STRENGTH PROPERTIES OF PINUS RADIATA PLYWOOD AT ANGLES TO FACE GRAIN

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ABSTRACT
The strength and stiffness properties of Pinus radiata D. Don plywood at angles to the grain of the face veneer have been modelled by theoretical equations and compared with test results. Stiffness properties were calculated assuming linear elastic behaviour of each wood veneer in the plywood, and strength properties were determined using a two-dimensional plane stress failure criterion and assumed values for shear strength and axial strength of wood. Experimental values were close to the theoretical relationships derived for in-plane bending strength and stiffness, tension strength, and compression strength, and for stiffness bending perpendicular to the plane of the plywood. The derived relationships were used to calculate strength ratios which may be used for the design of plywood gusset plates and other components where stress is applied at angles other than parallel or perpendicular to the grain of the face veneer. Experimental results verify that the current code basic working stresses parallel to the grain derived from out-of-plane bending tests can be used for in-plane bending.

INTRODUCTION

Plywood is an assembled product comprising thin layers of wood bonded together with the grain direction of adjacent layers at right angles. Wood is anisotropic – that is, its properties vary with grain direction. It is strongest when stressed parallel to the grain and weakest when stressed perpendicular to it. Consequently, when wood veneer is glued together to form plywood the orientation of the grain of the face veneer determines for each construction (lay-up) which plies are most effective in resisting applied stresses. Any theory or analysis of plywood strength or stiffness must first deal with the strength and stiffness of the wood veneer. These results may then be applied to plywood of a particular lay-up. This is relatively straightforward for stiffness properties, but beyond the proportional limit, analysis relies on assumed failure criteria and stress and strain distributions.
In the gusset plates of portal frames, plywood acts in bending in the plane of the panel. In design it has been usual to assume that bending in the plane of the panel results in stress values similar to face load bending. This assumption needs to be verified since the stress distributions are different.

Plywood design is governed by the requirements of the plywood design code NZS 3615. The relatively low allowable stresses at 45° to the face grain given for plywood in bending, tension, and compression have been a cause for some concern since they apply directly to plywood gusset design. These apparently low values are derived from North American sources and may or may not apply to *P. radiata* plywood. The results of pilot compression tests agreed with the code value for compression and with a theoretical model. However, the same theory implied that bending and tension values should be double those which are currently allowed in the code. In view of these anomalies a test programme was established to provide design values for:

1. The tension strength of plywood at various angles to the face grain;
2. The compression strength of plywood at various angles to the face grain;
3. The in-plane bending strength and stiffness of plywood at various angles to the face grain;
4. The in-plane bending strength of plywood compared with the face load bending strength.

To obtain general design values from the results of testing on a limited number of plywood lay-ups, a theory is necessary which can reproduce the test results by calculation. To this end, governing equations were derived for *P. radiata* plywood and their predictions were compared with experimental results.

**OUTLINE OF EXPERIMENTAL PROGRAMME**

**Test Material**

Two batches of 10 sheets each of 5-ply 12.5-mm AA-grade plywood were supplied by N.Z. Forest Products Ltd. Clear specimens were cut from the first batch according to the cutting plan shown in Fig. 1.

Each sheet provided:

1. 700 × 75-mm specimen cut at 45° to face grain
2. 1200 × 75-mm specimens cut at 45° to face grain
3. 1200 × 75-mm specimens cut at 30° to face grain
4. 1200 × 75-mm specimens cut at 15° to face grain
5. 1200 × 75-mm specimens cut at 0° to face grain
6. 700 × 75-mm specimen cut at 0° to face grain
7. 700 × 150-mm specimen cut at 0° to face grain
8. 2400 × 150-mm specimen cut at 0° to face grain

From the second batch an additional 1200 × 75-mm specimen at 75° was obtained for in-plane bending tests.
Tests were carried out on the Baldwin, Warner and Swasey, Weidemann Universal Testing Machine (Serial No. MA-1B 1279).

