POLYNOMIAL TAPER EQUATION FOR *PINUS CARIBAEA*

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ABSTRACT

A stem taper model involving a high-order polynomial for plantation *Pinus caribaea* Morelet var. *hondurensis* Barrett & Golfari (Caribbean pine) grown in Queensland was fitted in three stages. In the first stage, the stem profile for each tree was modelled using functions of under-bark diameter and height as the dependent and independent variables respectively. In the second stage, the parameter estimates from these individual tree regressions were subjected to a principal component analysis. The first two principal components were then modelled using total height and diameter at breast height as the independent variables. In the third stage, total height and diameter at breast height under bark were modelled in terms of predominant height, and diameter at breast height over bark. Using these equations, and the inverse transformation from the principal components to individual tree regression coefficients, individual tree profiles and volumes were predicted from height and diameter at breast height.

The maximum average diameter bias for the final model, using predominant height and diameter at breast height over bark as predictive variables, was 2.5 mm. The average tree volume bias, based on coefficients modelled using predominant height and over-bark diameter at breast height, was 0.13%. Apart from its accuracy, another advantage of this model is its ability to accommodate taper changes with tree size.

Keywords: taper model; volume; stem form; *Pinus caribaea*.

INTRODUCTION

Taper equations are functions which can provide estimates of the diameter at any height along the bole, the height of any predetermined diameter, and the volume between any two points on the stem. Numerous taper models have been reported over several decades. The early models were relatively simple for ease of computation and generally involved only one independent variable (e.g., Behre 1923). With the advent of the computer, taper models became more sophisticated and their accuracy improved. Max & Burkhart (1976) divided the stem into three segments and modelled each segment using quadratic polynomials, while Liu (1980) applied cubic spline functions to model stems divided into several segments. Other authors (e.g., Bruce *at al.* 1968; Goulding & Murray 1976; Gordon 1983) have used higher order polynomials containing several terms to model the entire stem using a single function.

Because no joining points are required, these models are, in general, algorithmically less complex than models developed using the segmented approach. In all of these taper equations, relative diameter or a function of relative diameter has been modelled in terms of relative height.

In contrast, Fries (1965), Fries & Matern (1966), and Liu & Keister (1978) have used another approach. In their studies, diameters at several consistent positions relative to the height were subjected to a principal component analysis. One or more multiple regression equations were subsequently developed, using the eigenvectors as the dependent variables and the relative heights and their powers as the independent variables.

In all of these studies, the data from a number of sample trees are used simultaneously to determine the model coefficients. Consequently, the models predict a constant relative tree shape for all trees regardless of size. In other words, as pointed out by Demaerschalk & Kozak (1977) most popular taper models are biased and the pattern of bias changes from one size class to another. Thus, it may be necessary to develop a series of discrete functions to accommodate taper changes with size, which is inconsistent with the continuous nature of such changes.

In order to overcome this problem, Real & Moore (1988) modelled the trees individually and predicted the individual tree regression coefficients using tree variables (viz total height, shape quotient, and crown ratio). This paper presents a taper model suitable for *Pinus caribaea* which has been based on the model developed by Real & Moore (1988). Height and diameter at breast height have been used to predict the individual tree regression coefficients.

NOTATION

For convenience, relevant notation is summarised below:

h = height from base of tree

d = diameter under bark at a given height (h)

H_T = tree total height

H_P = predominant height (i.e., height of the 50 tallest trees per hectare measured on the basis of one tree per 0.02-ha unit)

$DBHob$ = diameter at breast height over bark

$DBHub$ = diameter at breast height under bark

X = $(H_T - h) / (H_T - 1.3)$

Y = $(d / DBHub)^2 - X^2$

b_i = the i^{th} regression coefficient

$\mathbf{b}$ = vector of regression coefficients

p_i = the i^{th} principal components

$\mathbf{p}$ = vector of principal components

$\mathbf{E}$ = matrix containing the eigenvectors (arranged in columns) of the covariance matrix for the regression coefficients

$\mathbf{E}^T$ = transpose of $\mathbf{E}$.

DATA

A total of 730 trees from Ingham (latitude 18° 39') in northern Queensland were destructively sampled. The trees were selected from routinely managed plantations to cover the range of size classes. The diameter under bark for each tree was measured at 0.20, 0.50, 1.30 m (*DBHub*), and then at approximately 3-m intervals in the middle of the internodes. Total heights (H_T), predominant heights (H_P), and diameters at breast height over bark (*DBHob*) were also recorded. Heights and diameters were measured in metres (to the nearest decimetre) and centimetres (to the nearest millimetre) respectively.

