MINIMUM TOTAL COST: AN IMPROVED WEIGH SCALING STRATEGY

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(First received for publication 9 November 1977; in revised form 11 September 1978)

ABSTRACT

The usual statistical sampling technique of choosing a sample size to produce an estimate with a specified error limit can be improved upon in situations where indirect costs resulting from estimation errors can be evaluated. The estimation of the weight to volume conversion factor in weigh scaling is such a situation and this paper presents the formulae needed to determine sample sizes that will minimise total scaling costs for both simple random sampling and stratified random sampling. Using cost and production data that is representative of weigh scaling for a New Zealand Forest Service conservancy, the minimum total scaling cost strategy is compared with the 2½ percent error strategy in terms of sample sizes, variable and total scaling costs, and standard error attained. The comparisons illustrate the differences between the strategies when considering various stratum classifications and stumpage rates.

The minimum total cost strategy produces significant savings compared to the current method and produces more accurate estimates (i.e. smaller standard error) of more valuable forest products which is an intuitively desirable characteristic.

INTRODUCTION

Stumpage paid for timber removals from a forest has traditionally been calculated on a volume basis. On the other hand, the net weight of timber removed can be more easily estimated, especially during truck hauling operations, through the use of weighbridges. Thus volume estimates can be obtained at low cost by using weight to volume conversion factors. This weight-to-volume conversion is of special importance to both buyers and sellers of timber from New Zealand forests because of the large sums of money involved.

This paper reviews the current method of converting weight to volume estimates as practised by the New Zealand Forest Service conservancies and presents a rational methodology which will minimise the measurement workload and will satisfy both the buyer and the seller that an unbiased and precise estimate of log volume is obtained.

Current Weigh Scaling Method

Over the years the New Zealand Forest Service has, for estimating the volume of timber removals, developed methods that are fast, relatively free from error, and which do not interfere too much with the logging operations. To begin with, the net weight of wood, bark and foreign matter is found by weighing loaded trucks on strategically located weighbridges and deducting the tare weight of the vehicles from the gross weight. A weight-to-volume conversion factor is then applied to the net weight to obtain a volume estimate. The conversion factor is derived by finding the volume from sectional measurements of previously weighed loads that have been selected according to a specified sampling scheme. The volume/weight ratio is subject to error due to variation in moisture content, wood density and the relative amount of foreign material and bark present in the loads. As a result, different factors are estimated for each forest compartment, species, season and for each product mix that is removed from the forest.

The conversion factors meet specified maximum error requirements (e.g. $\pm 2\frac{1}{2}\%$ with 95% probability) and ensure that at least the minimum number of loads will be sampled in order to produce a conversion factor of specified standard error.

The successful use of this method is determined by the conservancy's ability to choose acceptable error levels for various quantities and values of forest products. For example, it seems logical that a product with a high unit value should be estimated more accurately, with smaller error, than a low value product. However, conservancies, and decision makers in general, may have difficulty in assigning appropriate error levels because the value varies with the proportions of peelers, sawlogs and smallwood that come from a forest. A more appealing strategy is the one presented here which determines the sample size which minimises the total scaling cost.

THE MINIMUM TOTAL COST STRATEGY

Total scaling costs include both direct and indirect costs. Direct costs are of two sorts: fixed costs such as weighbridge depreciation, maintenance, and staffing, which do not vary with the number of sampled loads of wood to be sectionally measured; and variable costs such as the machine and labour costs incurred in unloading, measuring and reloading logs that are sectionally measured.

Indirect scaling costs on the other hand are costs that are incurred because the estimate of wood volume is in error. Such costs would be the dollar loss in stumpage incurred by the Forest Service or the customer because the estimate was respectively either under or over the actual volume removed.

The objective of the minimum total cost procedure is to establish for log populations conversion factors which are satisfactory to both the buyer and the seller of logs and whose derivation incurs the minimum scaling costs. To be acceptable to the buyer and seller both should be satisfied that the *value* of forest produce is fairly assessed. It follows that the estimate provided for high value products should be more precisely determined than that for low value products. If the estimate of wood volume is in error *cost* is incurred. An underestimate results in a cost to the seller in stumpage lost, while an overestimate results in a cost to the buyer in overpayment. These costs can be considered equal to the standard error of the weight/volume conversion factor multiplied by the total value of the log product. This is a measure of the indirect cost incurred by either buyer or seller for a given conversion factor.

