outside the range of these two values were very extreme points of the distribution as shown by boxplots of Tukey (1977). Most of these trees had some degree of deformity due to the presence of fire scar, knot, bulge, partial death of the stem, coppice, or epicomic growth, reflecting partly the frequent fire disturbance in the regrowth forests where the samples were taken (Bi & Hamilton 1998). The number of outlying trees accounted for between 0% and less than 6% of the sample trees among the species. In total, outlying trees accounted for less than 4% of all trees for all species combined. These outlying trees, if included, would have had an undue influence on parameter estimation, particularly with the presence of multicollinearity in the log-transformed Equation 1. Therefore they were excluded from further analysis.

The initial parameter estimates showed that two of the four coefficients, a_1 and a_3, were negative for each species, and so the two components associated with a_1 and a_3 in Equation 1 became the denominator

$$R_h = \frac{\sin^{a_2}(\pi/2\, h^{1/2})\, \sin^{a_4}(\pi/2\, h^{1/4})}{\sin^{a_1}(\pi/2\, h)\, \sin^{a_3}(\pi/2\, h^{1/3})} \tag{8}$$

It is clear from the above expression that no logical values can be obtained when h equals zero. As h tends to zero, the numerator and the denominator both go towards zero, but with a difference in speed depending on the estimated coefficients. This difference would result in illogical predictions of R_h when relative height is very close to zero. The lower limit of relative height below which the volume ratio equation gives illogical predictions was calculated using numerical methods for each species. This limit of relative height varied from 0.000 001 to 0.007 915 among the 25 species. For most species the limit is below 0.002, i.e., 0.2% of the total tree height. Although not a concern in practical application, it was still desirable to have a limit as low as possible and uniformly applicable to all species. It was identified that the values of the lower limit and the illogical behaviour below it were largely attributable to the lack of data at very low relative heights for some species and the presence of multicollinearity in the log-transformed Equation 1. So for each tree, a volume ratio at a