The distribution of the trees according to predominant-height and diameter-at-breast-height classes is shown in Table 1.

TABLE 1–Distribution of sample trees according to predominant-height and diameter-at-breast-height-over-bark (DBHob) classes

DBHob	Predominant height class (m)						Totals
class (cm)	5–10	10–15	15–20	20–25	25–30	30–35	
5–15	18	24	3	7	3	1	56
15–25	43	74	101	89	89	44	400
25–35	0	2	18	55	989	83	256
35–45	0	0	0	1	2	15	18
Totals	21	100	122	152	192	143	730

The stands from which the sample trees were obtained were extremely variable. They had been subjected to a variety of silvicultural practices, especially with respect to thinning regimes and fertiliser applications, and were planted on a variety of soil types having different site qualities. In addition, some stands were severely damaged by a cyclone in 1985. Consequently, the intact stems which were sampled varied considerably in both size and form.

TAPER MODEL

First Stage—Estimation of Individual Tree Regression Coefficients

The taper model developed by Real & Moore (1988) was as follows:

$$Y = b_1(X^3 - X^2) + b_2\,(X^8 - X^2) + b_3\,(X^{40} - X^2)$$

An examination of the predicted profiles for the sample trees indicated that the Real & Moore (1988) model consistently under-estimated diameters towards the tops of the larger trees (greater than approximately 25 m) and was not sufficiently flexible to cope with the variability in the region of the central stem. It was satisfactory for most of the medium-sized and smaller trees. This is illustrated by the three typical trees in Fig. 1.

Real & Moore (1988) demonstrated that the first term in their model (i.e., $X^3 - X^2$) influenced diameter estimation along the entire stem but principally at the tip. Empirical investigations, using the data for *P. caribaea*, revealed that adjustments to this term could improve the fit at the tip of the tree. As a consequence, it was modified by substituting a function of H_T and *DBHub* for the index of the first element of this term (i.e., X^3). This

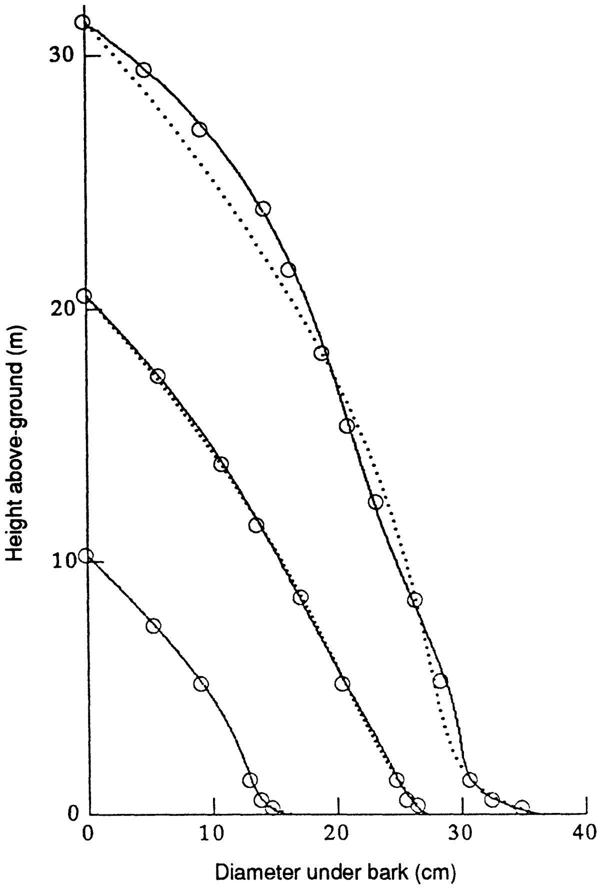

FIG. 1–Examples of the fit of the Real & Moore (1988) model to a large, medium, and small tree (dotted lines), and the modified model (solid lines) to the same trees using coefficients based on individual tree regressions.

function was determined using non-linear regression analysis, making the power of the first element a parameter in the model, and modelling this parameter estimate in terms of H_T and *DBHub*.