The relevance of the direct cost of scaling on the acceptability of the sample size is dependent on the scaling costs being shared. This is not generally the case in New Zealand. In some cases the weighbridge is owned and operated by the seller and in others by the buyer, and in all cases the actual log sectional measurement cost is born by the seller. The methodology assumes that these direct scaling costs should be shared by both (a not-unreasonable assumption). The minimum total cost approach is not new, and was proposed by Blythe (1945) for use in the scaling of sawlogs.

There are two quite different situations for which conservancies must calculate conversion factors. The first and less common situation, is where a conservancy has many small customers, each buying a single species from a single compartment within a given season. This is the "one customer-one stratum" situation and can be treated as a simple random sampling problem. The direct and indirect scaling cost formulae, as well as the formula to estimate the sample size to minimise total scaling costs in this situation, are listed in Table 1. Derivation of the formula to estimate the sample size for minimum total scaling costs is presented elsewhere*. The second more common situation encountered is the one where a conservancy has only a few customers and each customer buys one or more species in a number of compartments throughout the year. This is the "one customer-many strata" situation and can be treated as a stratified random sampling problem. The cost and sample allocation formulae used in this situation are listed in Table 2, and the derivation of the formulae are also given elsewhere*.

TABLE 1—Cost and sampling formulae to be used in "One Customer-One Stratum" situations

	$TC = DC + IC$	(1)
where	TC is total scaling cost per year	
	DC is direct scaling cost per year	
	IC is the average indirect scaling cost per year	
	$DC = LF + Vn$	(2)
where	L is total loads weighed per year	
	F is the fixed cost of weighing a load	
	V is the cost of sectionally measuring a load	
	n is the number of loads sectionally measured	
	$IC = 1.25\ W.L.R\sqrt{S^2/n}$	(3)
or	$IC = CS/\sqrt{n}$	(3a)
where	W is average weight/load (tonnes)	
	R is stumpage rate $/m³	
	S^2/n is the variance of the estimated mean volume/weight ratio	
	n is the number of loads sectionally measured	
	$C = 1.25$ W.L.R.	
	$n = (CS/2V)^{2/3}$	(4)

TABLE 2—Cost and sampling formulae to be used in "One Customer-Many Strata" situations

	$DC = LF + V\Sigma n_h$	(5)
	$IC = 1.25W.L.R.(\Sigma P^2_h S^2_h/n_h)^{1/2}$	(6)
or	$IC = C(\Sigma P^2_h S^2_h/n_h)^{1/2}$	(6a)
	$n = (C(\Sigma P_h S_h)^2/2V)^{2/3}$	(7)
and	$N_h = nP_h S_h/\Sigma P_h S_h$	(8)
where	N_h is the number of loads sampled in the h^{th} stratum	
	S_h is the standard deviation of the ratio in the h^{th} stratum	
	P_h is the fraction of the total loads in the h^{th} stratum	
	$N = \Sigma n_h$ — total sample size	

and all other notation is as previously defined in Table 1.

* Supporting supplement, obtainable from the Editor on request.

COMPARISON OF CURRENT AND MINIMUM TOTAL COST SCALING METHODS

In order to gain an insight into the practical differences that may exist between the current method and the minimum total cost method, actual scaling data were obtained and used to test the two methods. In particular, analyses were performed to answer the following questions:

1. How important are differences in stumpage rates on the optimum sample size and total scaling costs under New Zealand scaling conditions?

2. What is the relative efficiency of the minimum total cost method compared to the current method in terms of total scaling costs?

3. What degree of stratification should be carried out? In other words, is it necessary to stratify on such factors as species, season, compartment or not?

The analysis of the more usual one customer-many strata formulae is described in this paper. A more detailed description of this analysis as well as the one customer-one stratum analysis may be found in the supporting supplement.

The data used in the analysis (see Acknowledgments) included the weight and volume of 188 truck loads that were sectionally measured in five three-month periods between 1 April 1976 and 30 June 1977. Each load was identified with respect to species, forest, compartment number, season, log type (export or domestic) and stand age. The following are some of the statistics associated with these data.