**Bending in-plane**

One of each 1200 × 75-mm specimen was tested in bending with a central point load over a span of 1050 mm (L/D = 14) with the specimen held upright and restrained from buckling on the compression edge using roller bearings at four points along the span. The 2400 × 150-mm specimen was likewise tested over a span of 2100 mm. The cross-sectional dimensions were measured prior to test. The rate of loading in these tests maintained a crosshead movement of 7 mm per minute.

**Bending flat**

Each 700-mm long specimen was tested in bending with a central point load over a span of 450 mm. Cross-section was measured prior to testing. The rate of crosshead movement was 14 mm per minute.

From automatic measurement of deflection and load in the above tests, the modulus of elasticity and modulus of rupture for each test specimen were calculated, using the full cross-section properties.
Tension test

The remaining 1200-mm specimens were tested in tension. Maximum tensile load was recorded and the stress calculated on the measured full cross-section properties.

Results

The results appear as appropriate in the Figures which follow. Data for 3- and 7-ply plywood were from C.R. Hellawell's tests (unpubl. data) done in the mid 1960s.

COMPARISON OF IN-PLANE AND FLAT BENDING MODES

The basic working stress in bending given in the plywood design code (NZS 3615:1981) is derived from bending tests perpendicular to the plane of the panel, parallel to the face grain. In order to justify the use of this stress for in-plane bending parallel to the face grain, both flat and in-plane tests were carried out. The veneer stresses were calculated as 89.1 MPa for flat bending and 86.52 MPa for in-plane bending. The code basic working stress parallel to the grain can therefore be used for both modes.

ELASTICITY

Modulus of Elasticity of Wood

The elastic properties of wood and plywood have been the subject of much investigation. Hearmon (1948) discussed the effect of grain angle on the elastic properties of wood and used these results to predict the stiffness of plywood. His literature review is complemented by that of Kollman et al. (1975) who listed the elastic constants of wood for many species from a number of sources. From the literature, three conclusions can be drawn.

(1) Hankinson's equation:

\[
E_\theta = \frac{E_0 E_{90}}{E_0 \sin^2 \theta + E_{90} \cos^2 \theta}
\]

(1)

can be used to relate the modulus of elasticity of wood at any angle \(E_\theta\) to the moduli parallel \(E_0\) and perpendicular \(E_{90}\).

(2) The data for \(E_0\) and \(E_{90}\) are extremely variable. The ratio \(E_0/E_{90}\) ranges from 11 to over 60 for hardwoods and from 18 to 40 for softwoods.

(3) Nowhere in the literature is there any study of this ratio in \(P.\ radiata\). Traditionally the New Zealand plywood design code has assumed \(E_0\) to \(E_{90}\) to be 20. This was based on informed judgement (C.R. Hellawell, unpubl. data) and seems to be derived from early American practice. Section properties for \(P.\ radiata\) plywood in the current New Zealand plywood design code for stiffness and strength are calculated using this value. That is, elastic behaviour has been assumed to occur up to failure.
Bending Stiffness of Plywood

The stiffness of plywood in bending may be evaluated by summing the contributions computed for each veneer.

From a rearrangement of Equation (1) each veneer has the characteristic:

\[
\frac{E}{E_0} = \frac{1}{(\frac{E}{E_{90}} - 1) \sin^2 \theta + 1}
\]

(2)

Let \( B = \frac{E_0}{E_{90}} - 1 \)

then, for veneers parallel to the face grain (at angle \( \theta \) to the direction of stress)

\[
\frac{E}{E_0} = \frac{1}{B \sin^2 \theta + 1}
\]

(3)

and, for the crossbands, perpendicular to the face grain (at angle \( 90 - \theta \) to the direction of stress)

\[
\frac{E_{90}}{E_0} = \frac{1}{B \cos^2 \theta + 1}
\]

(4)

In-plane bending and axial stress

Define,

\[
T = \frac{\text{thickness of parallel plies in panel}}{\text{total thickness of panel}}
\]

then, for a unit width of panel,

\[
\frac{(EI)_\theta}{E_0 I_z} = T \frac{1}{B \sin^2 \theta + 1} + (1 - T) \frac{1}{B \cos^2 \theta + 1}
\]

(5)

where \( E_0 I_z \) is the stiffness of a piece of parallel-to-grain "veneer" of the same thickness as the plywood, and \( (EI)_\theta \) is the bending stiffness of the plywood at angle \( \theta \) to face grain. The stiffness ratio for axial loading has the same value as Equation (5).