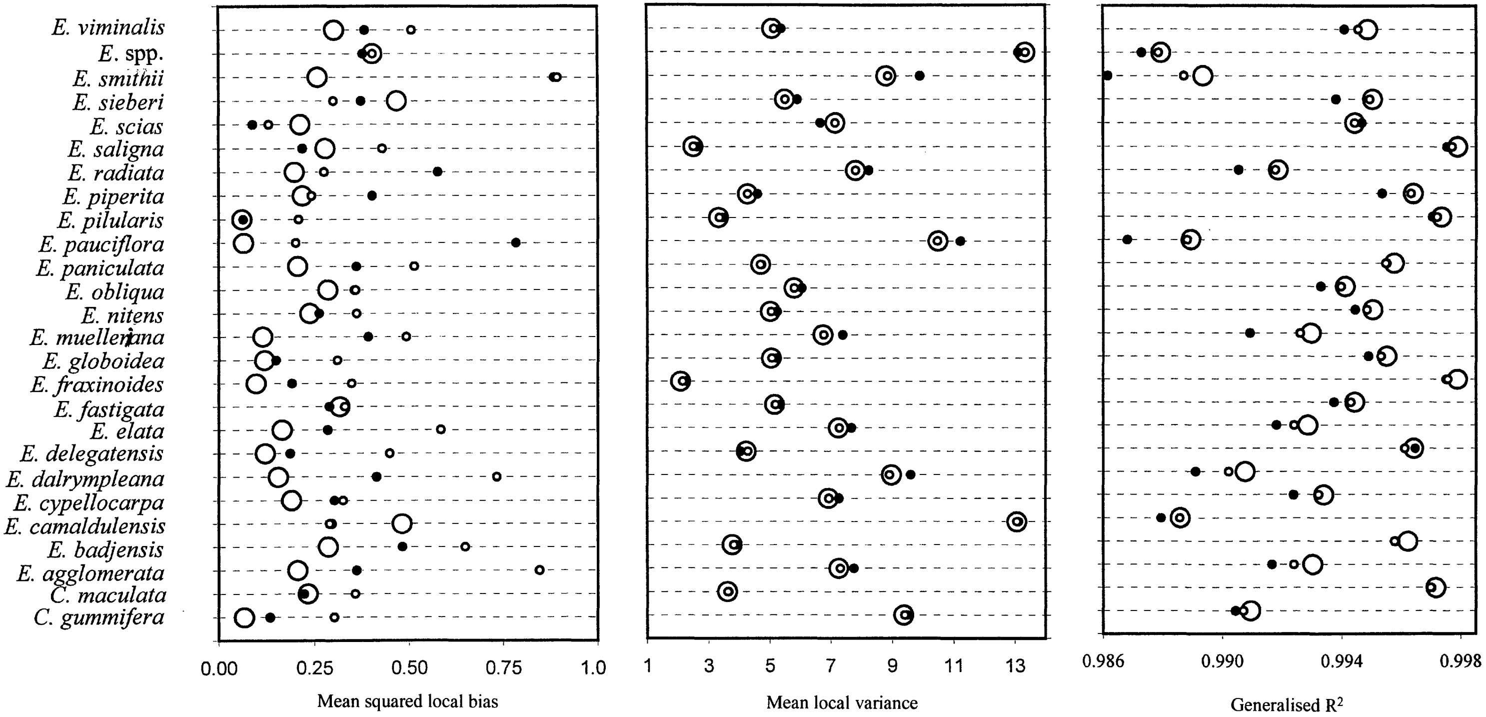

FIG. 1–Dot plots of mean squared local bias, mean local variance, and generalised R^2 for the trigonometric volume ratio equation (large circle), the nonlinear model form of Cao & Burkhart (1980) (small dot), and the power function of Forslund (small circle) across 26 species.

relative height very close to zero ($h = 1\times10^{-6}$) was obtained and inserted into the data set through quadratic interpolation using three data points closest to this relative height. With these additional data points in the data set, the equation was refitted for each species. Subsequent numerical analyses showed that the lower limit of relative height became less than 5×10^{-6} for all species.

One of the assumptions underlying least squares linear regression is that the error terms are independent, identically distributed, normal random variables. Since the taper data comprised multiple diameter measurements along each sample stem, positive autocorrelations existed among the residuals in addition to heteroscedasticity. When autocorrelation and heteroscedasticity are present, the least squares estimates of regression coefficients remain unbiased and consistent, although no longer efficient (Myres 1990). So the estimates can still be used for prediction but not for making statistical inferences. As found with taper functions, prediction accuracy is little affected by autocorrelation in the error term and multicollinearity among predictor variables in the equation (Kozak 1997). Even when the correlated error structure was accounted for in fitting taper models, the improvement in prediction accuracy was too small to be of practical importance (Williams & Reich 1997).

However, the systematic pattern of heteroscedasticity identified in this study may affect local accuracy of prediction to some extent. Since the variance of the residuals decreased with the predicted values, the contribution of data points from the base and lower stem to the least squares estimates was disproportionally greater than those from the middle and upper stem. When the distribution of residuals over the base and lower stem was not strictly normal, as was the case for some species, estimates might not give the least biased prediction over the middle and upper stem, for which the volume ratio equations would be often used in practice. To reduce the influence of these data points to the least squares parameter estimates, weighted least squares regression was used for estimating the regression coefficients using weight functions of the form $[100(1-h_i)]^{-k}$. Furnival's index of fit (Furnival 1961) was used to select the best weight function for each species among 41 values of k ranging from 0 to 4 with an even interval of 0.1. This index is based on transformed maximum likelihood values and takes the following form when weight functions are of the form $[100(1-h_i)]^{-k}$

$$I = \left[\mathit{anti}\log \frac{\sum_{i=1}^{n} \log \sqrt{[100(1-h_i)]^{-k}}}{n} \right]^{-1} S \tag{9}$$

where S is the least squares estimate of the standard error of the weighted error term, h_i is the relative height of the ith observation, and n is the total number of observations for each species. It provides a relative measure of the departures from linearity, normality, and homoscedasticity of residuals simultaneously, with a smaller value indicating a smaller departure and a larger value indicating a greater departure. The pattern of the index plotted against k was a concave curve for every species, showing that the minimum was within the range of k specified above.