Total number loads — 188
No. loads — *Pinus radiata* — 140
No. loads — *Pinus nigra* — 48
Average weight/load — 19.87 tonnes (range 7.8-35.5)
Average age of stands cut — 44.5 years (range 24-75)
Total number of compartments sampled — 23
Number of forests sampled — 4

Regression techniques (Freese, 1964) were used to analyse these data to determine how much the standard error of the estimated volume/weight ratio increased as the data were consolidated into larger and larger groupings. Table 3 summarises these initial results.

At this preliminary stage of the analysis it was apparent that the errors associated with the ratio estimate for *P. radiata* are twice as great as for *P. nigra* and therefore the ratios for the two species should be estimated separately.

Assuming the following to be a realistic set of scaling, production and cost figures for a customer, the optimum sample sizes and costs have been calculated and compared for the current and minimum cost methods, based on stumpage rates of \$5/m^3, \$10/m^3 and \$15/m^3.

Fixed scaling cost — \$1.60/load weighed
Variable scaling cost — \$16.50/load sectionally measured
No. loads *P. radiata*/year 4500
No. loads *P. nigra*/year 1500
Average tonnes/load 19.87

TABLE 3—Effect of stratification on standard deviation of ratio

Stratification	Standard Deviation of Ratio Estimate (m^3/t)		
	P. radiata	**P. nigra**	Combined
1. Species	0.0823	0.0368	0.0639
2. Species — Forest	0.0640	0.0308	0.0525
3. Species — Forest — Season	0.0608	0.0285	0.0475
4. Compartment — Species	0.0501	0.0235	0.0391
5. Compartment — Species — Season	0.0485	0.0227	0.0379

Table 4 lists the sample sizes, direct and indirect costs and standard error obtained using the current method. The total sample size to attain a $2\frac{1}{2}\%$ error of the estimated volume of wood was found using the stratified random sample optimum allocation formula for an infinite population (Freese, 1962).

$$n = t^2(\Sigma P_h S_h)^2/(0.025K)^2 \qquad (10)$$

where P and S_h are as previously defined in Table 2
and n is sample size
t is Student's t
K is the approximate value (m^3/tonne), of the conversion factor.

K was set at 1.00 and t was set at 2.00 thus permitting an approximate 5% probability of obtaining estimates with greater than $2\frac{1}{2}\%$ error. Allocation of numbers of samples per stratum (n_h) were found using Equation 8.

For example, the sample sizes from Equation 10 where the Conservancy is selling timber from eight strata (two species in each of four forests), and using the standard deviations listed in Table 3, would be:

$$\begin{aligned} n &= 2^2(\tfrac{1}{8} \times 0.0640 + \tfrac{1}{8} \times 0.0640 + \ldots + \tfrac{1}{8} \times 0.0308)^2/(0.025)^2 \\ &= 19.9 \end{aligned}$$

The sample allocation for a *P. radiata* stratum would be:

$$\begin{aligned} n_r &= 19.9 \times (\tfrac{1}{8}) \times 0.0640/(\tfrac{1}{8} \times 0.0640 + \tfrac{1}{8} \times 0.0640 + \ldots + \tfrac{1}{8} \times 0.0308) \\ &= 4.3 \end{aligned}$$

The direct scaling cost for the forest-species stratification option is found using Equation 5 where 4 of the 8 strata are *P. radiata* and 4 are *P. nigra.*

$$\begin{aligned} DC &= 6000 \times 1.60 + 16.50\ (4.3 + 4.3 + 4.3 + 4.3 + 0.7 + 0.7 + 0.7 + 0.7) \\ &= \$9930 \end{aligned}$$

The indirect cost for the forest-species stratification option with $\$5/m^3$ stumpage is found using Equation 6.

$$\begin{aligned} IC &= 1.25 \times 19.8 \times 6000 \times 5.00\ (4 \times (\tfrac{1}{8} \times 0.0640/4.3)^2 + 4 \times \\ &\qquad (\tfrac{1}{8} \times 0.0308/0.7)^2)^{\frac{1}{2}} \\ &= \$9290 \end{aligned}$$

The standard error attained is found using the formula

$$\begin{aligned} SE &= (\Sigma P^2{}_h S^2{}_h/n_h)^{\frac{1}{2}} \qquad (11) \\ &= (4 \times (\tfrac{1}{8} \times 0.0640/4.3)^2 + 4 \times (\tfrac{1}{8} \times 0.0303/0.7)^2)^{\frac{1}{2}} \\ &= 0.0125 \end{aligned}$$

Table 5 lists the sample sizes, direct and indirect costs and standard errors obtained using the minimum total cost method.