Equation (5) is shown in Fig. 2(a) as computed for \( E_0 / E_{90} = 20 \) and ratios of \( T \) of 0.5, 0.57, 0.6, and 0.67 which cover the range of construction plywoods currently available in New Zealand.

Figure 3 shows the curve for 5-ply \( (T = 0.6) \) superimposed on the moduli of elasticity determined from in-plane bending tests on 5-ply \( P. radiata \) plywood (see Experimental Programme). The y axis intercept of Equation (5) has been scaled to fit the mean value of the parallel-to-face-grain test results. The agreement between test and theory is self evident.
Figure 2(b) shows the sensitivity of Equation (5) to changes in the E ratio. As the ratio increases, the theory approaches the parallel-plies-only model (used with modification factors in North America) which is only really applicable at 0° or 90°. The E ratio value of 20 gives the best fit.

**Bending perpendicular to the plane of the panel**

When plywood is face loaded the outer plies carry most of the load. The contribution of each veneer must be determined from its position in the cross-section and its thickness. Assuming linear strain, the moment of inertia of the parallel veneers ($I_1$) is given for 5-ply plywood by

$$I_1 = \frac{t^3}{12} \left[ 1 - \left( \frac{3}{5} \right)^3 + \left( \frac{1}{5} \right)^3 \right] = 0.792 I_g$$

The contribution of the perpendicular veneers ($I_{II}$) is given by

$$I_{II} = \frac{t^3}{12} \left[ \left( \frac{3}{5} \right)^3 - \left( \frac{1}{5} \right)^3 \right] = 0.208 I_g$$

This can be expressed in general terms for a symmetrical plywood panel of $n$ plies of equal thickness as

$$I_1 = I_g \sum_{j=0}^{n-1} \left( \frac{n-2j}{n} \right)^3 (-1)^j$$

$$= I_g \Sigma$$

and $I_{II} = I_g (1 - \Sigma)$

From Equation (5) it follows that, for a plywood panel of unit width, bending flat

$$\frac{E I_g}{E_0 I_g} = \frac{\Sigma}{B \sin^2 \theta + 1} + \frac{1 - \Sigma}{B \cos^2 \theta + 1}$$

This curve is plotted in Fig. 4 for the range of constructions of *P. radiata* plywood and is superimposed on the results of bending tests normalised as a proportion of stiffness bending parallel to the face grain.
FIG. 2—In-plane bending stiffness of plywood. (a) Percentage of parallel veneer in panel cross-section is shown on each curve. (b) Ratio of $E$ parallel to $E$ perpendicular for the veneer is shown on each curve.
FIG. 3—Modulus of elasticity v. face grain angle.

FIG. 4—Stiffness of plywood bending flat. Number of plies is shown on each curve.
STRENGTH

Beyond the limit of proportionality the relationships between stress and strain become difficult to determine. There are, however, theories of failure which may be applied to stresses at ultimate loads. Norris (1942, 1943) described the use of Mohr’s circle in determining the stresses at any angle to the grain and suggested the use of Hankinson’s formula for computing the strength. However, failure loads are dependent on the magnitude of the direct stresses parallel to and perpendicular to the grain and the shear stresses. Hankinson’s formula does not include a shear stress term. Van der Put (1982) investigated a general failure criteria which utilised tensor algebra to deal with the three-dimensional problem in wood. Veneer stress and strain can, however, be reduced to a simpler two-dimensional problem. Norén (1964) described one failure criterion which included shear stress and which forms the basis for the following analyses.