Because the equations were fitted through weighted least squares regression using different weights for different species, and the weighted equations were fitted without an intercept term, a generalised R^2 was calculated according to Equation 7 to show the percentage of variation in volume ratio explained by the regression equations.

Model Testing

To assess the predictive accuracy of the estimated volume ratio equations, the leave-one-out resampling approach was adopted. Although data splitting has been proposed as a way of validating regression models (Snee 1977; Miller 1984; Picard & Cook 1984; Picard & Berk 1990), it has been demonstrated to be a costly approach, in terms of both the predictive accuracy of the fitted model and the precision of the estimates of that accuracy (Roecker 1991). To maximise predictive accuracy while retaining a reliable estimate of that accuracy, it is recommended that the entire sample be used for both model development and assessment (Roecker 1991).

For each species with N sample trees, the equation was fitted N times using the same weight function. Each time, all data points of one tree were removed from the fitting process and predicted volume ratios were obtained for them using the coefficients estimated from the remaining data. The difference between the observed and predicted volume ratio was taken as the error of prediction. For each species, the observed volume ratio was plotted against the predicted values. If the two are identical, all data points will fall on the line of unity. The vertical distance from a data point to the line showed the size of prediction error relative to the predicted value. The accuracy of prediction was also evaluated over local relative height classes for each species. The range of relative height between 0 and 1 was divided into 10 even intervals. Within each interval, the mean, the 2.5th, and the 97.5th percentiles of prediction error were calculated to indicate the bias and precision of the volume ratio prediction.

RESULTS

The estimated coefficients of the volume ratio equation for the 25 species together with fit statistics are shown in Table 1. Volume ratio curves derived from the estimated equations (Fig. 2) showed that the relative height of centroid (i.e., the centre of volume of the tree bole dividing the total stem volume into two halves) varied between 0.21 and 0.26 among the species, with *E. camaldulensis* Dehnh. and *E. pauciflora* Sieber having the largest proportion of stem volume in the lower 21% of the stem. For all species the lower half of the stem accounted for 80% or more of the total stem volume, and the lower 60% of the stem accounted for about 90% or more of the total stem volume (Table 2).

The pattern of observed volume ratio plotted against predicted values showed no marked departures from the line of unity for all species (Fig. 3). The vertical spread of the data points at a given relative height showed the size of prediction error relative to the predicted value. There was little local bias of prediction for all species, and the 95% confidence limits of prediction error were mostly within 10% (Fig. 4). For species such as *E. muelleriana* Howitt and *E. smithii* R.T.Bak., the confidence interval was asymmetric at certain relative height intervals, reflecting a skewed error distribution within these relative height ranges.

DISCUSSION

The volume ratio equations reported here are a part of a system of equations for stem volume and taper prediction being developed for native forest tree species of New South Wales. These volume ratio equations are to be used together with the total stem volume equations for these species (Bi & Hamilton 1998) for predicting stem volume to any height

TABLE 1–Parameter estimates and fit statistics of volume ratio equations for the 25 species. The number of measurements and the number of sample trees used in the estimation were also included.