Using the species-forest stratification option as an example, with stumpage set at $5/m³, the total sample size using Equation 7 is:

$$n = (1.25 \times 19.89 \times 6000 \times 5.00 (\tfrac{1}{8} \times 0.0640 + \tfrac{1}{8} \times 0.0640 + \tfrac{1}{8} \times 0.0640 + \ldots + \times 0.0308)^2/2 \times 16.50)^{2/3}$$
$$= 116.6$$

Using Equation 8, the sample to be taken in each *P. radiata* stratum is

$$n_r = 116.6 \times \tfrac{1}{8} \times 0.0640/(\tfrac{1}{8} \times 0.640 + \tfrac{1}{8} \times 0.0640 + \ldots + \tfrac{1}{8} \times 0.0308)$$
$$= 25.1$$

The sample size allocated to each *P. nigra* stratum is 4.0.

Direct and indirect scaling costs and attained standard errors have been calculated in exactly the same way as for the current method using Equation 10.

By comparing Tables 4 and 5, some insight is obtained into the answer to the questions raised at the start of this study.

According to the minimum total cost method, larger samples of both *P. radiata* and *P. nigra* loads are required than indicated according to the current method, and the sample size increases significantly as the stumpage rate increases from $5/m³ to $15/m³.

TABLE 4—Scaling costs when sampling with 2½% error limit

		Stratum Classification				
		Species	Species & Forest	Spec. For. Season	Comp't. & Species	Comp't., Spec. Season
(a) No. Strata/Year		2	8	32	40	60
(b) Initial Standard Deviation						
	P. radiata	0.0823	0.0646	0.0608	0.0501	0.0485
	P. nigra	0.0368	0.0308	0.0285	0.0235	0.0227
(c) Optimum Sample Size/Stratum						
	Species					
	P. radiata	28.0	4.3	0.92	0.35	0.22
	P. nigra	4.2	0.7	0.13	0.16	0.10
All Strata/Year		32.2	19.9	17.0	12.1	11.4
(d) Annual Scaling Cost ($)						
Stumpage	Type of Cost					
$5/m³	Direct	10131	9930	9877	9800	9788
	Indirect	9323	9290	9390	9317	9308
	Total	19454	19220	19267	19117	19096
$10/m³	Direct	10131	9930	9877	9800	9788
	Indirect	18645	18580	18780	18634	18616
	Total	28776	28510	28657	28434	28405
$15/m³	Direct	10131	9930	9877	9800	9788
	Indirect	27968	27870	28169	27951	27925
	Total	38099	37800	38046	37751	37713
(e) Standard Error Attained (m³/t)						
Stumpage						
$5/m³		0.0125	0.0125	0.0125	0.0125	0.0125
$10/m³		0.0125	0.0125	0.0125	0.0125	0.0125
$15/m³		0.0125	0.0125	0.0125	0.0125	0.0125

TABLE 5—Scaling costs using the minimum total cost method

		Stratum Classification				
		Species	Species & Forest	Spec. For. Season	Comp't. & Species	Comp't., Spec. Season
(a) No. Strata/Year		2	8	32	40	60
(b) Initial Standard Deviation						
	P. radiata	0.0823	0.0640	0.0608	0.0501	0.0485
	P. nigra	0.0368	0.0308	0.0285	0.0235	0.0227
(c) Optimum Sample Size/Stratum						
Stumpage	Species					
$5/m³	**P. radiata**	119.2	25.1	6.0	2.9	1.9
	P. nigra	17.8	4.0	0.8	1.3	0.9
All Strata/Year		137.0	116.6	110.8	98.8	96.8
$10/m³	**P. radiata**	189.2	39.9	10.0	4.5	3.0
	P. nigra	28.2	6.4	1.3	2.1	1.4
All Strata/Year		217.4	185.1	175.8	156.8	153.7
$15/m³	**P. radiata**	247.9	52.2	12.5	5.9	3.9
	P. nigra	37.0	8.4	1.6	2.8	1.8
All Strata/Year		284.9	242.5	230.4	205.5	201.33
(d) Annual Scaling Cost ($)						
Stumpage	Cost Type					
$5/m³	Direct	11861	11524	11428	11230	11197
	Indirect	4520	3851	3689	3241	3159
	Total	16381	15375	15117	14471	14356
$10/m³	Direct	13187	12654	12501	12187	12136
	Indirect	7176	6106	5723	5189	5032
	Total	20363	18760	18224	17376	17168
$15/m³	Direct	14301	13601	13402	12991	12992
	Indirect	9402	8005	7685	6791	6625
	Total	23703	21606	21087	19782	19547
(e) Standard Error Attained (m³/t)						
Stumpage						
$5/m³		0.0061	0.0052	0.0049	0.0043	0.0042
$10/m³		0.0048	0.0041	0.0038	0.0035	0.0034
$15/m³		0.0042	0.0036	0.0034	0.0030	0.0030