Anisotropic Properties of Wood Veneer

Figure 5 shows a sheet of veneer subjected to plane stress with the principal stress $\sigma_a$ and $\sigma_b$ in directions a and b where a makes an angle $\theta$ with the grain direction. An element of veneer within the sheet is aligned parallel to the grain direction. Consequently this element is stressed parallel and perpendicular to the grain by stresses $\sigma_0$ and $\sigma_{90}$ and a shear stress $\tau$. The relationship between these stresses is illustrated by Mohr’s circle (Fig. 6) and is described by

$$
\sigma_0 = \sigma_a \cos^2 \theta + \sigma_b \sin^2 \theta
$$
$$
\sigma_{90} = \sigma_b \cos^2 \theta + \sigma_a \sin^2 \theta
$$
$$
\tau = \frac{\sigma_a - \sigma_b}{2} \sin 2\theta
$$

If the element is subjected only to stresses in direction a, then $\sigma_b = 0$ and Equations (8) reduce to

$$
\sigma_0 = \sigma_a \cos^2 \theta
$$
$$
\sigma_{90} = \sigma_a \sin^2 \theta
$$
$$
\tau = \sigma_a \sin \theta \cos \theta
$$

FIG. 5— An element of veneer subjected to principal stresses $\sigma_a$ and $\sigma_b$ at angle $\theta$ to grain direction.
Strength of plywood

The assumed failure criterion is expressed as

\[
\left( \frac{\sigma_0}{\sigma_{1,U}} \right)^2 + \left( \frac{\sigma_{90,U}}{\sigma_{11,U}} \right)^2 + \left( \frac{\tau}{\tau_{11}} \right)^2 = 1
\]

(10)

which Norris (1962) described as being an empirical formula fitted to data from tension, compression, and shear tests on yellow poplar plywood.

\(\sigma_0\) is now the stress in an element in the direction parallel to the face grain and \(\sigma_{1,U}\) is the strength of the plywood in that direction. Likewise the subscript \(\text{II}\) refers to the perpendicular direction.

Axial strength and in-plane bending

For a plywood element subjected to uniform axial strain throughout thickness of the panel:

\[
\begin{align*}
\sigma_{1,U} &= T \sigma_{0,u} + (1 - T) \sigma_{90,u} \\
\sigma_{11,U} &= T \sigma_{90,u} + (1 - T) \sigma_{0,u}
\end{align*}
\]

(a) (b)

(11)

where \(\sigma_a\) is the veneer strength at 0° or 90° to the grain as subscripted, and \(T\) the proportion of parallel veneers in the cross-section. Norén (1964) assumed \(\sigma_{90,u} = 0\) and, while this is acceptable for tension, the strength of the perpendicular veneer may contribute significantly to the over-all strength of the panel for compression and bending.
Let $A = \frac{\sigma_{0,u}}{\sigma_{90,u}}$

where $A$ is thus defined as the axial stress ratio.

Then $\sigma_{1,u} = \left[ T + \left( \frac{1-T}{A} \right) \right] \sigma_{0,u}$ (12a)

$\sigma_{11,u} = \left[ \frac{T}{A} + (1-T) \right] \sigma_{0,u}$ (12b)

Let $S = \frac{\sigma_{0,u}}{\tau_{0,u}} = \frac{\text{axial strength of veneer parallel to grain}}{\text{shear strength of veneer parallel to grain}}$

where $S$ is thus defined as the shear stress ratio.

The shear strength of the plywood is given by

$\tau_{U} = T \tau_{0,u} + (1-T) \tau_{90,u}$

but $\tau_{0,u}$ is complementary and equal to $\tau_{90,u}$.

Therefore

$\tau_{U} = \frac{\sigma_{0,u}}{S}$ (12c)

Substitution of Equations (12) and (9) into (10) leads to

$$\frac{\sigma_{3}}{\sigma_{0,u}} = \sqrt{\left( \frac{\cos^2 \theta}{T + \frac{1-T}{A}} \right)^2 + \left( \frac{\sin^2 \theta}{A} \right)^2 + \left( \frac{\sin \theta \cos \theta}{1-S} \right)^2}$$

(13)

where $\frac{\sigma_{3}}{\sigma_{0,u}} = \frac{\text{strength of plywood panel at angle } \theta \text{ to face grain}}{\text{axial strength of veneer parallel to the grain}}$

Equation (13) may be plotted for different values of $T$(lay-up), $A$, and $S$ to determine the effect of each variable on the strength of the plywood.