Species	a_1	a_2	a_3	a_4	R^2	No. of measurements	No. of trees
Corymbia							
C. gummifera Hill&Johnson	–3.728	48.646	–123.208	86.629	0.991	1 808	149
C. maculata Hill&Johnson	–3.721	46.403	–113.653	78.228	0.997	3 201	212
Eucalyptus							
E. agglomerata Maiden	–4.828	66.050	–172.822	122.621	0.993	2 068	117
E. badjensis de Beuzev&Welch	–5.508	68.412	–172.301	119.962	0.996	526	32
E. camaldulensis Dehnh.	–2.739	33.267	–82.084	58.341	0.989	1 940	109
E. cypellocarpa Johnson	–3.763	49.870	–128.365	91.258	0.993	3 879	273
E. dalrympleana Maiden	–5.110	71.491	–191.424	138.021	0.991	221	20
E. delegatensis Baker	–3.897	50.467	–126.571	88.090	0.996	4 983	315
E. elata Dehnh.	–5.199	68.221	–177.193	125.727	0.993	392	25
E. fastigata Deane&Maiden	–4.390	56.771	–145.871	103.433	0.994	12 644	933
E. fraxinoides Deane&Maiden	–4.632	61.421	–157.973	111.290	0.998	1 201	75
E. globoidea Blakely	–3.290	45.486	–117.145	83.060	0.995	4 752	338
E. muellerana Howitt	–4.253	58.572	–154.287	110.493	0.993	2 024	138
E. nitens (Deane&Maiden) Maiden	–4.663	60.644	–156.110	110.512	0.995	703	55
E. obliqua L'Her.	–4.040	53.935	–139.469	99.140	0.994	6 848	543
E. paniculata Smith	–4.499	55.912	–138.313	95.359	0.996	658	49
E. pauciflora Sieber	–4.304	55.754	–146.183	105.640	0.989	399	41
E. pilularis Smith	–3.166	41.116	–102.583	71.765	0.997	2 524	148
E. piperita Smith	–2.215	32.244	–83.870	60.700	0.996	3 001	194
E. radiata Sieber	–4.144	54.143	–140.817	101.004	0.992	3 011	286
E. saligna Smith	–3.114	42.238	–106.856	75.048	0.998	436	24
E. scias Johnson&Hill	–2.936	36.606	–88.374	60.856	0.994	301	23
E. sieberi L.Johnson	–2.533	36.330	–94.589	68.134	0.995	17 637	1192
E. smithii Baker	–4.668	64.759	–172.253	123.789	0.989	997	47
E. spp.	–4.118	52.953	–135.244	95.684	0.988	702	51
E. viminalis Labill	–5.621	74.831	–195.644	138.959	0.995	1 285	106