However, the model still lacked the necessary flexibility to cope with the variability in the region of the central stem, indicating that an additional term was required. Real & Moore (1988) demonstrated that the second term in their model (i.e., $X^8 - X^2$) controlled the lower 30% of the bole. Thus, after testing powers of X from four to seven, $(X^5 - X^2)$ was selected as the most suitable term. Consequently, the model became:

$$Y = b_1(X^{5.0 - 0.12H_T + 0.033\,DBHub} - X^2) + b_2(X^5 - X^2) + b_3(X^8 - X^2) + b_4(X^{40} - X^2)$$

which can also be expressed as:

$$d^2 = DBHub^2\left\{X^2 + b_1(X^{5.0-0.12H_T+0.033DBHub} - X^2) + b_2(X^5 - X^2) + b_3(X^8 - X^2) + b_4(X^{40} - X^2)\right\}$$

The improved fit of the model is also illustrated in Fig. 1.

Second Stage—Prediction of Individual Tree Regression Coefficients from H_T and *DBHub*

In order to model the individual tree regression coefficients (b_i) in terms of H_T and *DBHub*, it was necessary to exclude 15 trees which had extremely aberrant individual tree regression coefficients and unusual profiles. These trees, which had either very large or small height to diameter ratios, were excluded from these and subsequent analyses. The first-stage regression coefficients for the remaining 715 stems were subjected to a principal component analysis using the covariance matrix for **b**. That is:

$$\mathbf{p} = \mathbf{E}^T \mathbf{b}$$

The covariance matrix and the eigenvectors for this transformation are given in the Appendix. The means for the four individual tree regression coefficients (b_1 to b_4) were –4.04, 1.44, –0.501, and 0.0355 respectively. The eigenvalues corresponding to the four eigenvectors were 27.5, 2.47, 0.157, and 0.000 402 respectively. Estimates ($\hat{p}_i$) of the first two principal components were obtained using stepwise multiple regression analysis, with H_T and *DBHub* as the independent variables. It was only necessary to model the first two principal components since together they explained 99.5% of the variation for b. The equations are given below, together with the coefficients of determination (r^2) and the residual mean squares (*RMS*) for each equation. All regression coefficients in both equations were statistically significant ($p < 0.05$).

$$\hat{p}_1 = -3.58 - 0.391 H_T + 7.37 \frac{DBHub}{H_T} \quad (r^2 = 0.503, RMS = 13.7)$$

$$\hat{p}_2 = 4.36 - 0.936 H_T + 0.409 DBHub + 0.0264 H^2{}_T - 0.0103 DBHub^2 \; (r^2 = 0.494, RMS = 1.26)$$

Since the matrix **E** is orthogonal (i.e., the inverse and transpose are identical), an estimate ($\hat{\mathbf{b}}$) of **b** was obtained using the following inverse transformation.

$$\hat{\mathbf{b}} = \mathbf{E}\,\hat{\mathbf{p}}$$

For this reverse transformation, the mean values were used for the third and fourth principal components (–0.000 345 and 0.0140 respectively).

Third Stage—Prediction of H_T and *DBHub*

In order for the model to be used in practice, it was necessary to estimate volume and taper from the routine field measurements of *DBHob* and H_p. Thus, *DBHub* and H_T were also modelled using H_p and *DBHob* as independent variables. The model used to predict H_T, which was developed by Vanclay (1982), has two desirable properties. H_T is 1.30 m for a *DBHob* of 0 cm and H_T increases asymptotically towards H_p approximately as *DBHob* increases. Thus, estimates of H_T and *DBHub* were determined using the equations given below. Coefficients of determination (r^2) and residual mean squares (*RMS*) are also given. All regression coefficients in the models were statistically significant ($p < 0.05$).

$$\hat{H}_T = 3.44 + 0.984\, H_p - \frac{3.44 + 0.984 H_p - 1.3}{1 + 0.0458 DBHob + 0.00425 DBHob^2} \quad (r^2 = 0.949, RMS = 2.04)$$

$$\widehat{DBHub} = e^{-0.445 + 1.07 \ln DBHob} \; (r^2 = 0.971, RMS = 0.0024)$$

In Fig. 2, which shows the three trees presented in Fig. 1, the shape averaging is illustrated which occurred, particularly for the large tree, when predicted coefficients were used.