**Bending perpendicular to the plane of the panel**

The moment of resistance of any panel cross-section will depend on the compressive and tensile veneer strengths parallel and perpendicular to the grain, and on the panel construction. In order to simplify computation a linear stress distribution is assumed (as is usual for modulus of rupture calculation).

We recall that in general

$$Z = \frac{2L}{t}$$
It follows from Equation (6) that
\[ Z_1 = Z_g \Sigma \]
\[ Z_{11} = Z_g (1 - \Sigma) \]
whence, Equations (11) become
\[ \sigma_{1,1} = \Sigma \sigma_{0,1} + (1 - \Sigma) \sigma_{90,1} \]  
\[ \sigma_{11,1} = \Sigma \sigma_{90,1} + (1 - \Sigma) \sigma_{0,1} \]  \hspace{1cm} (14a)
\hspace{1cm} (14b)

The strength equation for bending flat is therefore the same as Equation (13) with \( T \) replaced by \( \Sigma \).
\[ \frac{\sigma_\mu}{\sigma_{0,\mu}} = \sqrt{ \left( \frac{\cos^2 \theta}{1 - \Sigma} \right)^2 + \left( \frac{\sin \theta \cos \theta}{\Sigma} \right)^2 + \left( \frac{\sin^2 \theta}{1 - \Sigma} \right)^2 } \]  \hspace{1cm} (15)

However, in bending perpendicular to the grain the outer perpendicular veneer is often ignored in section property calculation since it is of low tensile strength. The value of \( \Sigma \) must accordingly be reduced by the second moment of area of the outer ply. That is
\[ I_{11} = I_g (\Sigma - \frac{3(n - 1)^2}{n}) \]  \hspace{1cm} (16)

Likewise, the distance to the extreme fibre is reduced and
\[ Z_{11} = Z_g \left( \frac{n}{n - 2} \right) \left( \Sigma - \frac{3(n - 1)^2}{n} \right) \]  \hspace{1cm} (17)

In bending at 45° to the face grain, the full cross-section resists the applied stress. Somewhere between 45° and 90° it becomes prudent to ignore the outer ply. If Equation (15) is computed for both full and reduced section values of \( Z_{11} \) and the greater value is chosen as the strength ratio above 45° face grain angle, then the stress ratio of plywood bending flat may be plotted as a smooth curve for various values of \( \Sigma \) (lay-up section property), \( A \), and \( S \).

**APPLICATION OF THE STRENGTH THEORY TO P. RADIATA PLYWOOD**

Equation (13) is plotted in Fig. 7 and 8 for different values of \( A \) and \( S \). These figures show that the equation is not sensitive to small changes in \( A \) above about 20, but low values of \( A \) increase the panel strength for grain angles below about 15° and above 65°. Therefore, for bending and tension the ratio \( A \) is not critical. In compression it should be determined with care. This is difficult, since strength in compression perpendicular to the grain is usually given at the proportional limit. The equation is very dependent on the chosen shear stress ratio except within about 3° of 0° and 90°. The ratio of \( S \) should be carefully considered for all stress modes.
Hinds & Reid (1957) have published data for the following air-dry (12% m.c.) *P. radiata* clear properties, both parallel to and perpendicular to the grain.

- **Shear parallel to grain**: 10.7 MPa
- **Tension parallel to grain**: 90.0 MPa*
- **Compression parallel to grain**: 40.7 MPa (ultimate)
- **Compression parallel to grain**: 25.6 MPa (proportional limit)
- **Bending (MOR)**: 76.0 MPa
- **Tension perpendicular to grain**: 2.82 MPa
- **Compression perpendicular to grain**: 5.92 MPa (proportional limit)

The ratios A and S for each stress mode can be determined as:

<table>
<thead>
<tr>
<th>Stress Mode</th>
<th>A</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tension</td>
<td>32</td>
<td>8.5</td>
</tr>
<tr>
<td>Bending</td>
<td>27</td>
<td>7.5</td>
</tr>
<tr>
<td>Compression</td>
<td>4.3</td>
<td>4</td>
</tr>
</tbody>
</table>

![Graph](image)

FIG. 7—Strength as a function of axial stress ratio. Ratio A is shown on each curve.