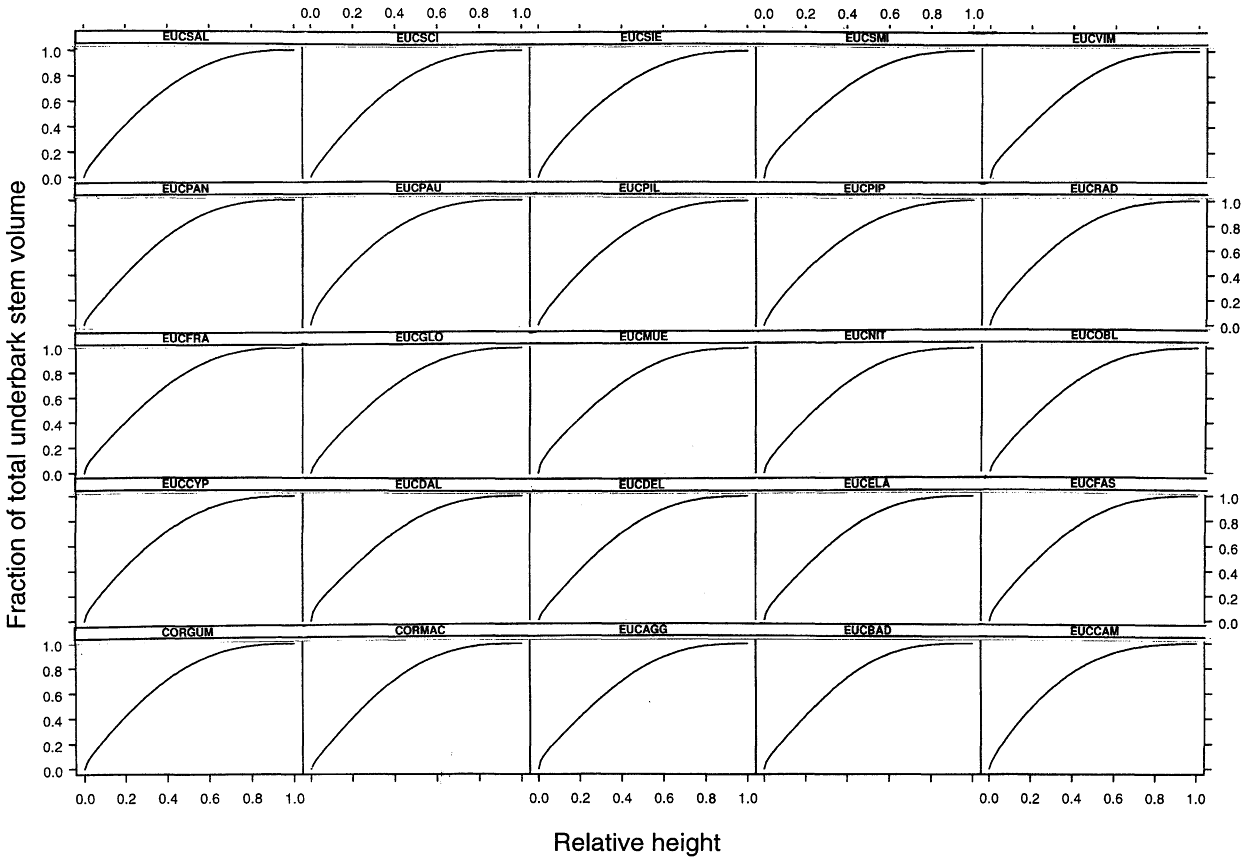

FIG. 2–Predicted volume ratio in relation to relative height for the 25 species as indicated by the six-letter code in the top strip of each panel.