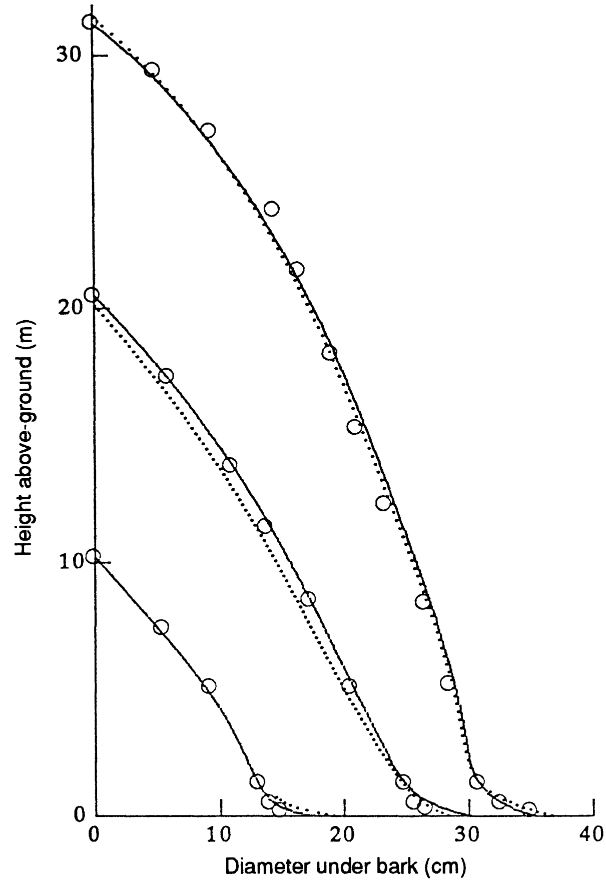

FIG. 2–Fit of the modified model to the trees in Fig.1, based on coefficients estimated from total height and diameter at breast height under bark (solid lines) and from predominant height and diameter at breast height over bark (dotted lines).

VOLUME

Total volumes (cubic metres) for individual trees were determined as follows using integration:

$$\text{Volume} = \frac{\pi}{40\,000}\int_{h_1}^{h_2} d^2\,dh$$

$$= \frac{\pi DBHub^2}{40\,000}\,(1.3 - H_T\left[\frac{X^3}{3}\quad(1 - b_1 - b_2 - b_3 - b_4)\right.$$

$$\left. + b_1\,\frac{X^{6.0\,-\,0.12H_T\,+\,0.033DBHub}}{6.0 - 0.12H_T + 0.033DBHub} + b_2\,\frac{X^6}{6} + b_3\,\frac{X^9}{9} + b_4\,\frac{X^{41}}{41}\right]_{h_1}^{h_2}$$

MODEL EVALUATION

Diameter

Aggregate difference (*AD*) and root mean square (*RMSE*) for the diameters of each tree were used as measures of bias and prediction respectively (Real & Moore 1988). These statistics were calculated for each tree for five sections of equal length and then averaged over all trees.

$$AD = \frac{\sum_{i=1}^{i=n} (Ai - Ei)}{n}$$

$$RMSE = \sqrt{\sum_{i=1}^{i=n} (A_i - E_i)^2 / n}$$

where A_i is the actual (i.e., measured) diameter, E_i is the estimated diameter, and n is the number of diameter measurements for each section for each tree.

The *AD*s and *RMSE*s were calculated using coefficients based on individual tree regression equations (developed using all sample tree data) for both the Real & Moore (1988) model and the modified model. For the modified model, these statistics were also calculated using the modelled coefficients at both levels of prediction (i.e., stages 2 and 3). The *AD*s and *RMSE*s, expressed in absolute terms and as percentages of the mean observed diameter for each stem section, are presented in Table 2.

As indicated previously, it is evident from Table 2, that the Real & Moore (1988) model consistently under-estimated diameters for the upper part of the stem. The *AD*s and *RMSE*s based on individual tree regressions using the modified model were the smallest. Both the *AD*s and the *RMSE*s determined using coefficients predicted from H_T and *DBHob* were generally smaller than those determined using coefficients predicted from H_p and *DBHob*.

Volume

Using integration, total volumes were determined for each tree using the coefficients obtained from the individual tree regressions. These volumes were compared with the total volumes determined using the standard method currently being used by the Queensland Forest Service for estimating the volumes of sample trees. This method, which has been described by Vanclay & Shepherd (1983) and which is based on the technique developed by Grosenbaugh (1966), was used to determine the "true" taper and volume for each sample tree. It involves the fitting of appropriate (parabolic, hyperbolic, or conic) shapes to three diameter-height pairs, interpolating diameter estimates for heights midway between the measured heights and using Newton's formula to calculate the volume of each frustum. The almost perfect agreement between these two estimates of individual tree volume is demonstrated by the following equation. The coefficient of determination (r^2) and the residual mean square (*RMS*) are also given.