* This value is not given, but estimated from tests on clear plywood (C.R. Hellawell, unpubl. data).
It is assumed that these ratios are typical, even if it may be argued that the absolute values of the strength properties are not.

It is now possible to substitute the appropriate values (a), (b), or (c) in Equations (13) or (15) to compare predicted strengths with observed test results.

**Tension**

Figure 9 shows the results of tension tests on 5-ply *P. radiata* at 0°, 15°, 30°, and 45° and Equation (13) plotted with ratios (a). Although the curve lies above the observed results the agreement is satisfactory.

**Compression**

The ratio panel strength at 45° panel strength at 0° is calculated from (c) as 0.62. Using the results of compression tests on 108 pieces of plywood (Bier unpubl. data) the observed value is 0.54. The lower observed value may be due to buckling of particularly the outer veneers. When a value of S of 4.8 is used, Equation (13) matches the observed value.

**In-plane bending**

Using the compression ratios (c) and the tension ratios (a) two curves may be drawn which outline a strength envelope within which all bending failures should occur.
These curves are shown in Fig. 10 together with the modulus of rupture values determined by test. It is evident that bending strength is governed mainly by tension failure. It is therefore appropriate to use the ratios (a) in the strength model for both in-plane bending and tension.

![Graph of tensile strength vs. face grain angle.](image)

**FIG. 9**—In-plane strength of plywood. Tensile strength v. face grain angle.

**Bending perpendicular to the plane of the panel**

The curves for bending flat, determined using ratios (b) are shown in Fig. 11 superimposed on the average values of test results normalised as a portion of strength parallel to grain. There is adequate agreement for 5-ply but inadequate data (only 90° values) for 3-ply or 7-ply. The model requires refinement to allow for the non-linear stress distribution and change in neutral axis depth. The discontinuity at 45° implies that the outer ply cannot be ignored. The curve for 3-ply includes the outer veneer to avoid a nonsensical prediction of a greater strength at 90° than parallel to the face grain. This stress mode is not often encountered with different face grain angles in practice and further work is probably unwarranted.

**CONCLUSIONS**

For direct stress and bending in the plane of the panel, Equations (5) and (13) predict the stiffness and strength of plywood at various angles to face grain and have been confirmed by experiment.

For bending perpendicular to the plane of the panel, the stiffness at angles to the face grain can be predicted by Equation (7).
The basic working stress given for plywood at 45° to grain in NZS:3615 is conservative, and should have different values for bending in-plane and bending flat.

**DESIGN IMPLICATIONS**

The stress ratio of different lay-ups of plywood at any angle to the grain may be calculated from Equation (13) (or (15). Dividing the equation for a specific lay-up by the ratio at 0° yields

\[
\frac{\sigma_\theta}{\sigma_{0,\theta}} = \frac{\sigma_\theta}{\sigma_0}
\]

which is the ratio of panel strength at angle $\theta$ to the face grain to the panel strength parallel. This ratio is plotted for in-plane stress in Fig. 12 and can be defined as the face grain orientation factor for strength, $K_F$.

Values of the $K_F$ were calculated for each lay-up at 0°, 15°, 30°, 45°, 60°, 75°, and 90° and are given in Table 1.

There are three possible approaches to the design of plywood elements at angles to the stress direction.
FIG. 11—Strength of plywood bending flat. Number of plies is shown on curve.

FIG. 12—Strength as a function of panel construction. Percentage of parallel veneer in each panel is shown on each curve.
TABLE 1 - Face-grain orientation factor (strength) for given grain angle