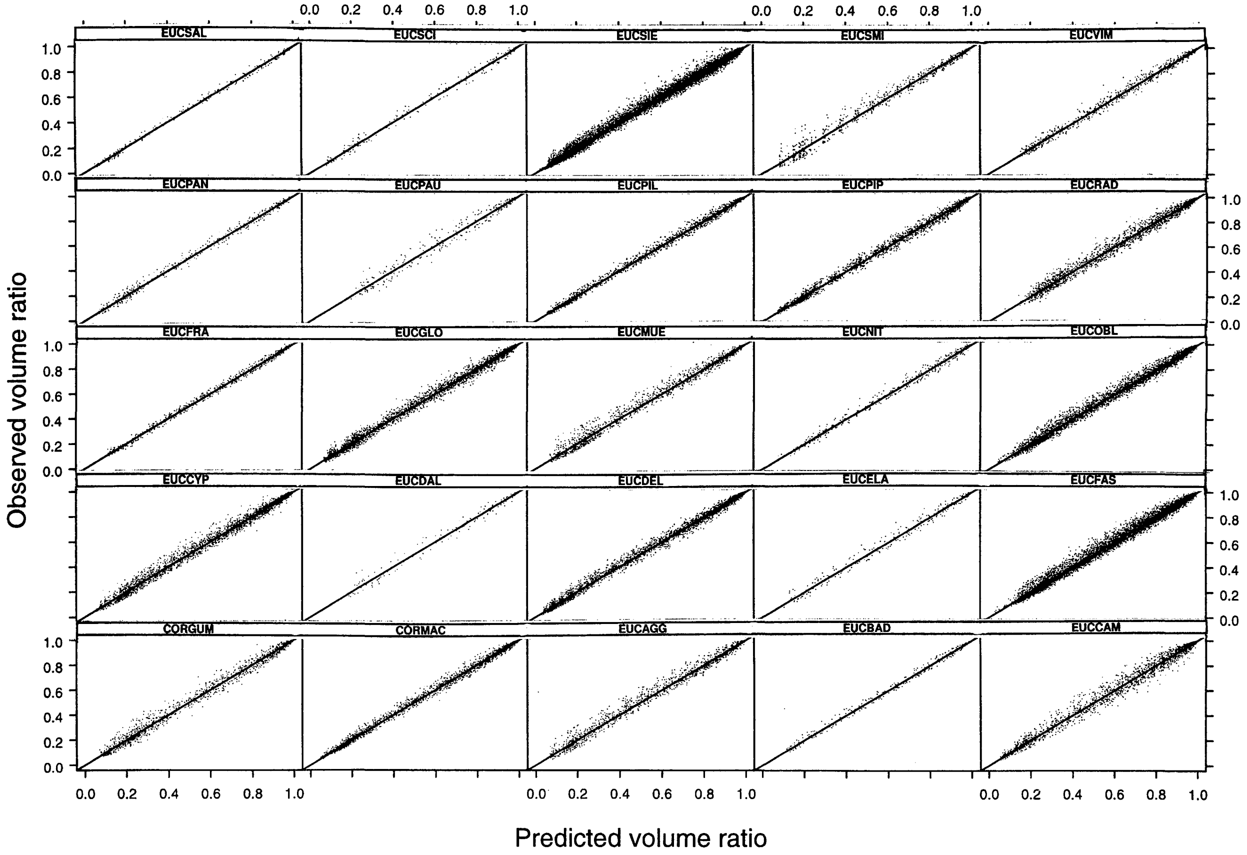

FIG. 3–Multipanel display of observed volume ratio *v.* predicted volume ratio for the 25 species as indicated by the six-letter code. The diagonal line in each panel is the line of unity.