Taper model volume = 0.000 332 + 0.9997 current standard volume
(r^2 = 0.999, *RMS* = 0.000 002)

TABLE 2–Diameter bias and predictability for trees modelled using (i) the Real & Moore (1988) model for individual tree regressions, (ii) modified model for individual tree regressions, (iii) coefficients predicted from total height and diameter at breast height under bark (DBHub), and (iv) coefficients predicted from predominant height and diameter at breast height over bark (DBHob)

	Section (% height)	Mean diameter (cm)	Aggregate difference (cm)	Aggregate difference (%)	Root mean square error (cm)	Root mean square error (%)
(i) Real & Moore (1988)						
	0–20	19.5	0.02	0.11	0.16	0.80
	20–40	14.7	–0.06	–0.42	0.16	1.10
	40–60	12.4	–0.10	–0.78	0.20	1.57
	60–80	8.8	0.28	3.18	0.46	5.23
	80–100	3.8	0.45	11.65	0.52	13.63
(ii) Modified model						
	0–20	19.5	–0.01	–0.07	0.10	0.51
	20–40	14.7	0.00	–0.02	0.08	0.56
	40–60	12.4	–0.01	–0.07	0.10	0.81
	6—80	8.8	0.02	0.23	0.14	1.62
	80–100	3.8	0.02	0.47	0.25	6.45
(iii) Total height and DBHub						
	0–20	19.5	–0.13	–0.65	1.06	5.42
	20–40	14.7	0.02	0.11	0.81	5.50
	40–60	12.4	–0.07	–0.60	0.97	7.80
	60–80	8.8	–0.07	–0.77	0.87	9.87
	80–100	3.8	0.01	0.36	0.52	13.56
(iv) Predominant height and DBHob						
	0–20	19.5	–0.15	–0.79	1.17	6.0
	20–40	14.7	0.16	1.11	0.90	6.12
	40–60	12.4	–0.25	2.05	1.24	8.42
	60–80	8.8	–0.18	–1.99	1.24	14.04
	80–100	3.8	0.05	1.26	1.21	31.64

The tangent of the slope of this regression line was not significantly different from one, while the intercept was not significantly different from zero ($p < 0.05$).

Individual tree total volumes, based on individual tree regressions, were then compared with the corresponding volumes estimated using predicted regression coefficients. By regarding the volumes obtained using individual tree regressions as the actual values (A_i) and volumes based on the appropriate modelled coefficients as the estimated values (E_i), the relative mean bias (*RMB*) for volume was defined as:

$$RMB = \frac{\sum_{i=1}^{i=n} (A_i - E_i)}{\sum_{i=1}^{i=n} A_i}$$

where n is the number of trees. These relative mean biases were determined for several H_p and *DBHob* classes, as well as for all stems combined. They are presented in Table 3, expressed as percentages.

TABLE 3–Volume biases, expressed as a percentage of the volume from individual tree regressions, based on (i) coefficients predicted using total height and diameter at breast height under bark, and (ii) coefficients predicted using predominant height and diameter at breast height over bark (DBHob); n = numbers of trees in each size class.

DBHob class (cm)	Predominant height class (m) <20			20–30			>30			Totals		
	(i)	(ii)	n	(i)	(ii)	n	(i)	(ii)	n	(i)	(ii)	n
<20	-2.66	2.23	140	1.29	-2.00	60	-2.95	-11.30	10	-1.15	-0.41	210
20–30	0.94	1.82	98	-0.23	0.19	259	-0.08	-1.19	78	0.02	0.24	435
>30			0	-1.99	-1.51	25	1.20	0.68	45	0.28	0.05	70
Totals	-0.29	1.96	238	-0.37	1.96	344	0.61	-0.40	133	-0.02	0.13	715

The over-all relative mean biases for volume were small for both levels of prediction. These were -0.02% using coefficients predicted from H_T and $DBHub$ and 0.13% for the coefficients predicted from H_p and $DBHob$. For the various size classes, the range of relative mean biases (from -11.3% to 2.2%) resulting from coefficients predicted using H_p and $DBHob$ exceeded the range (from -3.0% to 1.3%) of those predicted using H_T and $DBHub$. However, the -11.3% bias was based on only 10 stems and represented an average bias of only 0.019 m^3/tree.