Correspondingly, the direct scaling cost for the minimum total cost method is higher than for the current method ranging from approximately \$1,700 more for the \$5/m^3 stumpage — two strata category to approximately \$4,200 more in the \$15/m^3 stumpage — two strata category.

On the other hand, when indirect costs are also included and the total cost is considered, the minimum total cost method produces a savings over the current method from approximately \$3,000/year when stumpage is \$5/m^3 to approximately \$18,200/year when stumpage is \$15/m^3. The standard error attained also diminishes as the stumpage increases.

The relative efficiency, expressed as the ratio of the total cost of the current method divided by the total cost of the minimum total cost method, increases with both an increase in stumpage and an increase in stratification (Table 6).

Stratification apparently does not have as pronounced effect on either the direct or total scaling costs as the stumpage rate. However, by stratifying on the combination of compartment, species and season, costs are minimised for both the current method and the minimum total cost method.

TABLE 6—Relative efficiency in terms of costs for the Minimum Total Cost method

	Stratum Classification				
Stumpage	Species	Species & Forest	Spec.-For. Season	Comp't. Species	Comp't. Spec. Season
$5/m^3	1.19	1.25	1.27	1.32	1.33
$10/m^3	1.41	1.52	1.57	1.64	1.65
$15/m^3	1.61	1.75	1.80	1.91	1.93

DISCUSSION

The results of this study are specific to the particular scaling situation described here and are determined by the standard deviations encountered in the Canterbury Conservancy and the set of simulated cost and production data. In other conservancies, or with different cost data, the direct and total scaling costs may display a different trend as loads are stratified on the basis of species, forest, season and compartment, and a different stratification policy may prove to be optimum. Thus an analysis similar to the one described here should be carried out separately for each conservancy.

It is apparent from these results, however, that as the value of the product (hence indirect cost) increases, sample size should also increase; but that stumpage rate does not necessarily affect the choice of stratification policy.

While the use of the minimum total cost method incorporates the basic principle of making more precise estimates of higher value products, and is an improvement on the current method, it does not resolve one of the most perplexing problems associated with weigh scaling. That problem is the delay in obtaining a factor for the current production until after the sample has been taken. This delay would be eliminated if factors could be predicted *a priori* through the use of equations relating the volume/weight ratio to concomitant variables associated with wood density, bark density and foreign matter included in the load. Unfortunately such equations, that are completely stable over time and location, have not yet been found. However, when such equations are developed, they too should incorporate the concept of minimum total cost.

ACKNOWLEDGMENTS

The author gratefully acknowledges the assistance of the N.Z. Forest Service Canterbury Conservancy staff in providing the data used in this report. Cost and production data have been generalised to reflect more nearly the New Zealand situation as a whole and must not be construed to reveal the actual cost picture for the Canterbury Conservancy.

The author also expresses appreciation to Dr A. G. D. Whyte, of the School of Forestry, University of Canterbury, and one of the referees for constructive criticism of this paper.

REFERENCES

BLYTHE, R. H. 1945. The economics of sample size applied to the scaling of sawlogs. **Biometrics Bull. 1:** 67-70.

FREESE, F. 1962. Elementary forest sampling. **U.S.D.A. Forest Service, Agricultural Handbook 232:** 91.

——— 1964. Linear regression methods for forest research. **U.S.D.A. Forest Service, Research Paper FPL 17:** 136.