<table>
<thead>
<tr>
<th></th>
<th>0°</th>
<th>15°</th>
<th>30°</th>
<th>45°</th>
<th>60°</th>
<th>75°</th>
<th>90°</th>
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<tr>
<td><strong>Compression strength</strong></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4-ply</td>
<td>1.0</td>
<td>0.83</td>
<td>0.66</td>
<td>0.60</td>
<td>0.66</td>
<td>0.83</td>
<td>1.0</td>
</tr>
<tr>
<td>7-ply</td>
<td>1.0</td>
<td>0.81</td>
<td>0.62</td>
<td>0.56</td>
<td>0.59</td>
<td>0.73</td>
<td>0.85</td>
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<tr>
<td>5-ply</td>
<td>1.0</td>
<td>0.80</td>
<td>0.60</td>
<td>0.54</td>
<td>0.57</td>
<td>0.69</td>
<td>0.79</td>
</tr>
<tr>
<td>3-ply</td>
<td>1.0</td>
<td>0.77</td>
<td>0.56</td>
<td>0.50</td>
<td>0.51</td>
<td>0.60</td>
<td>0.66</td>
</tr>
<tr>
<td><strong>Tension strength and in-plane bending strength</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4-ply</td>
<td>1.0</td>
<td>0.69</td>
<td>0.49</td>
<td>0.43</td>
<td>0.49</td>
<td>0.69</td>
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<tr>
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<td>0.64</td>
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<td>0.57</td>
<td>0.77</td>
</tr>
<tr>
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<td>0.62</td>
<td>0.42</td>
<td>0.36</td>
<td>0.40</td>
<td>0.53</td>
<td>0.68</td>
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<td>0.32</td>
<td>0.34</td>
<td>0.43</td>
<td>0.52</td>
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<tr>
<td><strong>Bending strength perpendicular to the plane of the panel</strong></td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>3-ply</td>
<td>1.0</td>
<td>0.45</td>
<td>0.22</td>
<td>0.13</td>
<td>0.09</td>
<td>0.08</td>
<td>0.07</td>
</tr>
<tr>
<td>4-ply</td>
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<td>0.31</td>
<td>0.23</td>
<td>0.20</td>
<td>0.18</td>
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</tr>
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<td>5-ply</td>
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<td>7-ply</td>
<td>1.0</td>
<td>0.61</td>
<td>0.40</td>
<td>0.34</td>
<td>0.38</td>
<td>0.50</td>
<td>0.65</td>
</tr>
<tr>
<td>9-ply</td>
<td>1.0</td>
<td>0.63</td>
<td>0.42</td>
<td>0.36</td>
<td>0.41</td>
<td>0.56</td>
<td>0.74</td>
</tr>
</tbody>
</table>

(1) Capacity approach: The product of allowable stress times section property parallel to the grain is modified by the appropriate grain angle factor

\[ \text{Strength } \theta = K_F \text{ strength} \]

(2) Section property modification: The section property is multiplied by the modification factor

\[ Z_\theta = K_F \cdot Z \]

(3) Allowable stress modification: Using the section property parallel to the face grain, the allowable stress is modified by an additional K factor appropriate to the required direction of stress

\[ F_\theta = F \cdot K_F \]

Section properties have traditionally been calculated assuming linear elastic behaviour. For strength parallel to the grain they are close to values calculated using Equation (13). The use of section properties for plywood parallel to the face grain determined from Appendix A of NZS 3615:1981 is therefore justified.

Similar procedures can be used to determine the stiffness at any grain angle using Equation (5) to provide the required K factor.

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REFERENCES


SYMBOLS USED
A Ratio of axial strengths of veneer = \frac{\sigma_{90}}{\sigma_{90}}
B Function of ratio of moduli of elasticity of veneer = \frac{E}{E_{90}} - 1
E Modulus of elasticity
F Basic working stress
I Second moment of area
K_F Face grain orientation factor
S Ratio of axial strength to shear strength of veneer = \frac{\sigma_{0}}{\sigma_{u}}
T Ratio = \frac{\text{Thickness of parallel plies in balanced panel}}{\text{Thickness of panel}}
t Thickness of plywood
Z Section modulus
\Sigma Summation of second moments of area of parallel plies in a panel, see Equation (6)
\sigma Direct stress
\tau Shear stress

Subscripts
0 Property parallel to grain of veneer
90 Property perpendicular to grain of veneer
a Direction of principal stress at angle to grain of \theta
b Perpendicular to direction a
i Property parallel to face grain of plywood
II Property perpendicular to face grain of plywood
\Phi Failure value (strength) of plywood
u Failure value (strength) of veneer
e Gross, dimensions based on full cross-section
d Denote properties and stresses parallel and perpendicular to the face and grain as currently used in design. The parallel values incorporate a contribution from perpendicular veneers and vice versa.