Discussion

A three-stage procedure was used to develop the taper model presented here.

In the first stage, individual tree regressions were determined using all sample tree measurements. This model could be used as a replacement for the current method for the determination of sample tree taper and volume. It gave almost identical results but has the advantage of being algorithmically simpler.

The second stage involved the prediction of the individual tree regression coefficients obtained in the first stage, using total height and diameter at breast height under bark. The level of prediction is appropriate where bark thickness and individual tree heights are known. It has the potential for use as a research tool where reasonably intensive individual tree measurements are justified. For example, it could be used in the breeding programme to identify trees having superior volume and taper. It could also be used to determine the effects of silvicultural practices such as planting density or various thinning regimes on volume and taper.

Finally, diameter at breast height under bark and total height were modelled in terms of diameter at breast height over bark and predominant height. This level of prediction was the least accurate, but could be used where stem taper and volume are required from these routine stand measurements for the purposes of marketing or resource evaluation.

It should be noted that the use of principal component analysis for the construction of this model is very different from its use by Fries (1965), Fries & Matern (1966), and Liu & Keister (1978). Here, the technique has been used to facilitate the modelling of the regression coefficients (b_is) and to establish associations between the estimates of these coefficients. Although they did not state it explicitly, it would appear that Real & Moore (1988) recognised that it was necessary to establish such links. This is evident from the fact that they modelled their b_2 in terms of $\hat{b}_1$. My attempts to imitate their technique or to model the coefficients independently proved unsuccessful. However, it was possible to establish the necessary associations by modelling the coefficients simultaneously via the use of principal component analysis. The successful use of the technique in this situation, however, depends on linear relationships between the regression coefficients. In this instance, bivariate relationships between all the b_is were either linear or almost linear.

Where comparisons were possible, both diameter bias and predictability results compared favourably with those presented by Real & Moore (1988). Average diameter bias results based on coefficients derived from individual tree measurements were very satisfactory and were less than measurement errors. Diameter biases determined using the modelled coefficients, although larger, would still be adequate for most purposes. Similarly, while the predictability decreased for modelled coefficients, it was worst towards the top of the tree which is the least valuable part.

Despite the accuracy and reliability of the model, there are disadvantages. In particular, two diameter measures below 1.3 m with one between 0.2 and 0.3 m were required for its development. If these measures were not included it was possible for the predicted profile to turn inwards toward the tree axis at the base. Using predicted coefficients, it was possible for the predicted square of the diameter to become negative near the very top (3–4%) of the stem, indicating that the predicted profile had crossed to the other side of the tree axis. This was not a serious problem in that it occurred for only five trees with the worst having a "negative diameter" of 1.4 cm. This had only a slight effect on the volume estimates for the trees affected. Additionally, the model forces the predicted stem profile through total height and diameter at breast height under bark, which means that these data need to be either measured or predicted very accurately. This is not as difficult for the diameter as it is for height. The heights of standing trees are difficult to measure accurately and if the trees are felled the tops often break. Moreover, the equation used to predict total height had a larger residual variation than that used to predict diameter at breast height under bark.

Despite some shortcomings, the model has, with the exception of relatively few trees, proved sufficiently resilient to cope with the highly variable sample tree population used. It has shown sufficient promise to justify further developmental work. In particular, it needs to be validated using *P. caribaea* from other stands at other locations in Queensland. Since bark thickness and the relationship between predominant and total heights are sensitive to geographic, silvicultural, and site quality effects, it is very likely that separate fits would be required for the third stage. It may also be possible to improve the accuracy of the model by including additional individual tree attributes such as form quotient or crown ratio.

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APPENDIX

COVARIANCE MATRIX FOR THE INDIVIDUAL TREE REGRESSION COEFFICIENTS AND ITS EIGENVECTORS

Covariance matrix				
	b_1	b_2	b_3	b_4
b_1	19.6	–11.2	3.33	–0.093 7
b_2	–11.2	9.66	–2.58	0.044 2
b_3	3.33	–2.58	0.873	–0.019 0
b_4	–0.0937	0.0442	–0.0190	0.001 03

Eigenvectors of covariance matrix			
1	2	3	4
0.828	–0.559	–0.004 57	0.004 78
–0.539	–0.815	0.212	0.009 07
0.156	0.151	0.976	0.030 4
–0.003 79	0.005 49	–0.031 3	0.999