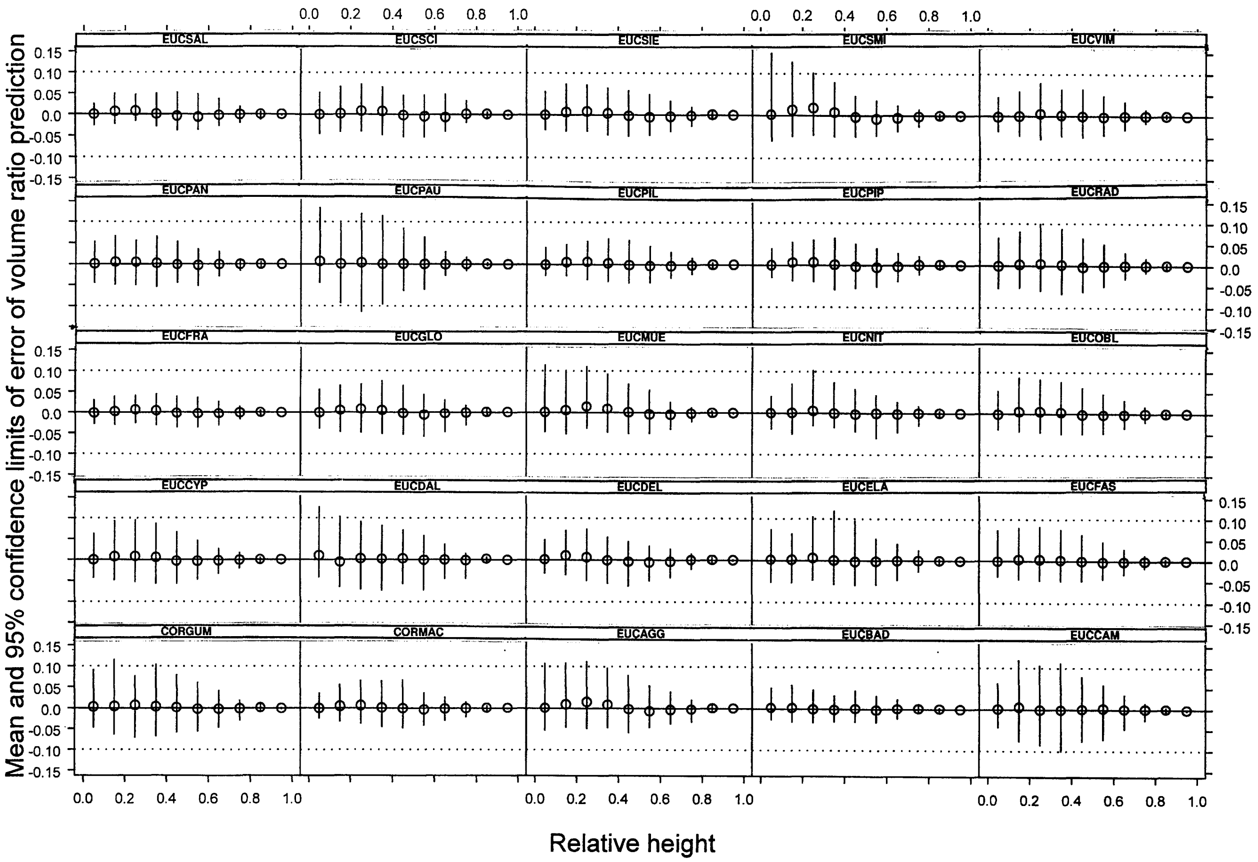

FIG. 4—Bias and precision of volume ratio prediction along relative height classes for the 25 species as indicated by the six-letter code. The empty circles represent the mean of prediction error for the corresponding relative height classes. Within each relative height class, 95% of the prediction error falls within the interval shown by the vertical bar.

TABLE 2–The relative height of centroid (h_c), and volume ratios at 0.5 and 0.6 relative height ($R_{0.5}$, $R_{0.6}$) derived from the volume ratio equations for the 25 species.

Species	h_c	$R_{0.5}$	$R_{0.6}$
Corymbia			
C. gummifera	0.248	0.821	0.899
C. maculata	0.254	0.819	0.899
Eucalyptus			
E. agglomerata	0.256	0.810	0.893
E. badjensis	0.246	0.836	0.915
E. camaldulensis	0.213	0.855	0.921
E. cypellocarpa	0.239	0.827	0.903
E. dalrympleana	0.246	0.817	0.897
E. delegatensis	0.256	0.815	0.896
E. elata	0.241	0.831	0.907
E. fastigata	0.235	0.836	0.911
E. fraxinoides	0.257	0.813	0.896
E. globoidea	0.255	0.807	0.889
E. muellerana	0.241	0.821	0.899
E. nitens	0.241	0.830	0.908
E. obliqua	0.245	0.822	0.900
E. paniculata	0.256	0.822	0.903
E. pauciflora	0.206	0.860	0.926
E. pilularis	0.247	0.820	0.898
E. piperita	0.241	0.814	0.891
E. radiata	0.223	0.844	0.916
E. saligna	0.258	0.807	0.888
E. scias	0.250	0.820	0.898
E. sieberi	0.242	0.814	0.891
E. smithii	0.235	0.825	0.902
E. spp.	0.233	0.837	0.911
E. viminalis	0.250	0.822	0.903

limit. To estimate the merchantable volume of a tree with a given diameter and height, calculate the total stem volume first using equations given by Bi & Hamilton (1998). Then divide the stump height and merchantable height of the tree by its total height to obtain the corresponding values of relative height for calculating volume ratios using the volume ratio equations. Finally, multiply total stem volume by the difference in volume ratio between the two relative heights to obtain the merchantable volume. This calculation can be extended to estimate the volumes of multiple logs from the same stem when trees are harvested for multiple products during integrated logging, which has been a part of the management system of native forests such as those in south-east New South Wales since 1969 (Bridges 1983). In addition, the volume ratio equations will prove to be useful in forest inventory where yield estimates by product categories are required. For predicting log-end diameters, trigonometric variable-form taper equations have been derived from the volume ratio equations for these species and the work will be reported elsewhere.

Among five common solids of revolution of interest to foresters, the relative height of centroid of conoid is 0.2063 and that of second-degree paraboloid is 0.2929 (Wood *et al.* 1990). For all 25 native forest tree species in this study, the relative height of the centroid ranged from 0.206 to 0.258 (Table 2), suggesting that stem shapes vary between a conoid and a second-degree paraboloid among these species. This contrasts interestingly with the relative height of centroid for the exotic *P. radiata* which is approximately 0.3, a second-degree paraboloid (Wood *et al.* 1990).

ACKNOWLEDGMENTS

I am indebted to forestry staff in both New South Wales and Victoria for collecting the data in the field. I thank Mr. Shimin Cai for technical assistance. Dr Phil West and Dr Peter Cornish provided helpful comments.

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